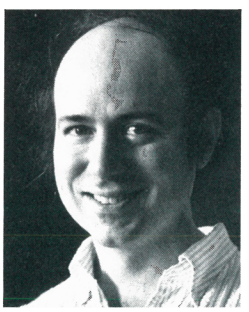
Why are there Analogies between Condensed Matter and Particle Theory?

Frank Wilczek

The idea that the microcosm somehow reflects or embodies the macrocosm is deeply appealing to the human imagination, and is prominent in prescientific and mystical thinking. In fact, there once appeared to be an overwhelming argument for such a connection, often quoted in alchemical texts: One could not conceive how objects as complicated and structured as plants and animals are known to be could issue from tiny seeds, except by growth from miniature templates; and the homunculus would necessarily contain the seeds of future generations, even smaller . . . . This argument may strike us as naive, but let us remember that the elements of a true molecular explanation of genetic encoding, deciphering and development are only just now emerging, and they are no less amazing and inspiring! In any case, we can still readily sympathize with William Blake's longing "To see a World in a Grain of Sand / And a Heaven in a Wild Flower, / Hold Infinity in the palm of your hand / And Eternity in an hour."

In classical physics, it is a remarkable fact that the form of the laws for large and small bodies is essentially the same. Newton went to great pains, and according to legend delayed for many years publishing what became the central results of the *Principia*, to prove the theorem that the gravitational force exerted by a spherically symmetric body is the same as that due to an ideal point of equal total mass at the body's center. This theorem provides quite a rigorous and precise example of how macroscopic bodies can be replaced by microscopic ones, without altering the consequent behavior. More generally, we find that nowhere in the equations of classical mechanics is there any quantity that fixes a definite scale of distance. The same is true of classical, Maxwellian electrodynamics. In this sense, classical physics embodies a perfect match between the microscopic and the macroscopic.

For this very reason, however, classical physics cannot account for salient



features of the actual world—specifically, the existence of atoms with definite sizes and properties.

The quantum revolution, as we know, changed all that. It is interesting that the reason for this change has often been misstated, or at least stated confusingly, starting with Max Planck himself. Planck was fascinated with the idea that, by combining his new constant h with the speed of light c and the gravitational constant G, one could form a definite length scale, $(Gh/c^3)^{1/2}$. This is indeed a remarkable length: the Planck length. It evaluates to about 10^{-35} m, and is thought to be the scale below which the effects of quantum gravity become significant. It has, however, nothing directly to do with the size of atoms, and thus far its role in physics has been more inspirational than constructive. For practical purposes the crucial length is not the Planck length, but rather the Compton wavelength h/mc, which one can construct using the definite (quantized) value of the electron mass. Also crucial is the quantized unit charge e, used to construct the dimensionless fine structure constant.

With the emergence of a fundamental length scale whose influence permeates every aspect of physical behavior, one might have anticipated that the theory of matter at larger scales (solid-state, or condensed matter, physics) and of matter at smaller scales (elementary particle, or high-energy, physics)—of macrocosm and microcosm—would irrevocably diverge. It is a profound, and at first sight astonishing, fact that this did not happen. One finds, instead, startling and far-reaching resemblances between phenomena at very different scales of time and distance, occurring in systems as different superficially as the electromagnetic ether and a crystal of diamond, or empty space and the inside of a metal, or the deep interior of a proton and a magnet near its Curie temperature.

Consider first the earliest history of quantum mechanics itself. Planck was led to discover his constant, which became supreme in the microworld, by analyzing an essentially macroscopic phenomenon: the behavior of the electromagnetic field at finite temperature (blackbody radiation). Planck's early use of his constant, however, was quite limited. He first introduced it as a parameter in an interpolation formula to fit the experimental results of Heinrich Rubens and Ferdinand Kurlbaum. He soon made a model for how their radiation spectrum could be achieved; in this model, the exchange of energy between atoms and radiation occurs only in discrete units proportional to h. Einstein, in work of almost supernatural genius, made analogies between Planck's formula and the corresponding formulas for gases of particles, and he insisted that the energy in radiation was not merely exchanged, but also propagated, in discrete units. In this way, the physical phenomenon underlying Planck's formula was stated in a universal fashion, independent of a detailed model of atoms: It was the existence of a new kind of elementary particle, the light-quantum, or photon. (Although this was the first step, a fully satisfactory derivation of Planck's formula required additional ideas, specifically stimulated emission and Bose statistics, and was not achieved until almost 20 years later.) Thus, Einstein was the first to predict the existence of a new elementary particle.

