Gravitational Radiation and the Validity of General Relativity

Observing the speed, polarization, and back influence of gravitational waves would subject Einstein's theory to new tests.

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While the detection of gravitational radiation may usher in a new era of "gravitational wave" astronomy (see the accompanying article by Barry Barish and Rainer Weiss, on page 44), it should also yield new and interesting tests of Einstein's general theory of relativity, especially in the radiative and strong-field regimes. Consequently, we are in an unusual situation. After all, we rarely think of *electromagnetic* astronomy as providing tests of Maxwell's theory. Neutrino astronomy may be a closer cousin: We can observe neutrinos to learn about the solar interior or about supernovae, while also checking such fundamental phenomena as neutrino oscillations. To some extent, the usefulness of astronomical observations in testing fundamental theory depends upon how well tested the theory is already. At the same time, since general relativity is the basis for virtually all discussion of gravitational-wave detectors and sources,[1] the extent of its "upfront" validity is of some concern to us.

Although the empirical support for the theory of general relativity is very strong, it is still not as solid as the support for Maxwell's theory, and only in the last 35 years or so have precise tests been feasible. Furthermore, general relativity has not been tested deeply either in its radiative regime or in the regime of strong gravitational fields, such as those associated with black holes or neutron stars. (See figure 1.) Most tests, such as those carried out in the Solar System, check the theory only in its weak-field, slow-motion, nonradiative limit. One famous exception, the Hulse–Taylor binary pulsar, does provide an important verification of the lowest-order radiative predictions of general relativity and is sensitive to some strong-field aspects. Still, important tests of gravitational radiation and its properties remain undone. Furthermore, interesting, well-motivated alternative theories to general relativity still exist that are in agreement with all observations to date. Gravitational-wave tests will remain of interest to us to the extent that they can further constrain the theoretical possibilities.

There are three aspects of gravitational radiation that can be subjected to testing:

▷ The polarization content of the waves (general relativity predicts only two polarization states, whereas other theories predict as many as six).

▷ The speed of the waves (general relativity predicts a speed the same as that of light, whereas other theories may predict different speeds).

▷ The back influence of the emitted radiation on the evolution of the source.

In this article, we discuss the three possibilities. First, though, we review the current status of tests of general relativity.[2,3]

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The Einstein equivalence principle

At the heart of gravitational theory is a concept called the Einstein equivalence principle, which modernizes Newton's postulate of the equivalence of gravitational and inertial mass. It states first, that bodies fall with the same acceleration regardless of their internal structure or composition (this piece of the Einstein equivalence principle is called the weak equivalence principle), and second, that the outcome of any local nongravitational experiment is both independent of the velocity of the free-falling reference frame in which it is performed (local Lorentz invariance) and independent of where and when in the universe it is performed (local position invariance).

The Einstein principle implies that gravitation must be described by a theory in which matter responds only to the geometry of spacetime. Such theories are called metric theories. General relativity is a metric theory of gravity, but so are many others, including the "scalar–tensor" theory of Carl Brans and Robert Dicke, a theory based on earlier work by Paul Jordan. Strangely enough, string theory—a leading contender for a unified theory of particle interactions and for a quantum theory of gravity—does not strictly satisfy the metric theory definition. In string theory, matter can respond weakly to gravitation-like fields, in addition to responding to geometry. Consequently, testing the Einstein equivalence principle is a way to search for new physics beyond standard metric gravity.

To test the weak equivalence principle, we can compare the accelerations a_1 and a_2 of two bodies of different composition in an external gravitational field. The resulting measurements will yield the difference in acceleration divided by the average acceleration, $2|a_1 - a_2|/|a_1 + a_2|$, called the Eötvös ratio after Roland, Baron Eötvös of Vásárosnamény, whose pioneering tests of the weak equivalence principle at the turn of the century formed a foundation for general relativity.

