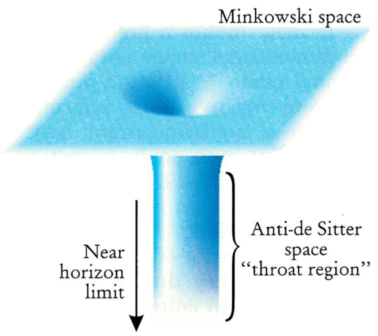
Duality Demonstrated on D-Branes

Juan Maldacena made clear the duality between two very different types of theories by formulating them both in terms of strange topological structures known as D-branes. D-branes are a subset of a larger class of objects known as *p*-branes (*p* is the object's spatial dimension), which are distortions of spacetime geometry, rather like dislocations in a crystal. If $p = 2$, the *p*-brane is an ordinary two-dimensional sheet, or membrane; if $p = 1$, it's a string and if $p = 0$, a point particle.

D-branes can have any dimension, but it may be easiest to visualize them as sheets. They are distinguished from other *p*-branes by being, loosely speaking, surfaces where the ends of open strings get stuck. (The strings that end on D-branes must satisfy Dirichlet boundary conditions; hence the name of these sheets.) The open strings give rise to particles whose dynamics take place on the brane. Joseph Polchinski (University of California, Santa Barbara) discovered that each D-brane carries a charge and hence is associated with a gauge potential, much as an electric charge is. On a single D-brane, one can formulate a one-color gauge theory, represented by the symmetry group U(1): Stacking N such D-branes gives a U(N) Yang–Mills theory.



REGION OF SPACETIME similar to a black hole, formed by stacking a large number of three dimensional hypersurfaces known as D-branes. The flat portion of the surface resembles Minkowski space, and the cylindrical portion (the throat region) is an anti-deSitter space, described by five-dimensional spheres with negative curvature. According to a new duality, the ten-dimensional spacetime deep in the throat, near the horizon of the black hole, is related to a four-dimensional gauge theory, which can be thought of as existing on the very remote boundary of the throat region.

To get a gauge theory in four dimensions, one needs three-dimensional (D3) branes, which can be embedded in ten-dimensional spacetime. In general, the gauge fields on these branes will interact with the gravity field in the surrounding space, but for low enough energies, the gauge field on the D3 brane is decoupled from gravity. Thus, for the gauge field side of his duality, Maldacena took N D3 branes.

These same D-branes can be viewed as a spacetime gravity solution that is similar to certain types of charged black holes. For the supergravity side of the duality, Maldacena used the curved spacetime around N D3 branes—the same configuration used by Klebanov and company for their cross sectional calculations. The metric around this stack takes the form shown in the figure above. Much of the hypersurface is nearly flat, and resembles Minkowski space. But a portion descends into a throatlike region, which continues indefinitely. One can think of this throat as leading to the horizon of a black hole; particles descending into it steadily lose potential energy, as they would were they to fall into a black hole. In ten dimensions, the metric there is described—formidably—as the product of five-dimensional anti–de Sitter (AdS) space and a five-dimensional sphere. AdS space is analogous to a sphere with negative curvature, rather like a multidimensional hyperboloid. The product of AdS space with a five-sphere is considered maximally symmetric. (John Schwarz, who with Michael Green invented the type of superstring theory in which D-branes arise, pointed out in 1983 that it admits this type of solution.[9])

Maldacena's contribution was to look at the supergravity solution deep in the throat region, where the energy is low. It turns out that, in this AdS space, the radius of curvature is proportional to $N^{1/4}$. N is the number of D3 branes, but it is also the number of colors in the gauge theory. That's the amazing part. The supergravity solution can be trusted the most when N is very large, and that translates precisely to the strong-coupling limit for the gauge theory. Maldacena points out that for large N, the supergravity in the bulk region (toward the center of the throat) is not influenced by the remote boundary, just as the gauge theory on the brane is not affected by gravity in the space around it.

roughly along the lines originally foreseen by 't Hooft.

Realization of the duality

Maldacena came to recognize the duality as a result of work that he and others had been doing on the entropy of black holes. The key to the duality is that black holes—or more generally, supergravity—can be described in terms of structures known as D-branes. One can also describe the low-energy dynamics of field theories in terms of D-branes. By thus formulating both string and gauge theories in terms of these strange structures, Maldacena was able to elucidate the connection between them. (See the accompanying box.)

Maldacena compared the supergravity and gauge theory solutions in the region where the parameter N is large. In gauge theory, large N (that is, many colors) corresponds to strong coupling. But how does this parameter relate to anything in supergravity? As discussed in the accompanying box, N turns out to be the number of D-branes in the formulation of the gauge theory and also of supergravity. In turn, the number of branes determines the radius of curvature of spacetime for the supergravity solution. Thus, taking the limit of large N carries the gauge theory into the strong-coupling limit and the supergravity theory into a region far from the (remote) boundary—a region where it can be trusted to give accurate results. More precisely, Maldacena demonstrated the validity of his conjecture for the case that g^2N is large, where g is the Yang–Mills coupling constant.

Maldacena was led in this direction by earlier work on black holes. Most notably, Andrew Strominger and Cumrun Vafa (both now at Harvard) used a description of black holes in terms of D-branes in their 1996 proof that the entropy for a certain type of black hole is related to its area in the predicted way. (See the news story in PHYSICS TODAY, March 1997, page 19.) Since that work, a number of researchers including Maldacena have been doing similar calculations for broader categories of black holes.

Klebanov and his colleagues had also studied the correspondence between supergravity and gauge theories, formulated on D-branes.[5] Specifically, they calculated the cross sections for entities such as gravitons to be absorbed by certain surfaces in the supergravity theory; these values agreed exactly with the corresponding calculations in the gauge field theory, where the values were related to two-point correlation functions. Thus, Klebanov and his group were exploiting some special cases of the duality be-