Quantum Theory without Observers—Part Two

The paradoxes of quantum theory can be resolved in a surprisingly simple way: by insisting that particles always have positions and that they move in a manner naturally suggested by Schrödinger's equation.

Sheldon Goldstein

Albert Einstein believed in the possibility of a quantum theory without observers—a version of quantum theory for which the notions of measurement, observation and observer are not invoked in its very formulation, but rather emerge from an analysis of more fundamental concepts. Niels Bohr believed that such a theory was "in principle" impossible. In part one of this article, I described one approach to such a theory, that of decoherent histories (DH). Although much progress has been made, it could be argued that this approach has not yet yielded a theory that is sufficiently well defined to provide decisive support for Einstein's view. The theories I discuss in this final part of the article are completely well defined and hence provide a conclusive refutation of Bohr's impossibility claims.

Reflection upon the problem of measurement—of macroscopic superpositions—very strongly suggests that for any quantum theory without observers there are two alternatives: Either the Schrödinger wavefunction is not right—that is, the Schrödinger evolution is not exact—or the Schrödinger wavefunction is not everything—that is, it does not provide us with a complete description of a physical system. (DH avoids the measurement problem by accepting, in effect, the wavefunction-is-not-everything possibility: The histories with which it is concerned are histories of quantum observables, not of wavefunctions, which play only a secondary, theoretical role.) The theories to which I now turn, spontaneous localization and Bohmian mechanics, may be regarded, respectively, as the simplest realizations of these two alternatives.

Spontaneous localization

The spontaneous localization (SL) approach, initiated by Philip Pearle around 1970, may be regarded as concerned with a minimal modification of the Schrödinger evolution in which wavefunctions of macroscopic systems behave in a sensible way. This goal proved elusive, but in 1985 a breakthrough occurred: GianCarlo Ghirardi, Alberto Rimini and Tulio Weber (GRW), by appreciating the privileged role somehow played by positions and thus focusing on the possibility of *spatial* localization, showed how to combine the Schrödinger evolution with spontaneous random collapses—given by "Gaussian hits" centered at random positions $\mathbf{x}$ occurring at random times t—to obtain an evolution for wavefunctions that reproduces the Schrödinger evolution on the atomic level while avoiding the embarrassment of macroscopic superpositions.[1]

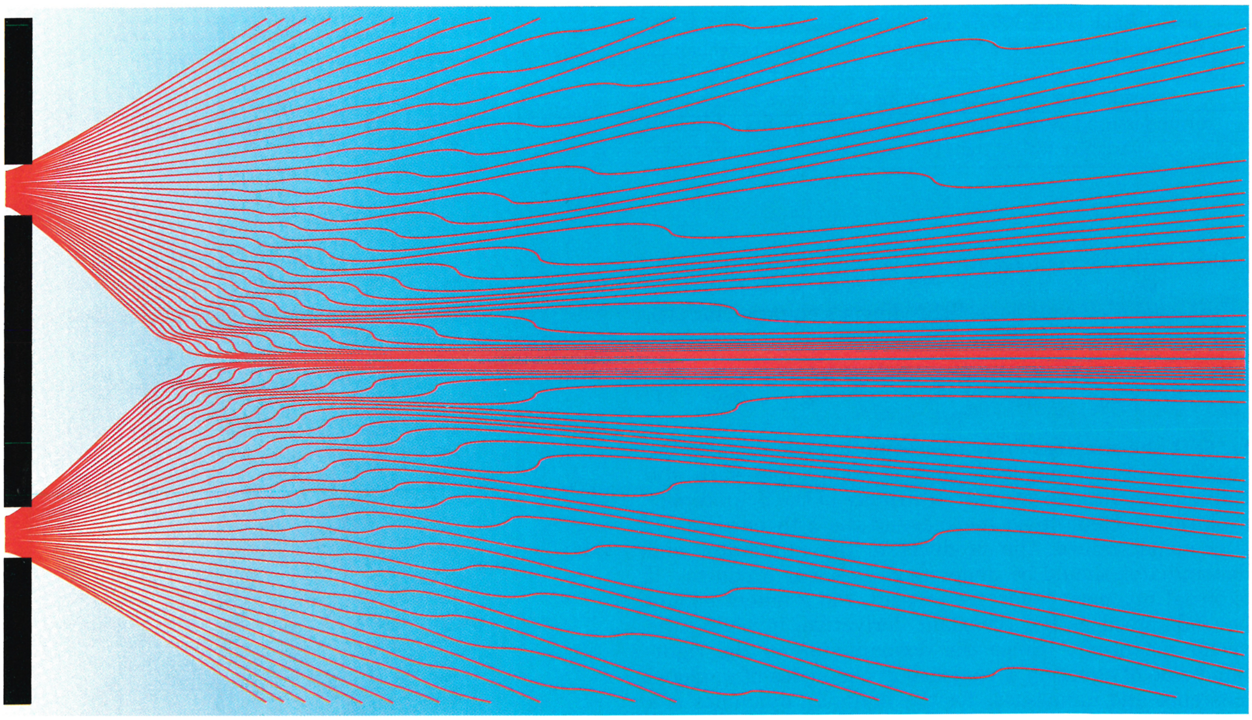
SHELDON GOLDSTEIN *is a professor of mathematics at Rutgers University in New Brunswick, New Jersey. Part one of this article appeared in last month's issue.*