The best test so far of the weak equivalence principle has been a series of experiments carried out at the

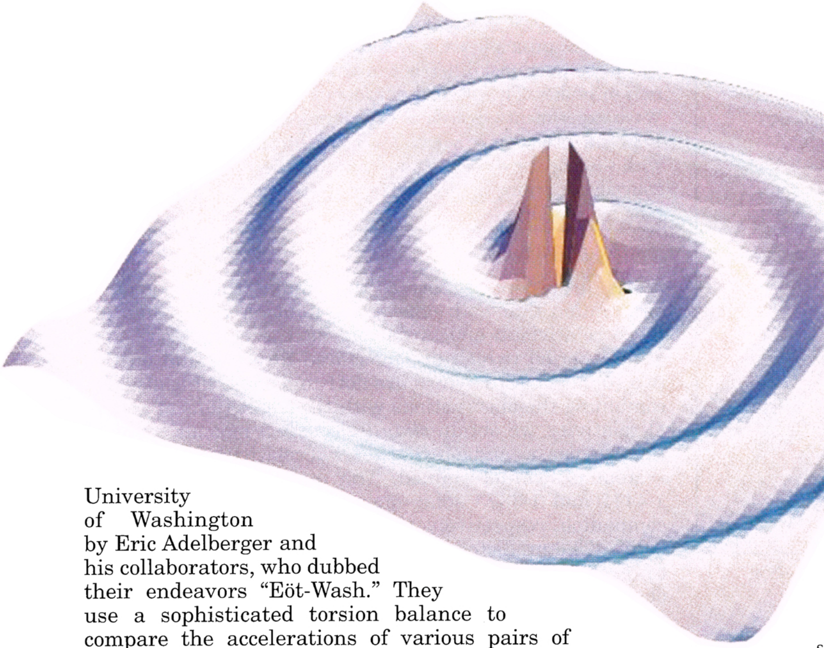

FIGURE 1. GRAVITATIONAL WAVES expected from an inspiraling binary system of neutron stars or black holes. Height above the plane represents the amplitude of one polarization mode of waves at a fixed moment of time. The amplitude decreases with distance, both because of the usual $1/R$ fall-off and because waves measured farther from the source were emitted earlier in its evolution, when the emission was weaker. The double-armed spiral pattern reflects waves from a rotating quadrupole source. Displacements induced in a detector are transverse to the radial direction. The peak at the center indicates the beginning of merger of the two objects, where the post-Newtonian approximation that was used to generate this plot breaks down and numerical solutions of Einstein's equations must be used. (Image courtesy of Laser Interferometer Gravitational-Wave Observatory.)

University of Washington by Eric Adelberger and his collaborators, who dubbed their endeavors "Eöt-Wash." They use a sophisticated torsion balance to compare the accelerations of various pairs of materials toward Earth, the Sun, and the Galaxy.[4] Another strong bound comes from lunar laser ranging (LURE), which checks the equality of acceleration of Earth and the Moon toward the Sun. Figure 2 summarizes key results. (See the article by Kenneth Nordtvedt, PHYSICS TODAY, May 1996, page 26.)

The best tests of local Lorentz invariance consist of "clock anisotropy" experiments. Latter-day versions of the classic 1887 experiments of Albert Michelson and Edward Morley, they involve looking for variations in the rates of clocks as their orientation changes with respect to Earth's 350 km/s velocity relative to the cosmic microwave background radiation. The frame of that background would be a preferred rest frame for physics if local Lorentz invariance were violated. In the Michelson–Morley experiments, the "clocks" being compared were those defined by light propagation along the two perpendicular arms of their interferometer. The modern versions of the experiments, which use laser-cooled trapped-atom techniques to compare the transition rates of atoms as a function of their orientation, have placed exquisite bounds—as tight as parts in 10^{26}—on anomalies. (For further discussion, see the article by Mark P. Haugan and Clifford M. Will, "Modern Tests of Special Relativity," PHYSICS TODAY, May 1987, page 69.)

Local position invariance requires, among other things, that the internal binding energies of all atoms be independent of location in space and time, when measured against some standard atom. If that requirement is fulfilled, an intercomparison of the rates of two kinds of clocks should be independent of time and of the local gravitational potential, and the measured frequency shift between two identical clocks at different locations should be directly related to the difference in gravitational potential between the locations. The best test of this principle to date has been the redshift experiment done in 1976 by Robert Vessot and Martine Levine of the Harvard Center for Astrophysics, in which they compared a hydrogen maser clock on a Scout rocket with another hydrogen maser clock on the ground.

