

Gravitational Self-Energy and the Equivalence Principle

The equivalence principle of general relativity asserts that, locally, gravitation is completely indistinguishable from the inertial “pseudo-force” one would experience in an appropriately contrived accelerated frame. This implies the precise equality of gravitational and inertial mass, irrespective of a body’s makeup. And, from special relativity, we know that a body’s inertial mass is given by its total energy in its rest frame.

Does any contribution δE , of whatever kind, to the rest energy of a body contribute $\delta E/c^2$ to its gravitational mass? General relativity says yes. But for more than 30 years, theorists and experimenters have asked whether gravitational self-energy—the negative potential energy of a gravitationally bound system—might not be an exception to the equivalence principle. In other words, is the energy of gravity itself a source of gravity like any other?

Now the 1 November issue of *Physical Review Letters* brings us a report by Eric Adelberger’s group at the University of Washington that looks like the best answer to date.¹ Having performed a laboratory-scale experiment intended to close a nagging loophole left by decades of exquisitely precise lunar laser ranging observations, the Seattle group concludes that gravitational-self-energy obeys the equivalence principle to within a part in a thousand.

Gravitational self-energy

It is easy to show that the Newtonian gravitational self-energy E_{GS} of a uniformly dense sphere of mass M and radius R is

$$E_{\text{GS}} = -\frac{3}{5}GM^2/R$$

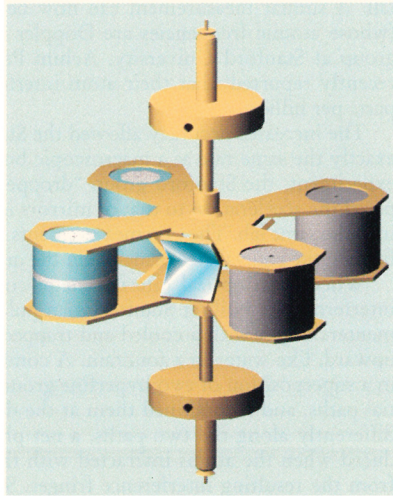
It’s the same undergraduate problem as calculating the electrostatic potential of a uniformly charged dielectric sphere—only with the opposite sign, because gravity is attractive.

Under what circumstances would E_{GS} be nonnegligible compared to a body’s rest energy? Dividing E_{GS} by Mc^2 we get

$$f \equiv -\frac{3}{5}GM/Rc^2 = -\frac{4}{5}\pi\rho GR^2/c^2$$

for the fraction of a body’s mass attributable to gravitational self-energy, where ρ is the density. For laboratory-size objects, f is completely negligible. But because f grows like R^2 , one might hope to see gravitational-self-energy violations of the equivalence principle in the orbits of astronomical

A latter-day Eötvös experiment preserves the sanctity of general relativity, at least for the moment.



bodies. For the Earth, $f = -4.6 \times 10^{-10}$. For the smaller, less dense Moon, it’s only -0.2×10^{-10} . In 1968, not long before the Apollo astronauts deployed reflecting corner cubes on the Moon for laser ranging measurements from the Earth, theorist Kenneth Nordtvedt at Montana State University pointed out that monitoring our distance from the Moon offered the best chance for detecting a gravitational-self-energy violation of the equivalence principle. (See his article in *PHYSICS TODAY*, May 1996, page 26.) Nordtvedt had earlier proven the surprising result that, in any modification of general relativity with additional tensor or scalar metric fields beyond the Einstein gravitational tensor field, the gravitational self-energy would violate the equivalence principle. In those days, the Brans-Dicke scalar-tensor modification of general relativity attracted considerable attention. (See the article by Clifford Will in last month’s *PHYSICS TODAY*, page 38.) And nowadays, with the

quantization of gravity high on the theoretical agenda, it is thought that any quantum theory of gravity must introduce scalar fields at some level. Further impetus for stringent testing of gravitational theory comes from the recent supernova evidence that the universal Hubble expansion appears to be speeding up.

Suppose, for the moment, that E_{GS} makes its full (negative) contribution to inertial mass, but contributes nothing (either positive or negative) to gravitational mass. In that case, at a given distance from the Sun, the Earth would experience an acceleration toward the Sun a few parts in 10^{10} greater than that of the Moon, in violation of the equivalence principle. But there’s no unanimity even about the sign of a possible violation. It might be that the ultimately correct modification of general relativity will have the gravitational self-energy making a *positive* contribution to gravitational mass.