Thus, as John Bell wrote, "any embarrassing macroscopic ambiguity in the usual theory is only momentary in the GRW theory. The cat is not both dead and alive for more than a split second."[2] Similarly, measurement pointers quickly point. Moreover, it is a more or less immediate consequence of the GRW dynamics that when a macroscopic superposition $\psi = \Sigma_\alpha \psi_\alpha$ collapses under the GRW evolution to one of its terms, the probability that ψ_α is the term that survives is $\|\psi_\alpha\|^2$, precisely as demanded by the collapse postulate of standard quantum theory.

It is tempting to say that with the SL approach, quantum mechanics is indeed fundamentally about the behavior of wavefunctions. I believe, however, that this is not quite right. The problem is that the purpose of any physical theory is to account for a pattern of events occurring in (ordinary three-dimensional) space and time. But the behavior of a wavefunction of a many (N) particle universe, a field on an abstract ($3N$-dimensional) configuration space, has in and of itself no implications whatsoever regarding occurrences in physical space, however sensible its behavior may otherwise be. As Bell noted, "It makes no sense to ask for the amplitude or phase or whatever of the wave function at a point in ordinary space. It has neither amplitude nor phase nor anything else until a multitude of points in ordinary three-space are specified."[2]

Therefore, Ghirardi rightly emphasizes the importance of specifying what he calls "the physical reality of *what exists out there*[3] [emphasis in original]." For this, he chooses the mass density function, which, for the simple GRW theory described here, can be identified with the mass-weighted sum $\Sigma_i\, m_i \rho_i(\mathbf{x})$, over all particles, of the one-particle densities ρ_i arising from integrating $|\psi|^2$ over the coordinates of all but one of the particles. (Because of subtle considerations related to the notion of "accessibility," Ghirardi's specific choice is actually the mass density averaged over a "localization volume.")

Bell proposed a strikingly different possibility: that the space–time points $(\mathbf{x},t)$ at which the hits are centered (which are determined by the wavefunction trajectory)



ENSEMBLE OF TRAJECTORIES for the two-slit experiment. (Adapted by Gernot Bauer from C. Philippidis, C. Dewdney, B. J. Hiley, Nuovo Cimento B **52**, 15, 1979.)

should themselves serve as the "local beables [Bell's coinage] of the theory. These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and as distinct from the 'observables' of other formulations of quantum mechanics, for which we have no use here). A piece of matter then is a galaxy of such events."[2] (Bell's proposal is not applicable to models involving continuous dynamical reduction.[3])

One can imagine, of course, many other choices, some better than others. The point I wish to emphasize, however, is that if we are to have a well-defined physical theory at all, some such choice must be made. Indeed, any quantum theory without observers, and arguably any physical theory with any pretense to precision, requires as part of its formulation a specification of the "local beables," of "what exists out there," of what the theory is fundamentally about—which I would prefer to call the *primitive ontology* of the theory. (It could be argued that the unease sometimes expressed about DH arises from the obscurity of its primitive ontology—or from its failure to commit in this regard.) Moreover, we must also specify, for a quantum theory, the relationship between the wavefunction and this primitive ontology, which for SL will be provided by a mapping or code connecting the evolution of the wavefunction to a story in space and time.

