The Stock Market: Brownian, Gaussian or Mandelbrotian

Fractals and Scaling in Finance: Discontinuity, Concentration, Risk

Benoit B. Mandelbrot
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Reviewed by Nigel Goldenfeld

On 19 October 1987, the Dow Jones industrial average, the widely followed proxy for the US stock market, declined by 23%, a move that some observers noted was a 20-standard-deviation event. This drop, which was almost twice as large as the famous stock market crash of 1929, was not an isolated incident, but one of a number of large drawdowns and bear markets in this century alone, the most recent of which, the October 1997 crash, was a "modest" 8% drop. Faced with these statistics, most of us would probably be prepared to agree that stock price changes are not Gaussian; nor are they examples of random-walk behavior. However, events such as the 1987 crash, World War I and the crash of 1929 are extreme and properly regarded as outliers. What about business as usual?

The answer, of course, depends on whom you ask. On one hand, Burton Malkiel's well-regarded semipopular book on finance, *A Random Walk Down Wall Street*, (Norton, 6th edition, 1996), takes its title and its theme from the notion that stock price changes follow a Brownian motion. Virtually every textbook on advanced finance takes the Brownian-motion description as its starting point, and the celebrated Black–Scholes formula for option prices is based upon this description.

On the other hand, there is Benoit Mandelbrot. No reader of PHYSICS TODAY can be unaware of the enormous impact made by Mandelbrot's *The Fractal Geometry of Nature* (Freeman, 1982), which introduced many to the notions of fractal dimensions, scaling and self-similarity and spawned a host of coffee-table imitations.

What is perhaps less well known, however, is that some of Mandelbrot's earliest forays into fractals involved a detailed analysis of the time series for cotton prices in New York. His shocking conclusion, published in 1963, was that the time series was in no way Gaussian. In fact, he argued, the departures from normality could be accounted for by using distribution functions with infinite variance, which are termed L-stable. Mandelbrot examined the convergence in sample number of the variance of the logarithm of the daily price changes and found erratic variation rather than convergence. Subsequently, his student Eugene Fama (who has himself enjoyed a distinguished career in finance) examined the time series for the 30 stocks in the Dow Jones industrial average, finding no exceptions to the long-tailed nature of the distributions observed.

The implications of these and subsequent findings are profound, yet it is fair to say that the work was practically ignored by economists and practitioners of finance. Even today, the problem of "fat tails" is swept under the rug by the vast majority of financial risk managers, even though the phenomenon is sufficiently widespread and well recognized as to have earned its whimsical name.

Mandelbrot's heirs are primarily physicists who enter the field of finance, recognize the fundamental importance of fat tails and then elaborate on and extend his suggestive results. This is something of an ironic development, as Mandelbrot takes pains to emphasize in *Fractals and Scaling in Finance*, and reflects the close intellectual relationship between finance and physics: The discovery of Brownian motion, usually attributed to Einstein's famous 1905 paper, was in fact anticipated by Louis Bachelier five years earlier, in his PhD dissertation on finance, "Theorie de la Speculation," which remained largely ignored by economists until the 1950s and '60s. Elements of Mandelbrot's work in the early 1960s, which superseded Bachelier's analysis just as it was becoming widely accepted, arguably anticipate some of the concepts of scaling and renormalization, which were a focal point of physics during the 1970s.

The concepts of fractional Brownian motion and multifractals, which are still frontier topics of research in physics and academic finance (as practiced by physicists), were introduced by Mandelbrot in the late 1960s and 1970s. And most recently, legions of physicists have found gainful employment on Wall Street as "quants," performing intricate calculations of price and risk of derivative securities, using sophisticated detailed models whose underlying premise remains that of Bachelier: Brownian motion (or more accurately, the logarithm of Brownian motion).

Fractals and Scaling in Finance is a characteristically idiosyncratic work. At once a compendium of Mandelbrot's pioneering work and a sampling of new results, the presentation seems modeled on the brilliant avant-garde film *Last Year in Marienbad*, in which the usual flow of time is suspended, and the plot is gradually revealed by numerous but slightly different repetitions of a few underlying events.

As Mandelbrot himself admits in the preface, the presentation allows the reader unusual freedom of choice in the order in which the book is read. In fact, I enjoyed this work most when I read it in random order, juxtaposing viewpoints and analyses separated in time by three decades and making clear the progression of ideas that Mandelbrot has generated. These include the classification of different forms of randomness, their manifestation in terms of distribution theory, their ability to be represented compactly, the notion of trading time, the importance of discontinuities, the relationship between financial time series and turbulent time series, the pathologies of commonly abused distributions (particularly the log–normal) and a catalog of the methods used to derive scaling distributions, both honest and fallacious.

Mandelbrot writes with economy and felicity, and he intersperses the more mathematical sections with frank historical anecdotes, such as the events that led up to his work on cotton pricing

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and the embarrassment caused by interpreting US Department of Agriculture data for weekly averages as "Sunday closing prices." There are many fascinating asides on a variety of topics, ranging from the importance of computer graphics in science to the distribution of insurance claims resulting from fire damage. In some places, the format of reprinted (but slightly edited) versions of classic papers allows Mandelbrot the surreal luxury of reviewing not only the content, but also the style and presentation of his work. And if all this were not enough, there are guest contributions from Eugene Fama, Paul Cootner and others.