Metric gravity and the post-Newtonian limit

In metric theories of gravity, the slow-motion, weak-field limit that incorporates the first corrections beyond Newtonian theory is called the post-Newtonian limit. Within this limit, it turns out that for a broad class of metric theories, only the numerical values of a certain set of coefficients in the spacetime metric vary from theory to theory. This framework, called the parametrized post-Newtonian (PPN) formalism, dates back to Arthur S. Eddington's 1922 textbook on general relativity; between 1968 and 1972, it was extended by Kenneth Nordtvedt of Montana State University and the author.[2] The PPN formalism is a convenient tool for classifying alternative metric theories of gravity, for interpreting the results of experiments, and for suggesting new tests of metric gravity.

The PPN formalism has ten parameters. For this discussion, the most important are γ, which is related to the amount of spatial curvature generated by mass; β, which is related to the degree of nonlinearity in the gravitational field; and the four parameters ξ, α_1, α_2, and α_3, which determine whether gravity itself violates a form of local position invariance or local Lorentz invariance. The combination $(1 + \gamma)/2$ governs both the deflection of light and a retardation in the propagation of light near a massive body (a retardation known as the Shapiro time delay, named for Irwin I. Shapiro of Harvard University). The "1/2" part of the coefficient corresponds to the so-called Newtonian deflection, which was derived two centuries ago by Henry Cavendish and later by Johann von Soldner. (It can also be derived by using the principle of equivalence.) The "$\gamma/2$" part comes directly from the warping of space near the massive body. The combination $(2 + 2\gamma - \beta)/3$ modulates the advance of the perihelion of planets such as Mercury. The combination $4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 - 2\alpha_2/3$ determines whether there is a violation of the weak equivalence principle for self-gravitating bodies such as Earth and the Moon—a phenomenon called the Nordtvedt effect. In general relativity, $\gamma - 1$, $\beta - 1$, and the remaining PPN parameters all vanish, as does the Nordtvedt effect.

Three decades of experiments, including the standard light-deflection and perihelion-shift tests; lunar laser

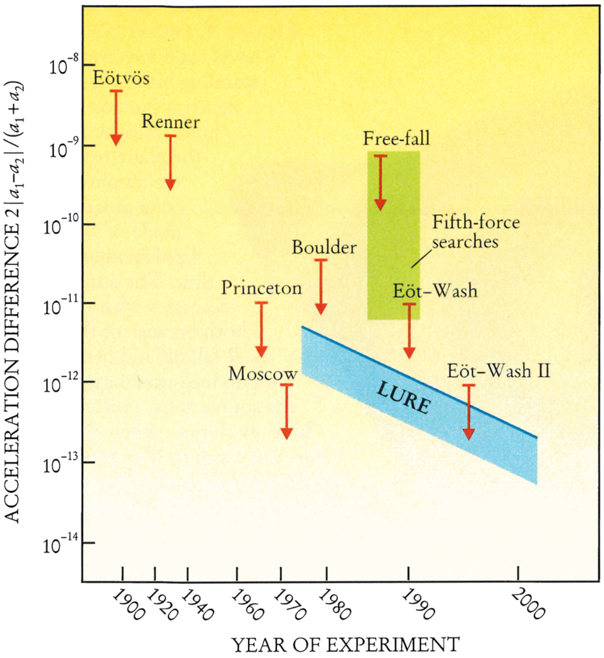

FIGURE 2. EÖTVÖS EXPERIMENTS. Selected tests of the weak equivalence principle are represented here by the bounds they set on the fractional difference in acceleration of different materials or bodies. The original purpose of the "free-fall" and Eöt-Wash experiments, as well as that of numerous others between 1986 and 1990, was to search for a fifth force. The diagonal line with shading shows current and potential bounds on the weak equivalence principle for Earth and the Moon from lunar laser ranging (LURE).

ranging, planetary and satellite tracking tests of the Shapiro time delay; and geophysical and astronomical observations, have placed bounds on the PPN parameters that are consistent with general relativity. The table on this page summarizes the results. To illustrate the dramatic progress of experimental gravity since the dawn of Einstein's theory, figure 3 shows a chronology of results for $(1 + \gamma)/2$. These results range from the 1919 solar eclipse measurements of Eddington and his colleagues to modern-day measurements that use very-long-baseline radio interferometry (VLBI) and orbiting astrometric satellites such as Hipparcos to tests of the Shapiro time delay.

