Physics Nobel Prize Goes to Tsui, Stormer and Laughlin for the Fractional Quantum Hall Effect

This year's Nobel Prize in Physics is shared by Robert Laughlin (Stanford), Horst Stormer (Columbia University and Bell Laboratories) and Daniel Tsui (Princeton), for their roles in the discovery and explanation of the fractional quantum Hall effect. In 1982, when Stormer and Tsui were experimenters at Bell Labs, they and their colleague Arthur Gossard discovered this totally unexpected quantum effect in the transport properties of two-dimensional electron gases at low temperature in strong magnetic fields.[1] (See PHYSICS TODAY, July 1983, page 19.)

Despite its phenomenological analogy to what would henceforth be called the "integral" quantum Hall effect, discovered two years earlier by Klaus von Klitzing in Germany, the fractional quantum Hall effect (FQHE) was a theoretical conundrum. While the integral effect could be explained rather straightforwardly in terms of the filling of successive "Landau levels"—essentially the Pauli exclusion principle—without having to worry about the Coulomb repulsion between electrons, nothing of the kind would do for the fractional effect.

But theorist Robert Laughlin, who had recently left Bell Labs for the Lawrence Livermore Laboratory, came to the rescue with astonishing promptness. A few months after the Tsui,

> In the 16 years since the totally unanticipated discovery of the first of the fractional quantum Hall states, these intriguing quantum liquids have spawned a thriving cottage industry for theorists and experimenters.

Stormer, Gossard discovery, Laughlin was able to explain[2] their puzzling result in terms of the formation of a novel ground state of the two-dimensional collectivity of electrons: an incompressible quantum liquid whose elementary excitations are quasiparticles of fractional charge.

"I predict," wrote Laughlin, disdaining the *Physical Review Letters* aversion to the first person singular, "the existence of a sequence of these ground states, decreasing in density and terminating in a Wigner crystal." Experimenters at Bell Labs and elsewhere soon found many more of these predicted fractional quantum Hall states, plus others that raised new theoretical issues.

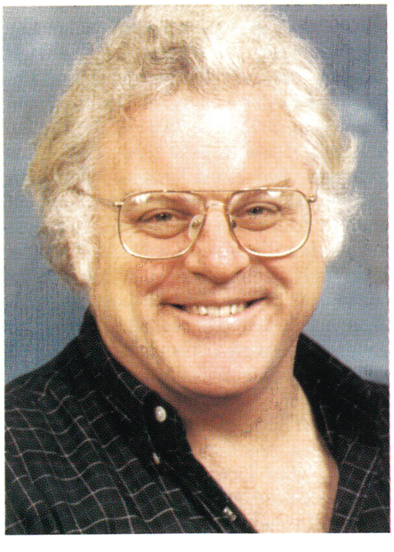
The Hall effect

The classical Hall effect is simply the lateral Lorentz-force pileup of carriers when an electric current **I** flows through a perpendicular magnetic field **B**, producing a transverse "Hall voltage" V_H in the direction $e\mathbf{I} \times \mathbf{B}$. Hall-effect measurements tell us about the nature of the objects carrying the current. For instance, an early manifestation of "holes" was the fact that the carrier charge e often turned out to be positive for metals on the right side of the periodic table.

Classically, the Hall voltage grows linearly with increasing **B**. What earned Klitzing the 1985 Nobel Prize was his 1980 discovery that, in the two-dimensional current flowing in the inversion layer of a silicon field-effect transistor at very low temperature and high magnetic field, this continuous classical growth is interrupted by a sequence of spectacularly flat quantized plateaus: Whenever the carrier density in the inversion layer and the perpendicular magnetic field threading through it correspond approximately to a small whole number ν of electrons per elementary flux quantum ($\Phi_0 = hc/e$), the Hall resistance $R_H \equiv V_H/I$ flattens out to $h/\nu e^2$ with a precision of better than a part in a million. (See PHYSICS TODAY, June 1981, page 17.) In the meantime, $h/e^2 \approx 25\,813\ \Omega$ has become an important metrological standard of electrical resistance.

The location of the plateaus made obvious sense: Every time the electron density in the plane became equal to a multiple of the flux quantum density,

HORST STORMER AND DANIEL TSUI



ROBERT LAUGHLIN

a Landau level of electrons executing tiny, quantized cyclotron orbits was filled to capacity, and any additional electron seeking admittance had to jump an energy gap to the next cyclotron energy level. The puzzling surprise was the width of the plateaus and their amazing precision in spite of all the imperfections in the two-dimensional inversion layer.