Different such specifications define different theories. They may also have different observable consequences. Moreover, the symmetries of the theory may depend critically on this specification. For example, with Bell's rather surprising choice, the GRW theory obeys a certain "relative time translation invariance" and becomes "as Lorentz invariant as it could be in the nonrelativistic version."[2] Thus a careful analysis of the symmetries of a theory demands a careful specification of its primitive ontology.

As a matter of fact, one would have to make a rather perverse choice to arrive at any empirical disagreement with the predictions arising from the choices of Ghirardi or Bell. It is clear, however, because of its abrogation of the Schrödinger evolution, that SL (in whatever version and with whatever choice of primitive ontology) must disagree somewhat with the predictions of orthodox quantum theory. In fact, by the uncertainty principle, the wavefunction localizations will increase the momentum space spread in the wavefunction and hence energy will tend to increase at a very small rate—so small, in fact, that this effect may be rather difficult to observe.

Bohmian mechanics

The last version of quantum theory without observers that I want to describe agrees completely with orthodox quantum theory in its predictions. Precise and simple, it involves an almost obvious incorporation of Schrödinger's equation into an entirely deterministic reformulation of quantum theory.

In the pilot-wave approach, quantum theory is fundamentally about the behavior of particles, described by their positions—or fields (described by field configurations) or strings (described by string configurations)—and only secondarily about wavefunctions. In this approach the wavefunction, obeying Schrödinger's equation, does not provide a complete description or representation of a quantum system. Instead, the wavefunction choreographs or governs the motion of the more fundamental variables.

Bohmian mechanics (or the de Broglie–Bohm theory), the simplest pilot-wave theory, is the minimal completion of Schrödinger's equation, for a nonrelativistic system of particles, into a theory describing a genuine motion of particles. For Bohmian mechanics the state of the system is described by its wavefunction $\psi = \psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)$, to-

JOHN BELL (1928–1990). For several decades, Bell was the deepest thinker on the foundations of quantum mechanics. His analysis of nonlocality and hidden variables revitalized the field. The implications of his work have been widely misunderstood as demonstrating the impossibility of hidden variables rather than the inevitability of nonlocality.

CERN

gether with the configuration Q defined by the positions $\mathbf{Q}_1, \ldots, \mathbf{Q}_N$ of its particles. The theory is then defined by two evolution equations: Schrödinger's equation for $\psi(t)$, and a first-order evolution equation

$$\frac{d\mathbf{Q}_k}{dt} = \boldsymbol{v}_k\,(\psi;\mathbf{Q}_1, \ldots, \mathbf{Q}_N) \equiv \frac{\hbar}{m_k}\,\mathrm{Im}\,\frac{\psi^*\nabla_{\mathbf{q}_k}\psi}{\psi^*\psi}\,(\mathbf{Q}_1, \ldots, \mathbf{Q}_N) \qquad (1)$$

for $Q(t)$, the simplest first-order evolution equation for the positions of the particles that is compatible with the Galilean (and time-reversal) covariance of the Schrödinger evolution.[4] Here m_k is the mass of the k-th particle. If ψ is spinor-valued, the products in the numerator and denominator should be understood as scalar products. If external magnetic fields are present, the gradient should be understood as the covariant derivative, involving the vector potential. (Since the denominator on the right-hand side of equation 1 vanishes at the nodes of ψ, global existence and uniqueness for the Bohmian dynamics is a nontrivial matter; it is proved in reference 5.) This deterministic theory of particles in motion completely accounts for all the phenomena of nonrelativistic quantum mechanics, from interference effects to spectral lines[6] to spin,[7] and it does so in a completely ordinary manner.

