

was incorporated into accessible computer programs.

Chemists demand a high level of accuracy because of the very small energy differences needed to understand chemical phenomena. To gain the desired accuracy in the density functional theory required a more sophisticated treatment of electron-electron correlations than is employed by the local density approximation. These correlations were introduced by the use of generalized gradients of the density, thanks to successive contributions by David Langreth, John Perdew and Axel Becke.

Currently, the accuracy of the density functional approach varies with the application. Kohn suspects that this approach, by its nature, may never be a theory for achieving great accuracy. Its unique advantage is its ability to deal with larger molecules, in which there is increasing interest. By contrast, the Hartree-Fock or other, more conventional calculations become much more difficult as the number of electrons in the problem increases.

By the early 1990s, the improved accuracy in the density functional theory and increased interest in large molecules had motivated Pople and others to incorporate the density functional theory into their computer programs.

Full careers

Kohn describes his early life as "turbulent". Born of Jewish parents in Vienna in 1923, he was just young enough to qualify for the last *Kindertransport* out of Nazi-occupied Austria when he was 16. After two years with a family in England, he was just old enough to be sent for detention in Canada as an "enemy alien." There, he eventually served in the Canadian armed forces during World War II. After the war, he attended the University of Toronto, earning a BA (1945) and an MA (1946) in applied mathematics. He got his PhD in physics under Julian Schwinger at Harvard University in 1948, staying on for two years as an instructor. From 1950 to 1960, Kohn was a professor of physics at the Carnegie Institute of Technology (today, Carnegie Mellon University) from 1950 to 1960 and at the University of California, San Diego, from 1960 to 1979. In 1979, he became the first director of the newly created Institute of Theoretical Physics at the University of California, Santa Barbara, guiding that institute through its first five years. In 1984, he became a professor of physics at UCSB and continues to be active as a research professor there.

Pople was born in the UK in 1925.

He earned his PhD in mathematics at the University of Cambridge in 1951, working under John Lennard-Jones. Pople remained at Cambridge until 1958, first as a research fellow and then as a lecturer in mathematics, all the while doing semiempirical studies of molecules. In 1964, after six years as superintendent of the basic physics division of the National Physical Laboratory in Teddington, England, Pople went to Carnegie Tech as a professor of chemical physics. In 1974, he became the John Christian Warner University Professor of Natural Sciences. He has been at Northwestern since 1986, where he continues to pursue his interests in molecular structure.

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Two-Dimensional Electron Gases Continue to Exhibit Intriguing Behavior

The startling discoveries of the integral and fractional quantum Hall effects were made in two-dimensional electron gases subjected to very high magnetic fields (see the story on page 17). Now, it appears, other surprises await us at lower fields. In a recent study, the longitudinal resistivity exhibited a strong anisotropy in a certain range of temperature and magnetic field: When plotted as a function of the magnetic field, the resistivity has a dip when the current flows in one direction and a strong peak when it flows in an orthogonal direction.¹ There's no *a priori* reason to think that these two directions are different.

When researchers from Caltech and Bell Laboratories, Lucent Technologies, reported these results last August at a conference on quantum Hall and mesoscopic systems held at the Institute for Theoretical Physics in Santa Barbara, California, the audience suggested ways to check for this or that possible confounding factor. Having survived further experimental scrutiny, the anisotropy is now attracting increasing interest from the theorists.

▶ Researchers have taken a closer look at some funny structure noted years ago in the resistivity of a quantum Hall sample at low magnetic fields. The prevailing explanation for what they see is that the electrons are forming charge density waves.

The leading speculation is that the anisotropy reflects the formation of theoretically predicted charge-density waves, with the electrons all lined up in rows.

Shades of the past

The work reported in Santa Barbara began last May, when Jim Eisenstein of Caltech set out to explore in more depth an effect that he and Robert Willett had seen in quantum Hall samples back in 1988 when both were working at Bell Labs. Similar anomalies were subsequently reported at the March 1993 meeting of the American Physical Society by Horst Stormer, who had been working with Daniel Tsui and Rui Du. Other groups also had evidence of anisotropic behavior, but no

one explored it further at the time.

For his second look, Eisenstein was joined by Michael Lilly and Kenneth Cooper of Caltech; they enlisted Loren Pfeiffer and Kenneth West of Bell Labs to prepare very clean, high-mobility samples. The samples were gallium arsenide/aluminum gallium arsenide heterojunctions, in which an electron gas forms at the interface.

The researchers zeroed in on the regions around filling factors of $\frac{1}{2}$ and $\frac{1}{2}$. By contrast, strong fractional quantum Hall states usually show up at filling factors less than 2, such as $\frac{1}{3}$ or $\frac{2}{5}$. The filling factor, ν , indicates the number of electrons for each flux quantum. Thus, one gets a filling factor of $\frac{1}{3}$ at a high value of the magnetic field, where there are three flux quanta for each electron. At these fields, even the very lowest Landau level ($N=0$) is not filled (it is filled when there is one electron of each spin for each flux quantum). One gets higher filling factors of $\frac{1}{2}$ and $\frac{1}{2}$ by decreasing the field.

The behavior of the two-dimensional gas at these higher filling factors

is summarized graphically in the figure at right, for data taken at 25 mK. The color of each curve is keyed to one of the two diagrams indicating the direction of the current. When the current is driven as shown in the purple diagram, sharp peaks appear in the longitudinal resistivity at filling factors of $\frac{9}{2}$, $\frac{11}{2}$, $\frac{13}{2}$ and higher. With the current driven at right angles, as shown in the red diagram, there is a minimum at the same value of the magnetic field where the peaks were seen. Moreover, the resistivity maxima are up to a hundred times greater than the resistivity minima.