That's where Laughlin, just a year out of graduate school at MIT, made his first mark in quantum Hall theory. He put forward an ingenious *gedanken* experiment, in which the two-dimensional current plane is a closed loop threaded by additional flux, to prove that the precision of the integral quantum Hall plateaus follows from the charge of the electron and the gauge invariance of its interactions. His idea that the plateaus are, in effect, measurements of elementary charge, was to become an important ingredient, two years later, of his otherwise quite different explanation of the FQHE, with its fractionally charged quasiparticle excitations.

More perfect interfaces

The study of two-dimensional electron gases in metal-oxide silicon field-effect transistors (MOSFETs) was pioneered by Alan Fowler, Webster Howard and coworkers at IBM in the 1960s. The system is effectively two-dimensional when the temperature is so low that motion normal to the plane is confined to the ground state. In the plane, by contrast, the electrons can wander freely. But the mobility of the electron gas in a MOSFET is limited by the disorder of the transistor's amorphous silicon oxide layer.

A crucial step in the development of clean, crystalline semiconductor interfaces with much higher electron mobilities was the invention of molecular beam epitaxy by Alfred Cho at Bell Labs in the early 1970s. At the end of the decade came another big step, the development of modulation doping by Stormer, Gossard and their Bell Labs colleague Raymond Dingle. The idea was to confine the implantation of dopant atoms to the aluminum gallium arsenide side of a GaAs/AlGaAs interface fabricated by molecular beam epitaxy, so that the active (gallium arsenide) side would acquire the donor's electrons without having to put up with its disruptive atomic-core scattering sites.

"When I came from Germany to Bell Labs as a postdoc in 1977," Stormer recalls, "Dan Tsui was already a great man in two-dimensional electron gas physics there, using silicon MOSFETs. Dan quickly saw that our modulation-doped gallium arsenide heterostructures could push 2D physics into a new realm. He grabbed me by the shirt—figuratively, of course—and said 'Come with me to the MIT [Francis Bitter] Magnet Lab, where we can do experiments with these high-mobility samples at very high fields and low temperatures.' "

Gossard was leading the fabrication effort toward ever higher electron mobilities. Late in 1981 he produced a GaAs/AlGaAs heterostructure with a record interface electron mobility of 9×10^4 cm^2/V s, an order of magnitude better than one can get with a silicon MOSFET. "That's the sample with which we found the first fractional quantum Hall states at the Magnet Lab," Stormer told us. "Gossard's wonderful specimen was crucial. With it, anyone could have done the experiment."

Fractional filling factors

The integral quantum Hall plateaus occur when the "filling factor" ν, the ratio of carrier electrons to magnetic flux quanta, is an integer—that is to say, when ν Landau levels are fully occupied. Does anything interesting happen when ν is less than 1?

"We wanted to look at this unexplored extreme quantum limit, where the flux quanta outnumber the electrons," Tsui told us. "Below $\nu = 1$, where all the single-particle Landau filling effects would be exhausted, we anticipated that electron–electron interaction effects would dominate. Work on fractional quantum numbers in other areas by theorists like Phil Anderson and Bob Schrieffer had led us to expect something interesting at, perhaps, $\nu = \frac{1}{2}$. We found nothing there, but what we did find, at $\nu = \frac{1}{3}$, was a complete surprise." (See the figure on page 19.)

As Tsui and Stormer lowered the temperature on their tiny GaAs/AlGaAs heterostructure (5 mm long and 0.4 mm wide), the expected integral quantum Hall plateaus began to appear at about 2 K. (In this experiment the magnetic field was varied while the interface electron density was fixed at 1.2×10^{11} /cm^2 by the nature of the heterostructure.) But then, when the temperature was reduced below 0.5 K, the experimenters saw something quite new in the vicinity of 15 tesla: an emergent plateau around $\nu = \frac{1}{3}$, with the corresponding Hall resistance $3h/e^2$.

The Hall resistance of the $\frac{1}{3}$ plateau, and of other fractional plateaus states discovered in the next few years, would prove to obey $R_{\rm H} = h/\nu e^2$ with precisions comparable to those of the integral quantum Hall plateaus. The FQHE also shares with the integral effect the appearance of prominent dips in the ordinary longitudinal resistance at the appropriate filling factors. (See the bottom panel of the figure.)