Note that, given an initial wavefunction ψ_0, the full Bohmian trajectory $Q(t)$ is determined by the initial configuration Q_0. Thus, given any probability distribution for the initial configuration, Bohmian mechanics defines a probability distribution for the full trajectory. Moreover, since the right-hand side of equation 1 is J/ρ, where J is the quantum probability current and ρ is the quantum probability density, it follows from the quantum continuity equation $\partial\rho/\partial t + \mathrm{div}\,J = 0$ that if the distribution of the configuration Q is given by $|\psi|^2$ at some time (say the initial time), this will be true at all times. Thus Bohmian mechanics provides us with probabilities for completely fine-grained configurational histories that are consistent with the quantum mechanical probabilities for configurations, including the positions of instrument pointers, at single times.

The pilot-wave approach to quantum theory was initiated, even before the discovery in 1925 of quantum mechanics itself, by Einstein, who hoped that interference phenomena involving particle-like photons could be explained if the motion of the photons were somehow guided by the electromagnetic field—which would thus play the role of what he called a *Führungsfeld*, or guiding field.[8] Although the notion of the electromagnetic field as guiding field turned out to be rather problematical, the possibility that for a system of electrons the wavefunction might play this role, of guiding field or pilot wave, was explored by Max Born in his early paper founding quantum scattering theory[9]—a suggestion to which Heisenberg was profoundly unsympathetic.

By 1927, an equation of particle motion equivalent to equation 1 for a scalar wavefunction had been written down by Louis de Broglie,[10] who explained at the 1927 Solvay Congress how this motion could account for quantum interference phenomena. However, de Broglie badly failed to respond adequately to Wolfgang Pauli's[10] objection concerning inelastic scattering, no doubt making a rather poor impression on the illustrious audience gathered for the occasion.

Born and de Broglie very quickly abandoned the pilot-wave approach and became enthusiastic supporters of the rapidly developing consensus in favor of the Copenhagen interpretation. Bohmian mechanics was rediscovered in 1952 by David Bohm, the first person genuinely to understand its significance and implications. (Unfortunately, Bohm's formulation involved unnecessary complications and could not deal efficiently with spin. In particular, Bohm's invocation of the "quantum potential" made his theory seem artificial and obscured its essential structure.[11]) The principal advocate of Bohmian mechanics during the sixties, seventies and eighties was Bell. Impelled by the evident nonlocality of Bohmian mechanics, Bell established, using the "no-hidden-variables theorem" based on his famous inequality, that nonlocality was unavoidable by any serious theory accounting for the quantum predictions.[12]

'Impossibility' of hidden variables

The possibility of a deterministic reformulation of quantum theory such as Bohmian mechanics has been regarded by almost all luminaries of quantum physics as having been conclusively refuted. For several decades, this refutation was believed to have been provided by the 1932 no-hidden-variables proof of John von Neumann,[13] despite the fact that, according to Bell,[14] von Neumann's assumptions (about the relationships among the values of quantum observables in a hidden-variables theory) are so unreasonable that "the proof of von Neumann is not merely false but *foolish* [emphasis in original]!" Although some physicists continue to rely on von Neumann's proof, it is interesting to note that in recent years it has been more common to find physicists citing Bell's no-hidden-variables theorem as the basis of this refutation—thus

A. PICCARD E. HENRIOT P. EHRENFEST Ed. HERZEN Th. DE DONDER E. SCHRÖDINGER E. VERSCHAFFELT W. PAULI W. HEISENBERG R.H. FOWLER L. BRILLOUIN
P. DEBYE M. KNUDSEN W.L. BRAGG H.A. KRAMERS P.A.M. DIRAC A.H. COMPTON L. de BROGLIE M. BORN N. BOHR
I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON
Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL

THE PARTICIPANTS of the Fifth Solvay Congress, 1927. (International Institute of Physics and Chemistry photo; courtesy of AIP Emilio Segrè Visual Archives.)

failing to appreciate that what Bell demonstrated with his theorem was not the impossibility of Bohmian mechanics but rather that its most radical implication—namely, nonlocality—was intrinsic to quantum theory itself.