His next step was almost equally remarkable, and wonderfully illustrates my theme. Einstein applied Planck's formula, which we could say describes the vibrations of the electromagnetic ether at finite temperature, to the analogous problem of the vibrations of a crystal. He found that it fit data on the specific heat of diamond at low temperature very well. The underlying physical phenomenon, of course, is that the vibrations are created and transmitted in discrete units:

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phonons. It was the beginning of the quasiparticle concept that came to dominate much of condensed matter physics. For crystals, the immediate consequence was that one could not have high-frequency vibrations of very small amplitude. Their absence suppresses the vibrational specific heat of diamond at low temperatures, just as it removes the threatened ultraviolet catastrophe in the photon specific heat for blackbody radiation.

Another analogy between elementary particle and condensed matter phenomena straddled the birth of the new quantum mechanics. Just before this, in 1923, Wolfgang Pauli, by analysis of spectroscopic data, was led to propose his exclusion principle: two electrons cannot occupy the same quantum state. He immediately applied this idea to explain the paramagnetism of metals. Subsequently, several physicists responded brilliantly to the challenge of working these schematic ideas into the modern theory of solids, especially by developing the band concept.

So far, this progress reads like a standard reductionist triumph—macroscopic behaviors were "reduced" to microscopic laws. While it was occurring, however, there was a remarkable, unexpected reverberation of these ideas back toward microphysics. When Paul Dirac developed his relativistic wave equation for the electron, he found a host of unphysical, negative energy solutions. Inspired by the exclusion principle and its successful applications, he proposed that, in apparently empty space, the negative energy states were in fact occupied. Excitations above this state could make "holes" in the Dirac sea, similar to the electron deficits that were an important part of chemical valence theory and became the holes of band theory. Today, of course, positrons and other antiparticles are part of the bread and butter of elementary particle physics, and holes are central players in solid-state electronics.

Finally, a more recent example: Starting in the late 1960s, Kenneth Wilson developed conceptual and mathematical tools for describing the self-similar behavior that occurs near second-order phase transitions.

Superficially, it may seem that nothing could be further from the problems of elementary particle physics. Yet this very set of ideas—the modern renormalization group—when applied to understanding the observed self-similar short-distance behavior of hadronic currents, led directly to the discovery and validation of the modern theory of the strong interaction, QCD.

I hope that you find these examples of the flow of ideas across boundaries of scale and substrate impressive; others will appear in subsequent Reference Frame columns. Clearly, I have described instances of what Eugene Wigner called "the unreasonable success of mathematics." Why should such things occur?

If one is looking for a rational explanation, one must first recognize that it is certainly not logically necessary for there to be any deep resemblance between the laws of a macroworld and those of the microworld that produces it. For example, the rules governing "Super Mario World," or any computer game world involving magical transformations and non-Newtonian jumping abilities, have very little in common with the rules governing the microworld of semiconductor electronics (or ultimately elementary particles) that generates it.

To make such a flow of ideas possible, the laws must have some special properties. What are these properties? An important clue is that they must be upwardly heritable. (There does not seem to be a standard phrase for this important concept; it deserves one.) That is, we require microscopic laws that, when consistently applied to large bodies, retain their character. And indeed, the most basic conceptual principles governing physics as we know it—the principle of locality and the principle of symmetry—are upwardly heritable. If the influence of elementary units is limited in time and space, this will also be true of assemblies of such units; if there is symmetry in the action of elementary units, there will also be symmetry in the action of assemblies (provided, of course, that the assemblies are themselves put together symmetrically).

The fact that these upwardly heritable principles are so powerful goes a long way toward explaining, *a posteriori*, why a flow of ideas from the microworld to the macroworld is possible. An additional feature helps explain the reverse flow. In the modern theory of elementary particles, we learn that empty space—the vacuum—is in reality a richly structured, though highly symmetrical, medium. Dirac's sea was an early indication of this feature, which is deeply embedded in quantum field theory and the Standard Model. Because the vacuum is a complicated material governed by locality and symmetry, one can learn how to analyze it by studying other such materials—that is, condensed matter. I believe that the upwardly heritable principles of locality and symmetry, together with the quasimaterial nature of apparently empty space, together underlie most and possibly all of the remarkable modern analogies between our theories of microcosmos and macrocosmos. ■