The binary pulsar

The binary pulsar PSR 1913+16, discovered by Russell Hulse and Joseph Taylor in 1974, provided important new tests of general relativity, especially for gravitational radiation and strong-field gravity. By precisely timing the pulsar clock, astrophysicists were able to measure the important orbital parameters of the system with extraordinary precision. Those parameters included the ones normally associated with a nonrelativistic Keplerian two-body orbit, such as the eccentricity e and the orbital period P_b, as well as relativistic parameters, such as the rate of advance of the periastron (the binary system analog of the perihelion), the combined effects of time-dilation and gravitational redshift on the observed rate of the pulsar, and the rate of decrease of the orbital period. The rate-of-decrease effect is a result of gravitational radiation damping; measuring it requires making a small correction for the effect of the galaxy's rotation on the distance to the pulsar (other possible sources of orbital damping, such as tidal friction, have been shown to be negligible).

If we assume that general relativity is provisionally correct and make the reasonable assumption that both objects are neutron stars, then all three relativistic effects depend on the eccentricity and orbital period (which are measured directly) and on the two stellar masses (which are not), and on nothing else. By combining the observations with the predictions of general relativity, we obtain *simultaneously* a measurement of the two masses and a test of general relativity, because the system is overdetermined. The masses turn out to be 1.4411 and 1.3873 solar masses for the pulsar and its companion, respectively, with an uncertainty of less than 0.05%. The predicted decrease of the orbital period owing to gravitational radiation damping agrees with the observed decrease to better than 0.3 %. The discovery of the binary pulsar garnered Hulse and Taylor the 1993 Nobel Prize in Physics.[5] Other binary pulsars, such as B1534+12, B2127+11C, and B1855+09, are also yielding interesting relativistic tests.[6]

Binary pulsar measurements also test the strong-field aspects of general relativity, because the neutron stars that make up the systems have very strong internal gravity, which contributes as much as several tenths of the mass-energy of each body at rest (compared to the orbital energy, which is only 10^{-6} of the system's mass-energy). In alternative theories, such as scalar–tensor gravity, the internal self-gravity effects can lead to qualitatively new phenomena, such as the emission of dipole gravitational radiation, whose damping effect on the orbit can be significantly different from that of the usual quadrupole radiation of general relativity. No such effects induced by internal energy occur in general relativity.

The return of scalar–tensor gravity

Among the alternative metric theories of gravity,

Current limits on the parameters of post-Newtonian gravity

Parameter	Upper limit on its absolute value	Observable effect	How tested
$\gamma-1$	3×10^{-4}	Light deflection	Very-long-baseline radio interferometry (VLBI)
	2×10^{-3}	Shapiro time delay	Mars radar ranging
$\beta-1$	3×10^{-3}	Perihelion shift of Mercury	Planetary radar ranging
ξ	10^{-3}	Anisotropy in gravity induced by distant matter	Gravimeter bounds on anomalies in Earth tides
α_1	4×10^{-4}	Anisotropy in gravity due to motion through universe	Lunar laser ranging
α_2	4×10^{-7}	Precession of spin of body moving through universe	Solar spin alignment in relation to ecliptic plane
α_3	2×10^{-20}	"Self" acceleration of spinning body moving through universe	No anomalies in pulsar $\dot{P}$ statistics
η	10^{-3}	Violation of equivalence principle for massive bodies (Nordtvedt effect)	Lunar laser ranging

In terms of the six parametrized post-Newtonian parameters discussed in the text, $\eta=4\beta-\gamma-3-10\xi/3-\alpha_1-2\alpha_2/3$
All parameters listed vanish identically according to general relativity.