According to Richard Feynman, the two-slit experiment for electrons is "a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the *only* mystery [emphasis in original]."[15] This experiment, Feynman declared, "has been designed to contain all of the mystery of quantum mechanics, to put you up against the paradoxes and mysteries and peculiarities of nature one hundred per cent."[16] Added Feynman: "How does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it."[16]

But Bohmian mechanics is just such a deeper explanation (as is SL, of which, however, Feynman could not have been aware). It resolves the dilemma of the appearance, in one and the same phenomenon, of both particle and wave properties in a rather straightforward manner: Bohmian mechanics is a theory of motion describing a particle (or particles) guided by a wave. The illustration at the beginning of this article shows a family of Bohmian trajectories for the two-slit experiment. While each trajectory passes through but one of the slits, the wave passes through both; the interference profile that therefore develops in the wave generates a similar pattern in the trajectories guided by this wave.

Compare Feynman's presentation above with that of Bell:

> Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in [the] screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave–particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.[2]

Nonetheless, it would appear that because orthodox quantum theory supplies us with probabilities not merely for positions but for a huge class of quantum observables, it is a much richer theory than Bohmian mechanics, which seems exclusively concerned with positions. Appearances are misleading, however. In this regard, as with so much else in the foundations of quantum mechanics, the crucial observation has been made by Bell:

> [I]n physics the only observations we must consider are position observations, if only the positions of instrument pointers. It is a great merit of the de Broglie–Bohm picture to force us to consider this fact. If you make axioms, rather than definitions and theorems, about the "measurement" of anything else, then you commit redundancy and risk inconsistency.[2]

Bell's point here is well taken: The usual measurement postulates of quantum theory, including collapse of the wavefunction and correspondence of measurement probabilities to the absolute square of probability amplitudes, emerge as soon as we take seriously the equations of Bohmian mechanics and what they describe[6]—provided that the initial configuration of a system is random, with probability distribution given by $\rho = |\psi|^2$. Moreover, Detlef Dürr, Nino Zanghì and I have shown how probabilities for positions given by $|\psi|^2$ emerge naturally from an analysis of "equilibrium" for the deterministic dynamical system defined by Bohmian mechanics, in much the same way that the Maxwellian velocity distribution emerges from

STILL PICTURES, MARK EDWARDS

DAVID BOHM. Some of Bohm's ideas about quantum mechanics and the nature of physical reality—for example, regarding the implicate order—were rather speculative. But his deterministic version of quantum mechanics is quantum theory's most lucid and straightforward completion.

an analysis of classical thermodynamic equilibrium.[4] Thus, with Bohmian mechanics the statistical description in quantum theory indeed takes, as Einstein anticipated, "an approximately analogous position to the statistical mechanics within the framework of classical mechanics."

Reality and the role of the wavefunction

Bohmian mechanics is, it seems to me, by far the simplest and clearest version of quantum theory. Nonetheless, with its additional variables and equations beyond those of standard quantum mechanics, Bohmian mechanics has seemed to most physicists to involve too radical a departure from quantum modes of thought. The approaches of spontaneous localization and decoherent histories have achieved much wider acceptance among physicists, SL because it ostensibly involves only wavefunctions, effectively collapsing upon measurement in the usual textbook manner, and DH because it apparently is defined solely in terms of standard quantum mechanical machinery—that is, the quantum measurement formulas of the orthodox theory, involving wavefunctions and sequences of Heisenberg projection operators.

However, SL clearly involves equations beyond those of orthodox quantum theory, and, as I've argued, DH must also be regarded in this way. I have also argued that neither for DH nor even for SL can the wavefunction be regarded as providing the complete description of a physical system. Thus, while there are significant differences in detail, the three approaches discussed in this two-part article have much more in common than is usually acknowledged. Each involves additional equations and additional variables. The variables are the fundamental variables, describing the primitive ontology—what the theory is fundamentally about. Their behavior is governed by laws expressed in terms of the wavefunction, which thus simply plays a dynamical role.