In any case, decades of lunar laser ranging measurements have by now limited any equivalence-principle anomaly in the cyclically varying distance between Earth and Moon, as they circle each other and the Sun, to about a centimeter, at most.² However, these magnificent measurements of the time it takes for a laser pulse to bounce off the Moon and come back leave a small but annoying loophole. The Moon is considerably less dense than the Earth. Because the Moon lacks an extensive core of iron and nickel, its mean density is more like that of the Earth’s relatively light mantle. That raises the remote possibility that some composition-dependent violation of the equivalence principle may be masking the gravitational-self-energy effect that the lunar laser ranging measurements were looking for.

Plugging an unlikely loophole

In the gravitation business, one distinguishes between two variants of the equivalence principle: the strong and the weak. The strong equivalence principle asserts the strict equivalence of gravitational and inertial mass for all forms of matter and energy. But the weak equivalence principle (which is all you can test in a self-contained laboratory experiment) allows a possible exception for gravitational self-energy. It demands only that the acceleration of a body in a gravitational field not depend on its material composition—for example, its mean nuclear binding energy.

We have no particular reason,

either theoretical or observational, to expect any violation of the weak equivalence principle. But because the material compositions of the Earth and the Moon are so different, one could imagine that a violation of the weak equivalence principle between them just happens to be roughly equal and opposite to a gravitational-self-energy effect of the kind that the theorists *do* expect at some level. The Seattle group's laboratory experiment was intended to exclude (or confirm) just that sort of adventitious violation of the weak equivalence principle.

"You might object that a compositional effect that hides a gravitational-self-energy anomaly would be highly implausible," says Adelberger. "But physics is an empirical science. It's not philosophy. And the equivalence principle is so very important that you have to test it as well as you possibly can."

Testing the strong equivalence principle requires the monitoring of large astronomical bodies. But the weak equivalence principle was already being tested in the Budapest laboratory of Baron Roland von Eötvös early in the century. Like the baron, the Seattle group uses a sensitive torsion pendulum to look for differences in the gravitational interaction of different materials. In his honor, the group called its original 1987 instrument Eöt-Wash. The figure on page 19 shows its much-upgraded descendant, Eöt-Wash II, the rotating torsion pendulum with which the group is now testing the relative accelerations of miniature Earths and Moons toward the Sun. The original Eöt-Wash I, built in response to Ephraim Fischbach's provocative suggestion of a short-range force that mimics a small correction to gravity, did much to kill that so-called fifth force. (See PHYSICS TODAY, July 1988, page 21.)

Surrogate Earth and Moon

The four cylindrical 10-gram test bodies arrayed around the Eöt-Wash II torsion pendulum have identical dimensions and gold plating. But their different internal compositions are meant to provide surrogates of the Earth and Moon. "Because we don't want to restrict our results by any preconception of how a compositional anomaly might couple to gravity," explains Adelberger, "we use test masses that simply embody the known differences between Earth and Moon."

Their compositional difference is dominated by the difference between the Earth's mantle and its core. The two gray cylinders, representing the

Atom Interferometer Measures *g* with Same Accuracy as Optical Devices

The acceleration due to gravity, *g*, can be measured simply by timing how long it takes an object to fall. One can accomplish this with great precision by orienting an optical interferometer so that one of its arms is vertical. If the mirror in that arm is then allowed to fall part way, a Doppler shift in the reflected light signals its rate of fall. A similar measurement can now be done just as accurately with falling atoms (whose atomic frequencies are Doppler shifted), thanks to a long-term effort by a group at Stanford University. Achim Peters, Keng Yeow Chung, and Steven Chu recently reported¹ that their atom interferometer has determined *g* to within three parts per billion.

The increased accuracy allowed the Stanford team to test whether an atom falls at exactly the same rate as a macroscopic body. In a modern version of Galileo's classic experiment, the Stanford group "dropped" atoms in their interferometer and compared the acceleration to that of mirrors in a commercial optical interferometer taken into the same lab. The two measurements agreed to within seven parts in a billion, confirming the equivalence principle in the quantum regime (see the story on page 19).

Whereas an optical interferometer controls light beams with mirrors, the atom interferometer built at Stanford manipulates atoms with pulses of light. The experimenters began with a cooled and trapped cloud of atoms, and launched it vertically upward, like water in a fountain. A combination of three laser pulses put the atoms in a superposition of two hyperfine ground states, sent them along two different spatial paths, and recombined them at the detector. Because the wavefunctions evolved differently along the two paths, a net phase shift—which depends on *g*—was introduced when the atoms interacted with the laser pulses. The value of *g* was deduced from the resulting interference fringes. Such gravitationally-induced quantum interference was first observed in neutron interferometry nearly 25 years ago.^{2,3}

It was no small feat to eliminate the many sources of error needed to achieve parts-per-billion accuracies. Chu and his coworkers developed a design that is relatively insensitive to drifts of the lasers and incorporated an actively stabilized vibration-isolation system. Furthermore, they have corrected for a vertical gradient in *g* and for changes in *g* caused by ocean tides. The biggest systematic effect is the uncertainty in the correction due to Earth's rotation.