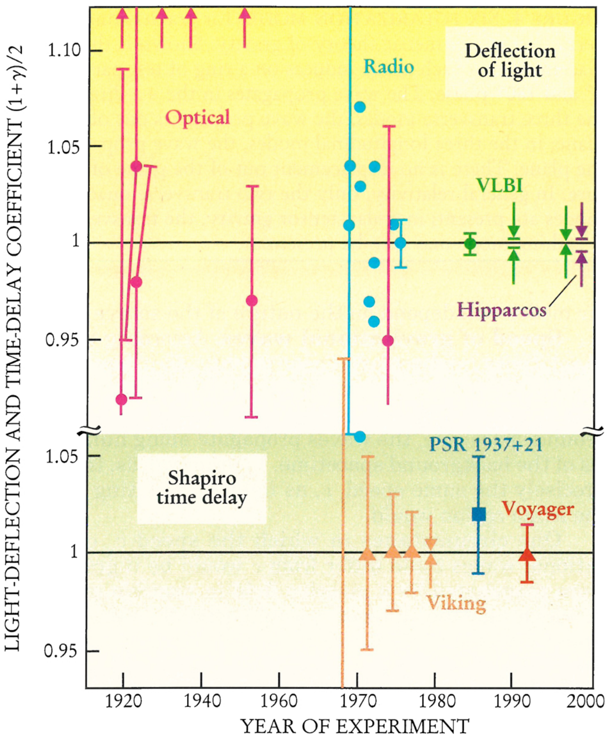

FIGURE 3. DEFLECTION AND DELAY OF LIGHT passing near the Sun. Plotted are values of the coefficient $(1+\gamma)/2$ based on observations of the deflection of light and of the Shapiro delay in the propagation of radio signals near the Sun (γ is a measure of the amount of spatial curvature generated by mass). General relativity predicts a value of unity for the coefficient. "Optical" denotes measurements of stellar deflection made during solar eclipses. "Radio" denotes interferometric measurements of radio-wave deflection. "VLBI" denotes very-long-baseline radio interferometry. "Hipparcos" denotes the optical astrometry satellite. Arrows indicate the anomalously large values from one of the 1919 eclipse expeditions and from other eclipse expeditions through 1947. Shapiro time-delay measurements with the Viking spacecraft on Mars yielded tests at the 0.1% level, and light-deflection measurements using VLBI have reached 0.03%.

scalar–tensor theories have played a special role. The most famous of the theories was the one developed and promoted in the 1960s by Brans and Dicke. In addition to the spacetime geometry described by a metric, $g_{\mu\nu}$, scalar–tensor theories postulate a scalar field Φ, that, in a standard representation, couples only to gravity itself, not to matter, thereby satisfying the requirements of metric gravity automatically. The "strength" of the scalar field is determined by a coupling constant ω such that the larger the value of ω, the weaker the scalar field. In the large ω limit, Brans–Dicke theory merges smoothly with general relativity, in that the differences between the two theories in all predictions vanish roughly as $1/\omega$. But because measurements of the deflection of light described above place the lower bound on ω at greater than 3000, Brans–Dicke theory has generally been regarded as all but dead.

During the past decade, however, new mutant strains of scalar–tensor gravity have emerged, their formulation motivated by string theory and by some models of inflationary cosmology, although they were studied in other contexts as early as 1968. In the new theories, the coupling ω is not a fixed constant but is a function of the scalar field Φ. So the theories can agree with experiment in the present Solar System—when Φ has values such that $\omega(\Phi) > 3000$—but may be very different from general relativity in the early universe, or in strong-field regimes such as neutron star interiors. In typical cosmological models, the scalar field evolves in such a way that $\omega(\Phi)$ is driven naturally to large (though finite) values in the present epoch, independently of its value in the early universe. In a sense, general relativity is a cosmological "attractor" for such theories. The fact that the present value of ω in some models could be as small as 10^4, suggests interesting and reachable goals for future experiments. Not only that, but as noted earlier, some string-inspired theories introduce direct weak couplings between matter and the scalar field—and by doing that, can violate the Einstein equivalence principle.[3]

Future experimental tests

Much of the discussion about future gravitational experiments focuses on ways to test these new versions of scalar–tensor gravity, in the hope of limiting or discovering new physics that might arise from strings or other models of unification. The following are four promising avenues of experimental work.

▷ In addition to improved ground-based Eötvös-type experiments, which could test the weak equivalence principle to the level of 10^{-14}, a proposed satellite test could reach the level of 10^{-18}.