Lilly and company noticed three other intriguing details. First, the peaks seen in one orientation grow nonlinearly as the temperature drops from 150 mK to 25 mK, although the widths do not get any narrower. Second, the anisotropy is seen only near half filling. Finally, the behavior at $\frac{9}{2}$, $\frac{11}{2}$ and $\frac{13}{2}$ is qualitatively very different from that at $\frac{7}{2}$ or $\frac{5}{2}$.

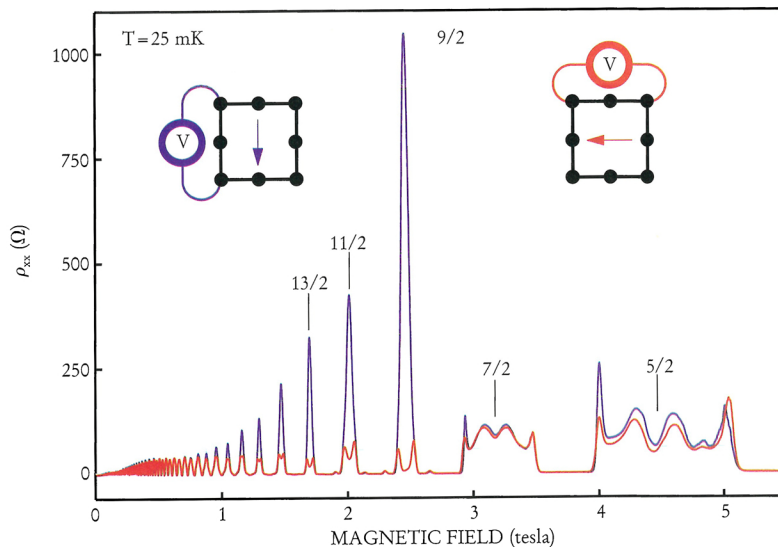
Is it an extrinsic effect?

Can this observed anisotropy be attributed to some geometric bias in the experiment, most notably to a gradient in the electron density within the sample? The researchers think not. Their sample was rotated during growth to minimize such gradients, so that the variation in density appeared to be less than 0.3%.

To check on the robustness of the effect, Lilly and company repeated their measurements on what are known as Hall bar samples, and found the same anisotropy. Unlike their original square samples, where one is not sure what the current distribution is, Hall bars are rectangular. The experimenters feel it's unlikely that some extrinsic effect could cause the sudden development at very low temperatures (below 150 mK) of strong temperature dependences and large anisotropies in just a few Landau levels above $N = 1$. Rather, they surmise, some previously unappreciated physics is at work. Eisenstein does add, however, some tiny extrinsic effect may also be present to break the symmetry and pick out a preferred axis.

What can it be?

The structure seen at these high filling factors indicates that electron interactions are at play, but they are clearly different from those that produce the quantum Hall plateaus at lower filling factors. One possible explanation for the anomalous behavior at $\frac{9}{2}$ and $\frac{11}{2}$ is the formation of charge-density stripes—entities predicted in the past two years by Alex A. Koulakov, Michael M. Fogler and Boris Shklovskii at the



DIFFERENT RESISTIVITIES IN DIFFERENT DIRECTIONS. The longitudinal resistivity for a two-dimensional electron gas exhibits peaks when the current flows in one orientation (blue curve and circuit diagram) and dips when the current flows at right angles to that (red). These effects are seen at filling factors of $\frac{9}{2}$, $\frac{11}{2}$ and $\frac{13}{2}$, but not at any filling factors below that. (Adapted from ref. 1.)

University of Minnesota² and later by Roderich Moessner and John Chalker at the University of Oxford.³ Both groups applied mean-field theory to the electron interactions at high Landau levels and predicted that the electrons would condense in some regions, leaving other regions free of charge.

The Minnesota theorists offer a qualitative explanation of how this charge separation comes about.² They treat the wavefunction of each electron in the partially filled level as a charged ring whose radius is that of a cyclotron orbit. The Coulomb interactions are largely screened, leaving only short-range interactions. It costs energy for two electron rings, or disks, to overlap, but that energy cost is independent of the extent of the overlap. If two orbits are to overlap, they may as well overlap a lot. Electrons thus tend to condense in either bubbles or stripes, the latter being favored at half filling. The filling factor is 1 inside a bubble or stripe, and 0 outside.

Charge-density waves had been predicted for two-dimensional gases back in 1979 by Hidetoshi Fukuyama, Philip Platzman and Philip Anderson,⁴ who used the Hartree-Fock approximation—a mean-field approach—to study electrons in the partially filled, lowest Landau level. It later turned out that the states suggested by Robert Laughlin (see the story on page 17) to explain the fractional quantum Hall effect are a bit lower in energy than the charge-density wave states predicted by Fukuyama, Platzman and Anderson.

At filling factors above $\nu = 4$, however, calculations show that the mean-field states do not lose out to the Laughlin states. In fact, Moessner and Chalker proved formally that the mean-field solution is exact in the limit of very high Landau levels.

While agreeing that uniaxial charge-density waves are currently the most likely explanation for the anisotropies, Bertrand Halperin (Harvard University) cited some observations that are hard to explain with a simple model. In addition, the experiments pose the question of what determines the preferred orientation of the charge-density stripes. Possibly, it's the sample itself: The formation of steps at the heterojunction interface or the slight miscutting of the substrate might determine a preferred direction. Lilly said he and his colleagues are planning several measurements with different samples to check out some of these possibilities.

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