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Earth's core, are made of iron, nickel, and chromium. The blue cylinders, representing the Moon (and the Earth's mantle), are mostly quartz (SiO₂), with some magnesium.

The four-cylinder array hangs like a little chandelier from a delicate fiber with a torsion constant of 0.03 ergs per radian of twist, yielding a free-oscillation period of about 15 minutes. Four right-angle mirrors are mounted on the pendulum between the test masses to reflect the laser beams that monitor the twist angle of the torsion fiber. The entire apparatus—fiber suspension and laser monitoring system—is continuously rotated on a laboratory turntable at a variable rate whose period is always set at some half-odd-integer multiple of the free oscillation period. One cannot completely suppress the pendulum's free oscillation. In the absence of any other perturbations, the oscillation would still exhibit thermal noise of order kT , corresponding to a torsion amplitude of a few microradians.

What the Seattle experimenters are looking for in their Fourier

decomposition of the twist angle's time dependence is a diurnal component that tracks the moving Sun, indicative of a differential acceleration of the test masses that violates the weak equivalence principle. On the other hand, signal components precisely at the instrument's rotation frequency, without regard to the Sun, are indicative of perturbation sources fixed in the laboratory frame—for example, magnetic fields, the instrument's tilt, and the local gravity gradient.

There are, unfortunately, spurious diurnal effects that perturb this delicate apparatus: Cars fill up and vacate the parking lot, and the Sun sequentially warms different slopes of the local hillside. Both these effects vary the building's tilt ever so slightly. Electric power use wanes after working hours. Happily, the man-made diurnal effects follow the 24-hour calendar day, which differs seasonally from the actual solar day by a few seconds. Over several months the two get out of phase by as much as 15 minutes. That helps the group filter out anthropogenic perturbations.

The torsion pendulum's tilt has been a major contributor to the experimental uncertainties. Any deviation from perfect horizontality generates a spurious signal at the instrument's rotation frequency. The group uses electronic level sensors that can sense tilts of a few nanoradians. A new innovation that has, in recent months, doubled the sensitivity of Eöt-Wash II is a continuous feedback system from the level sensors that constantly adjusts the lengths of the torsion pendulum's legs to compensate for any tilt.

Violation still unseen

After more than a year of running with the miniature Earths and Moons, the University of Washington group reports a fractional difference

$$\Delta a/a = 0.1 \pm 3.2 \times 10^{-13}$$

between the accelerations toward the Sun of the two kinds of test masses. This is, of course, a null result. The experiment finds no hint of a compositional anomaly that might be masking a gravitational-self-energy anomaly in the Earth-Moon system. The lunar laser ranging measurements over the years have also produced a null result, with slightly larger quoted errors than those reported for the Eöt-Wash II experiment. Combining the astronomical results with its own test of the weak equivalence principle, the Seattle group quotes a fractional upper limit of 5.5×10^{-13} on any differential acceleration that violates the strong equivalence principle.

If gravitational self-energy contributed absolutely nothing to gravitational mass, one would have a fractional differential acceleration between Earth and Moon toward the Sun of 4.4×10^{-10} . This "maximal" violation of the strong equivalence principle is almost a thousand times bigger than the upper limit the Seattle group gets by combining the lunar-laser-ranging null results with its own failure to find any compositional effect in the laboratory. "So we now have an unambiguous confirmation," Adelberger told us, "that gravitational self-energy obeys the equivalence principle, at least to about a part in a thousand. And we continue taking data to keep bringing that upper limit down."

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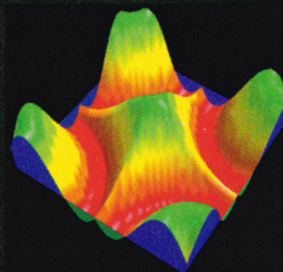
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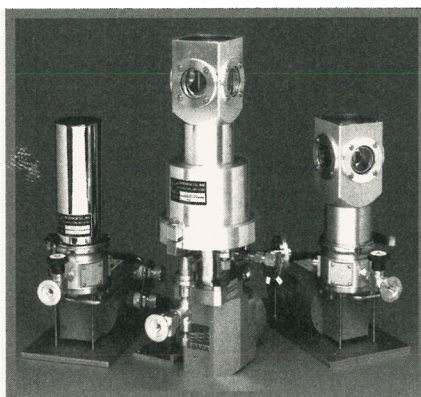
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