As to detail, Bohmian mechanics shows that if we don't insist upon patterning these laws upon familiar formulas such as those of the quantum measurement formalism, surprising simplicity can be achieved. GRW, particularly *à la* Bell, shows that these laws may be of a most unusual variety, with unexpected implications for the symmetry of the theory.[2] And DH introduces a fundamental, irreducible coarse graining. Furthermore, if it should turn out that more than one family satisfies the decoherence condition and suitable additional conditions, DH suggests that a fundamental stochastic theory need not assign probabilities to everything that can happen—for example, to histories of the form "h and $\tilde{h}$" where h and $\tilde{h}$ belong to different augmented decoherence condition families, while the history "h and $\tilde{h}$" belongs to no such family.

None of the theories sketched here is Lorentz invariant. There is a good reason for this: The intrinsic nonlocality of quantum theory presents formidable difficulties for the development of a Lorentz-invariant formulation that avoids the vagueness of the orthodox version. I believe, however, that such a theory is possible, and that the three approaches I've discussed in this two-part article have much to teach us about how we could go about finding one.

I am grateful to Karin Berndl, Jean Bricmont, Martin Daumer, Detlef Dürr, Gregory Eyink, GianCarlo Ghirardi, Rebecca Goldstein, Michael Kiessling, Joel Lebowitz, Eugene Speer, Herbert Spohn, James Taylor and Nino Zanghì for their comments and suggestions—as well as for their patience. This work was supported in part by a grant from the National Science Foundation.

References

1. G. C. Ghirardi, A. Rimini, T. Weber, Phys. Rev. D **34**, 470 (1986).
2. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge U. P., Cambridge, England (1987).
3. G. C. Ghirardi, in *Structures and Norms in Science*, M. L. Dalla Chiara *et al.*, eds., Kluwer Academic, Dordrecht, The Netherlands (1997).
4. D. Dürr, S. Goldstein, N. Zanghì, J. Stat. Phys. **67**, 843 (1992).
5. K. Berndl, D. Dürr, S. Goldstein, G. Peruzzi, N. Zanghì, Commun. Math. Phys. **173**, 647 (1995).
6. D. Bohm, Phys. Rev. **85**, 166, 180 (1952).
7. J. S. Bell, Rev. Mod. Phys. **38**, 447 (1966); reprinted in ref. 2.
8. E. P. Wigner, in *Quantum Theory and Measurement*, J. A. Wheeler, W. H. Zurek, eds., Princeton U. P., Princeton, N.J. (1983), p. 262.
9. M. Born, Z. Phys. **38**, 803 (1926); English translation in *Wave Mechanics*, G. Ludwig, ed., Pergamon, Oxford, England (1968), p. 206.
10. L. de Broglie, in *Electrons et photons: Rapports et discussions du cinquième conseil de physique tenu à Bruxelles du 24 au 29 Octobre 1927 sous les auspices de l'Institut International de Physique Solvay*, Gauthier-Villars, Paris (1928).
11. D. Dürr, S. Goldstein, N. Zanghì, in *Bohmian Mechanics and Quantum Theory: An Appraisal*, J. Cushing, A. Fine, S. Goldstein, eds., Kluwer Academic, Dordrecht, The Netherlands (1996).
12. J. S. Bell, Physics **1**, 195 (1964); reprinted in ref. 2.
13. J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Springer Verlag, Berlin (1932); English trans. by R. T. Beyer, *Mathematical Foundations of Quantum Mechanics*, Princeton U. P., Princeton, N.J. (1955).
14. N. D. Mermin, Rev. Mod. Phys. **65**, 803 (1993).
15. R. P. Feynman, R. B. Leighton, M. Sands, *The Feynman Lectures on Physics*, vol. 1, Addison-Wesley, New York (1963), p. 37–2.
16. R. Feynman, *The Character of Physical Law*, MIT Press, Cambridge, Mass. (1967). ■