▷ The Stanford–Lockheed–NASA Gyroscope Experiment, called Gravity Probe B, will measure the precession of an array of gyroscopes in Earth orbit. Although its primary science goal is a 1% measurement of the dragging of inertial frames that is caused by Earth's rotation (also called the Lense–Thirring effect), Gravity Probe B will also measure the precession caused by ordinary space curvature around the planet. Measurement of this effect could bound the coupling ω to 10^4 or higher. Launch of the mission is scheduled for late 2000.[7]

▷ Binary pulsars could yield bounds on scalar–tensor gravity, because that theory predicts dipole gravitational radiation. Unfortunately, the bound from the Hulse–Taylor system is only $\omega > 100$, because the near identity of the two neutron stars suppresses dipole radiation by symmetry. A suitable asymmetric system containing a black hole or a white dwarf as the pulsar's companion could yield bounds on ω as high as 10^4.

▷ Tests of the force of gravity at submillimeter ranges are being designed that could detect or bound new gravitation-like nonmetric couplings.

Gravitational-wave tests of gravitation theory

What could the direct observation of gravitational waves add to the list of tests and bounds? First, detection of the waves would in and of itself be a striking confirmation of general relativity, despite the fact that their existence is strongly supported by the binary pulsar. Here the situation is reminiscent of the case of neutrinos: the direct detection of neutrinos by Frederick Reines and Clyde Cowan in 1956 was an impressive discovery (worthy of the 1995 Nobel Prize in Physics) despite the preexisting confidence in their reality that beta decay engendered. Second, direct study of gravitational waves will check their properties as predicted by general relativity—properties that are only indirectly reflected in the damping of binary pulsar orbits. Third, gravitational waves are likely to carry the imprints of strong-gravity phenomena at the sources—

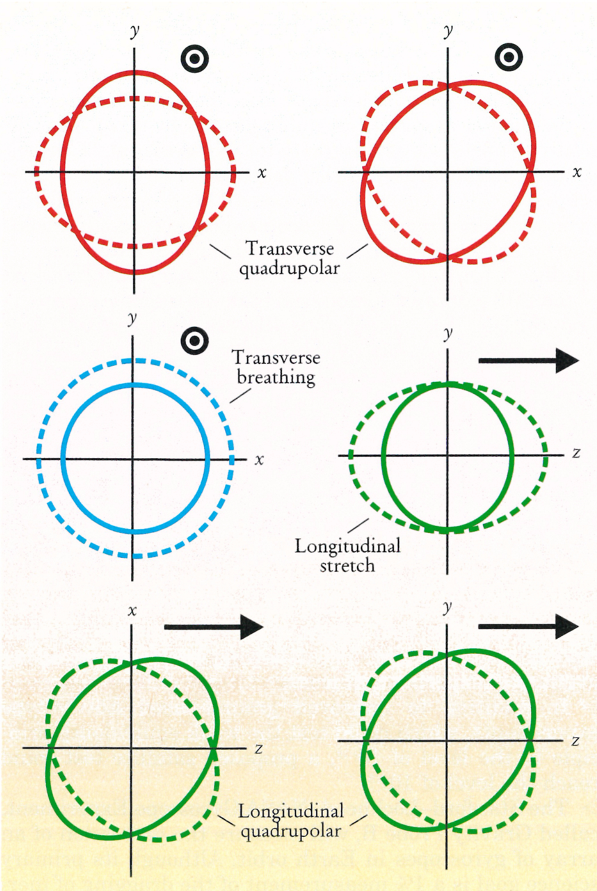

FIGURE 4. SIX POLARIZATION MODES for gravitational waves permitted in any metric theory of gravity. Shown is the displacement that each mode induces on a ring of test particles at 0° and 180° phase. The wave propagates in the $+z$ direction. In the three transverse modes, the wave propagates out of the plane; in the three longitudinal modes, the wave propagates in the plane. There is no displacement out of the plane of the picture. In general relativity, only the two transverse quadrupolar modes are present; in scalar–tensor gravity, the transverse breathing mode may also be present.

and study of those imprints could lead to tests of general relativity in the strong-field regime.