This volume is not intended to be a textbook of modern finance, and it will probably infuriate those seeking a balanced and systematic exposition. Some readers will be irritated by the admitted redundancy of the text and frequent lapses into informality. My favorite is the caveat on page 232: "Due to time pressure, the algebra in this section has not been checked through, and misprints may have evaded attention." Indeed, I noticed many misprints, but to criticize the volume on that account would be churlish. The reader who is open-minded and prepared to indulge one of our more influential and original thinkers will be amply rewarded.

All in all, this is a strange but wonderful book. It will not suit everyone's taste but will almost surely teach every reader something new. What more can one ask?

What Is Mathematics, Really?

Reuben Hersh
Oxford U. P., New York, 1997. 343 pp. $35.00 hc ISBN 0-19-511368-3

What Is Mathematics, Really? is not an introduction to the practice of mathematics. Nor is it a description of some of the interesting projects that currently occupy mathematicians. Nor is it a compendium of engaging puzzles and paradoxes. It does not fit the same mold as most of the books on the subject that are—either supposedly or actually—addressed to the lay public. Rather, it takes on precisely the question posed in the title, and this at the deepest, most metaphysical level.

In his attempts to characterize what mathematics precisely *is*, Reuben Hersh describes and comments on the various answers that have been proposed over the millennia. Among the schools of thought discussed are Platonism, which insists on the intrinsic reality of numbers and other mathematical notions, and formalism, which focuses on the "rule structure" of the subject and in fact (according to Hersh) discards all other aspects of mathematics.

Hersh rejects these two influential philosophical approaches in favor of a "social-historic cultural" characterization of the activity. To his credit, he lays his cards on the table. At the end of the first chapter, he sets forth two assertions: that mathematics is a "social-historic reality" and that there is no need to inquire beyond the social, historical and cultural meaning of mathematics. He points out that the first statement is almost a truism. The second is clearly controversial—ask anyone with a nodding acquaintance with the so-called culture wars surrounding modern critiques of science.

Hersh is anything but an enemy of conventional mathematics. A working mathematician and teacher, he evinces the highest respect for those in the trenches. His argument with much that has been said about mathematics is that it does not respect what actually goes on when a mathematician tries to advance the field.

Reading this book with an unjaundiced eye (and in the absence of a background in the subject beyond vague recollections of a semester-long introduction to metaphysics), I found it easy to sympathize with Hersh. The Platonic approach, taken literally, seems just a little far-fetched, and the claim of the formalists that all of mathematics reduces to set theory strikes me as an exercise in hubris. On the other hand, I am not particularly happy with the possibility that mathematics may not be about eternal verities, a prospect that Hersh accepts with equanimity. He likens the acceptance of the notion of mutable, dubitable mathematics to the expansion of the real number system required to accommodate the square root of minus one. While the analogy intrigues, I am afraid that I do not find the comparison of imaginary numbers to fictive mathematics particularly persuasive. Nevertheless, one cannot dismiss this proposal out of hand, and it is undeniably provocative.

There is a good deal more to *What Is Mathematics, Really?* than is mentioned above. A sizable portion of the book is devoted to thumbnail sketches of what has been said about mathematics by the most influential thinkers in the Western world, from Pythagoras, Plato and Aristotle, through Saint Augustine, Descartes, Kant, Hilbert and Russell, to contemporary commentators. Hersh's generally lucid descriptions of their reflections and views are often accompanied by parenthetical comments. Most are illuminating; some strike me as facile and possibly unfair. On the whole, however, it appears to me that the book does justice to the various attempts to understand the mathematical enterprise.

Other sections include thought-provoking discussions of intellectual processes that constitute mathematical thinking, primers on important concepts and trends, a list of criteria—essential, desirable and dispensable—for a philosophy of mathematics. The final section of the book consists of over 50 pages of background notes on mathematics. In many respects, reading this section was the highlight of my experience with *What Is Mathematics, Really?* Hersh has a talent for exposition that makes me wish he had written most of the books on math I've had to read.

My own discipline—physics—has its share of anomalies and puzzles, and the question of the nature of mathematics has direct relevance for those interested in what it is—really—that a physicist does. Reuben Hersh's fascinating, if not always satisfying, book should prove an enlightening and entertaining read for anyone who desires greater insight into the nature of the pursuit of fundamental knowledge.

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Traces of the Past: Unraveling the Secrets of Archaeology through Chemistry

Joseph B. Lambert
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Joseph Lambert is a distinguished practitioner of the relatively new field of archaeometry. In *Traces of the Past: Unraveling the Secrets of Archaeology through Chemistry*, he sets forth comprehensively the wide-ranging scope of that discipline insofar as chemical research is involved. In the process, he recounts the often-startling results produced by the partnership between the chemist and the archaeologist. This gem of a book focuses on chemistry and the extraordinarily close match between the interests and skills of the chemist and the needs of the archaeologist to interpret the results of field work—what comes out of the excavation.

Chemical analytical laboratory studies are now a routine feature of an archaeological dig, and Lambert, by and large, deals with the whole gamut