A new quantum liquid

"The data immediately suggested to me objects of charge $\frac{1}{3}e$," recalls Laughlin, "but I took a wrong first step, following too literally the ideas of Schrieffer, [Wu-pei] Su and [Roman] Jackiw. Thank God the Phys. Rev. Letters referee rejected this first effort, pointing out why it couldn't be right. Only after the Nobel Prize was announced did I learn that this providential referee had been Steve Kivelson [now at UCLA]. I could kiss him."

Soon thereafter, however, Laughlin was on the right track. Generalizing to a putative fractional quantum Hall ground state with filling factor $\nu = \frac{1}{m}$, he concocted a variational multiparticle wavefunction of the form

$$\psi \propto \prod_{j<k} (z_j - z_k)^m \times \exp\left(-\tfrac{1}{4} \sum_l z_l z_l^*\right).$$

The product is over all pairs of electrons in the interface, which is treated mathematically as the complex plane, z_j being the complex coordinate of the jth electron. If, therefore, an electron executes a complete loop around its neighbor, the wavefunction's phase changes by $2\pi m$. Electrons being fermions, it follows that m must be an odd integer. Indeed, it would be another five years before Stormer, Tsui and Gossard, working with Robert Willet and James Eisenstein, unearthed a fractional quantum Hall state with an *even* denominator.[3] (See PHYSICS TODAY, January 1988, page 17.) "That one, with filling factor $\frac{5}{2}$, is a puzzle we're still trying to crack," says Stormer. (For another ongoing problem, see the story on page 22.)

In Laughlin's wavefunction, the position of every electron represents a node of order m for every other electron, due essentially to Coulomb repulsion. This strict standoffishness results in an incompressible quantum liquid with the Coulomb energy minimized. Each node is associated with m flux quanta. The liquid is incompressible in the sense that adding just one more electron would generate m quasiparticle excitations of fractional charge e/m, at the cost of an energy gap.

"It was from the plasma-physics literature that I learned that my wavefunction described a liquid rather than a crystalline state," Laughlin told us. "After Bell Labs let me go in 1981, I was hired by Livermore to do plasma modeling. Until my security clearance came through, I was relegated to an annex called 'the cooler,' where I had the leisure to think about the puzzling new data of Tsui and company. But

Livermore plasma theorists Hugh DeWitt and Forest Rogers pointed me to analogous problems in the plasma literature whose solutions turned out to be crucial to figuring out what my variational wavefunction meant. That enabled me to convince myself that it really did describe the $\nu = 1/3$ data."

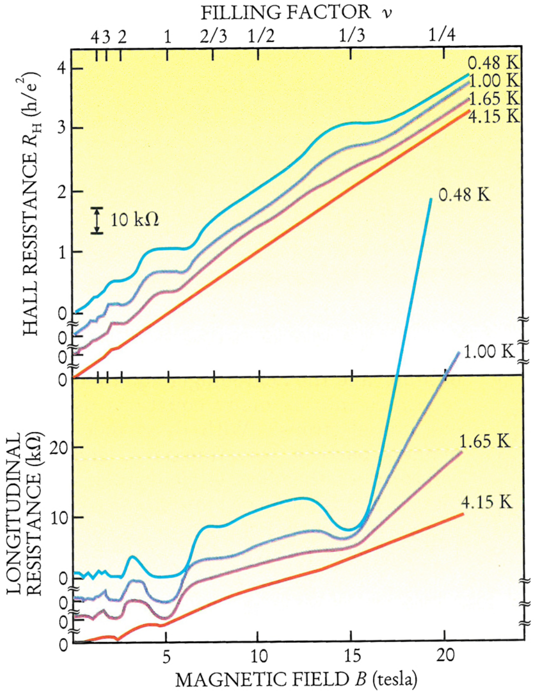
In 1984, Bertrand Halperin (Harvard) pointed out that the fractionally charged quasiparticles were neither fermions nor bosons, but "anyons." (See PHYSICS TODAY, November 1984, page 17.) Just last year, the reality of the charge $1/3\,e$ quasiparticle was vividly demonstrated in two experiments that measured Hall-current shot noise. (See PHYSICS TODAY, November 1997, page 17.)