Polarization of gravitational waves. A laser-interferometric or resonant-bar gravitational-wave detector measures local relative displacements of mirrors or mechanical elements, which can be related to a symmetric 3×3 strain tensor. The tensor, in turn, can be related directly to components of the Riemann curvature tensor of spacetime generated by the wave. The six independent components of the strain tensor can be expressed in terms of polarizations, which are modes of motion with specific transformation properties under rotations and boosts. Three are transverse to the direction of propagation, with two representing quadrupolar deformations and one representing a monopole "breathing" deformation. The other three are longitudinal, with one being an axially symmetric stretching mode in the propagation direction and the remaining two being quadrupolar (see figure 4).

General relativity predicts only the first two transverse quadrupolar modes, independently of the source; this behavior goes hand in hand with the notion that, at a quantum level, gravitational waves are associated with a spin-2 particle, the graviton. Scalar–tensor theories also predict the transverse breathing mode, a spin-0 mode. More general metric theories predict up to the full complement of six modes. A suitable array of gravitational antennas could delineate or limit the number of modes present in a given wave. If evidence were found of any mode other than the two transverse quadrupolar modes of general relativity, the result would be disastrous for the theory. On the other hand, the absence of a breathing mode would not necessarily rule out scalar–tensor gravity, because the strength of that mode relative to the quadrupolar modes will depend on the nature of the source.

Speed of gravitational waves. According to general relativity (and scalar–tensor gravity, as it happens), in the limit wherein the wavelength of gravitational waves is small in comparison to the radius of curvature of the background spacetime, the waves propagate along null geodesics of the background spacetime. In other words, they have precisely the same speed, c, as light propagating through the same region would.

One circumstance in which the speed v_g of gravitational waves could differ from c would be if gravitation were propagated by a massive field (a massive graviton), in which case the value of v_g would depend on the gravitational wavelength λ according to $v_g^2/c^2 = 1 - \lambda^2/\lambda_g^2$, where $\lambda_g = h/m_g c$ is the graviton Compton wavelength.

The most obvious way to check the speed of gravitational waves is to compare the arrival times of a gravitational wave and an electromagnetic wave from a single event, such as a supernova. For a source at a distance of 200 megaparsecs, a bound on the arrival time difference between electromagnetic and gravitational signals of one second would put a bound of 5×10^{-17} on a fractional speed difference. This scenario assumes, of course, that the source emits both gravitational and electromagnetic radiation in detectable amounts and that the relative time of emission can either be established to sufficient accuracy or be shown to be sufficiently small.

There is a situation, however, in which a bound on a hypothetical wavelength-dependent speed could be set by means of gravitational radiation alone. That is the case of an in-spiraling compact binary system containing neutron stars or black holes, a system that would be observed by laser interferometers such as the Laser Interferometer Gravitational-Wave Observatory (LIGO, discussed in the next article, by Barish and Weiss) and the European instrument known as VIRGO. In in-spiraling compact binary systems, gravitational-radiation damping drives the binary toward smaller separations and higher orbital frequencies, a process leading eventually to a catastrophic merger. Gravitational waves from the last few minutes of those in-spiraling systems will sweep through the sensitive bandwidth of LIGO/VIRGO detectors (10 to 500 Hz). We'll detect the radiation many years after the system "died." Because the frequency of the gravitational radiation sweeps from low frequency at the first moment of observation to higher frequency at the final moment, the speed of the waves emitted will vary from lower speeds at first to higher speeds (closer to c) at the end. The variation in speed will cause discernible distortion in the observed phasing of the waves. Even better tests of a wavelength-dependent speed could be obtained by observing supermassive in-spiraling double black hole systems (10^4 to 10^7 solar masses) in the centers of active galaxies by means of proposed space-based laser interferometric observatories[8] in the low-frequency band around 10^{-3} Hz.

Gravitational radiation back reaction. Binary orbits inevitably decay, because of the loss of gravitational