Other fractional states

In addition to the $1/m$ states for m odd, the Laughlin wavefunction also describes, without further ado, quantum Hall states with filling factors $1 - 1/m$. But experimenters soon found a proliferation of other odd-denominator FQHE states that sent the theorists scrambling. Among other things, theorists posited the filling of Landau levels with the fractionally charged quasiparticles or with "composite fermions."

In 1989, Jainendra Jain (SUNY, Stony Brook) introduced the notion of the composite fermion—an electron with two flux quanta firmly attached. He showed that a prominent but problematic class of FQHE plateaus could be regarded as *integral* quantum Hall states of such composite objects. Three years later, Halperin, Patrick Lee and Nicholas Read proposed that the very interesting state at $\nu = 1/2$, though it has no quantum Hall plateau, could be understood as a filled Fermi sea of composite fermions seeing no magnetic field—because they carry all of it on their backs. The effective mass of the composite fermions, they argued, would determine the sizes of the FQHE energy gaps. (See PHYSICS TODAY, July 1993, page 17.)

So Stormer, Tsui and Rui Du set out to weigh the composite fermions by measuring the energy gaps. Using specimens of exceptionally high quality grown at Bell Labs by Loren Pfeiffer and Ken West, they found that the effective mass of the composite fermion is an order of magnitude higher than that of an ordinary electron in GaAs.[4] "They're no more fictitious than Cooper pairs," Stormer argues, "and they give us a nice intuitive way of looking at the fractional states."



DISCOVERY OF THE FIRST fractional quantum Hall state in 1982 was signaled (top panel) by the unexpected appearance, at very low temperature, of a plateau around filling factor $\nu = 1/3$ in the growth of Hall resistance with increasing magnetic field normal to the GaAs/AlGaAs interface in which the two-dimensional current is flowing. The sequence of lower-lying plateaus indicates the already known *integral* quantum Hall states. The plateaus are accompanied (lower panel) by dips in the ordinary longitudinal resistance. (Adapted from ref. 1.)

Then and now

Laughlin's principal interests lie elsewhere. "For a long time now, I've been thinking about how the fundamental fermions of particle physics and their gauge interactions could have arisen out of the cosmological vacuum," he told us. "And I see the fractional quantum Hall effect as a deep and important precedent for our guidance. Its fractionally charged excitations are, I believe, related to the fractionally charged quarks of the standard model of particle physics."

Laughlin, born in 1950, is a California native. He joined the Stanford physics faculty in 1984. He got his PhD at MIT in 1979, working under John Joannopoulos on the quantum theory of glasses.

Tsui was born in China's Henan province in 1939, but he grew up in Hong Kong. He came to the US in 1958 to attend Augustana College, a Lutheran institution in Rock Island, Illinois. (At this point in our conversation, Tsui broke into a few bars of "Oh! The Rock Island Line is a mighty good road, . . .") His 1967 PhD thesis, under Royal Stark at the University of Chicago, was on the experimental study of the Fermi surfaces and band structure of ferromagnetic metals. A year later he went to Bell Labs. In 1982, he became a professor of electrical engineering at Princeton.

"For me," Tsui told us, "the important thing is the interplay between disorder and electron–electron interactions. The FQHE is, in some sense, the clean limit. But there's another limit, where both interaction and disorder are important. That's when the sequence of quantum Hall states terminates in an insulator state as the filling factor becomes ever smaller. If it were perfectly clean, you'd have a Wigner crystal, but there's always some disorder. Such an insulator might exhibit interesting new physics. That's something I'd like to spend a little more effort on."

Stormer was born in Frankfurt am Main in 1949. As a graduate student of Hans Queisser at the University of Stuttgart, he did his thesis research, on electron–hole droplets, at Grenoble's High-Magnetic-Field Laboratory and received his doctorate in 1977. Stormer became a professor of physics at Columbia at the beginning of this year, but he remains a part-time adjunct director at Bell Labs.

"When people ask me what the fractional quantum Hall effect is good for," Stormer told us, "I say, 'No, it won't revolutionize telecommunications.' I may, in deference to Bob Laughlin, mumble something about non-Abelian gauge fields, but I'm not really sure what that means. Our modulation-doping work, however, has led to the development of the high-electron-mobility transistors (HEMTs) widely used nowadays in cellular telephony. I'm as proud of that as I am of our discovery of the fractional quantum Hall effect."

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