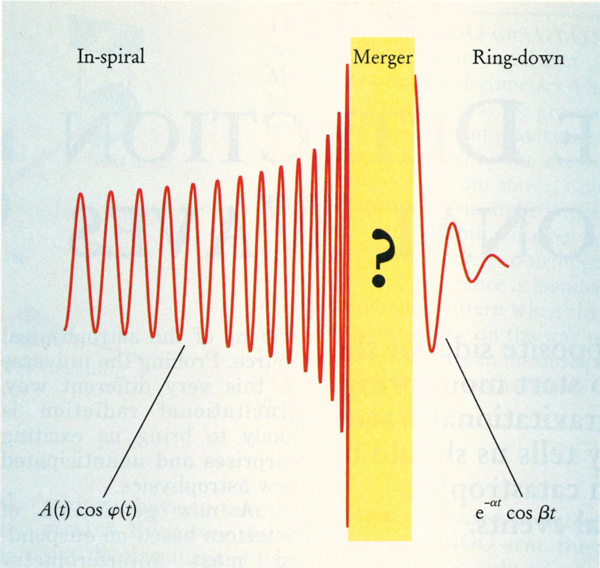

FIGURE 5. GRAVITATIONAL WAVEFORM expected from compact binary in-spiral, merger, and ring-down of a final black hole. During the in-spiral phase, a post-Newtonian approximation carried to high powers of v/c beyond Newtonian order accurately describes the orbit and waveform, with amplitude $A(t)$ and phase $\varphi(t)$ that evolve nonlinearly with time. The merger waveform is unknown at present; to determine it is the primary goal of numerical relativity. The ring-down waveform is a superposition of damped normal modes. For each mode, the damping coefficient α and frequency β have been thoroughly calculated by means of perturbation theory and have been cataloged as functions of the mass and spin of the black hole.

radiation energy. It is predicted that the Hulse–Taylor binary pulsar will reach a final in-spiral and merger in 240 million years. During the in-spiral phase, the motion of two compact bodies (neutron stars or black holes) can be described accurately by equations that treat Newtonian motion as the first approximation and include post-Newtonian corrections in increasing powers of v/c, where v is the orbital speed. The corrections include radiation back reaction. The evolution of the orbit is imprinted on the phasing of the emitted waveform, to which broadband laser interferometers are especially sensitive, because the data can be cross-correlated against theoretical templates derived from general relativity. Indeed, the sensitivity of the interferometers is expected to be so high that the equations of motion describing the orbit must be accurate to order $(v/c)^{11}$ beyond ordinary Newtonian gravity. Several groups are now engaged in the formidable task of deriving equations of motion from general relativity to that high order.[9]

This extraordinary accuracy will provide an opportunity to conduct further tests of general relativity. When spins and tidal effects can be ignored, the motion depends only on the two masses. As in the case of the binary pulsar, measuring the various post-Newtonian correction terms in the signal leads to a highly overdetermined situation, in which we can measure the two masses accurately, and simultaneously test general relativity. One additional test, for example, arises from a contribution—to the gravitational-wave signal and to the back reaction—known as the tail. The tail is a fundamentally nonlinear gravitational effect caused by backscattering of the outgoing gravitational waves off the local spacetime curvature generated by the binary system itself. The action of the tail results in a unique and expected contribution to the phasing of the waves, one that can be tested.

Scalar–tensor gravity can also be tested by means of such observations. The generation of dipole gravitational radiation by an asymmetric binary (for technical reasons, a binary consisting of a neutron star and a black hole is best) modifies the gravitational-radiation back reaction and the observed phasing of the waves. The fortuitous discovery of such a system could lead to a bound on the scalar–tensor coupling constant ω that exceeds current Solar System bounds.

Testing general relativity at strong fields

Finally, the in-spiraling and merger of two compact objects, or the core collapse in a supernova, involve the physics of spacetime curvature in the limit of strong, highly dynamical fields, as well as the formation and evolution of black hole event horizons. Although this physics is so complex that quantitatively precise tests of general relativity are not likely to be realized, making qualitatively striking tests may nevertheless be possible. For example, the gravitational-wave signal generated by the in-spiraling and merger of two compact objects to form a black hole, and the waves emitted during the ring-down of the final black hole in its discrete set of normal modes, will be imprinted with the masses and spins of the in-spiraling objects and the mass and angular momentum of the final black hole. The signal will reflect dynamical, strong-field general relativity in its full glory (see figure 5). Finding firm predictions for the waves to compare the observations against requires solving Einstein's equations in a regime in which post-Newtonian methods fail. Only large-scale numerical computation has a hope of yielding reliable results. This challenging task has been taken up by many "numerical relativity" groups around the world.[10] The discovery and study of the formation of a black hole by means of gravitational waves would provide a stunning test of relativistic gravity.

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