

LETTERS (continued from page 15)

quent speaker.' One should rather be able to say: 'He is a gentleman!'" Eugene Wigner was a gentleman in every sense of the word.

PAUL ROMAN

Ludenhansen, Germany

I would like to comment on Eugene Wigner's definition of a particle, which identifies an elementary particle as a unitary irreducible representation (UIR) of the Poincaré group, as discussed in Arthur Broyles's letter to the editor (September 1996, page 13) in response to David Gross's article on Wigner's legacy (December 1995, page 46). I believe there has been some misunderstanding about this definition that has a profound implication. I also am interested to see that Wigner's definition is equivalent to, albeit not in a literal sense, Broyles's preferred definition that "a particle is a point object that moves on a world line" (as Wigner emphatically told Broyles).

Wigner's definition not only is a correct concept but mathematically provides a unified approach to construct relativistic field equations for all integer and half-integer spins. Unfortunately, most books on field theory devote a lot of effort to providing a detailed classification of UIRs of the Poincaré group but neither give a simple physical explanation (except for referring to them as a representation that cannot be further subdivided) nor proceed to develop the relativistic field equations for all spins from the point of view based on Wigner's classification. Instead, they simply follow the historical development to introduce relativistic field equations (for example, Maxwell-Dirac equations) as if the development of UIRs were irrelevant to the construction of field equations.

Gross's article provides an accurate and a pictorial description of UIRs. However, a more down-to-the-earth explanation may be needed and would help to show that UIRs are equivalent to Broyles's preferred definition. Physically, an elementary particle is regarded as a stable, pointlike, structureless entity (structureless except for having mass, spin and other possible quantum numbers), which, in its free state, moves on a world line with momentum k .

Regardless of whether this notion of an elementary particle will continue to stand the test of time, mathematically it requires that all the solutions for the particle (which obeys the corresponding free-field equation) form an infinite dimensional solution space that is invariant under the ac-

tion of all elements of the Poincaré group, which consists of the usual Lorentz transformations, as well as translations in space and time (this is exactly what we mean by irreducibility). For example, if $\exp(ikx)$ is the solution for a spin-0 particle, it requires that the state $\exp(ik'x)$, with $k' = \Lambda k$, also be a solution. That is, there must be no other solutions forming an invariant subspace that cannot be reached by the action of the group elements. Thus, the infinite dimensional solution space $\{\exp(ikx), k^2 = m^2\}$ carries the UIR of the Poincaré group. And unitarity is required on the representation simply to preserve the norm (probability) of its inner product.

This requirement, in turn, imposes a profound restriction on the construction of relativistic field equations—that is, the equation should contain no invariant subspace. In principle, we can construct a field equation for spin- s by fixing the mass and the spin, which are the eigenvalues of two Casimir operators, P^2 and S^2 , respectively, by imposing field equations $(\square - m^2)\phi = 0$ and $[S_{op}^2 - s(s+1)]\phi = 0$, with ϕ to be some tensor or spinor of rank s that contains spin s and lower spins. Usually, ϕ is transformed according to a finite nonunitary irreducible representation $D(k,1)$ of the Lorentz group. However, Wigner's definition of an elementary particle allows us to replace the relativistic non-covariant second equation with a set of relativistic covariant subsidiary conditions, ϕ being a symmetric, traceless and divergenceless tensor or spinor of rank s . By taking away those relativistic invariant subspaces for lower spins, we guarantee that there are no more distinct invariant subspaces in the solution space of the field equations, as is confirmed by counting the number of independent components in ϕ to be $2s+1$. By further carrying out the Fierz-Pauli program to construct a single equation that contains both the main equation and the subsidiary conditions (to introduce interaction later in a consistent manner), we will have achieved our final goal of establishing a general free-field equation for arbitrary spin- s .¹ By substituting $s = 0, 1/2, 1, 3/2, 2, \dots$ in the general equation, we reproduce those familiar free-field equations: Klein-Gordon, Dirac, Maxwell, Rarita-Schwinger, Fierz-Pauli. . . .

Yet Wigner's remarkable definition is not restricted to free-field equations. It also serves as a guiding principle for introducing interactions among various fields. We can impose a mathematically consistent condition on a set of field equations to derive their mutual interactions, as well as self-interactions. This is based on the

same notion of Wigner's UIR as our requirement that the number of independent spin components $2s+1$ for each particle (or two helicity states for the massless particle) remains unchanged even in interaction. This simple requirement leads to the usual minimum coupling for spin $(1, 1/2)$ of the Maxwell-Dirac system, $SU(N)$ -type coupling for gauged spin-1 photons and Einstein's equation for the spin-2 system.²

This definition of an elementary particle is truly simple and ingenious. It leads directly to relativistic field theory, bypassing the nonrelativistic quantum mechanics. And from the relativistic field equation, we can derive the Feynman diagram rules and calculate scattering amplitudes. The probability interpretation of nonrelativistic quantum mechanics can be understood now from a different perspective by looking at every Feynman diagram as a real physical process—how an electron is scattered depends on whether it encounters a photon. Maybe Einstein was right—God does not play dice with us. Maybe it's a game of marbles instead.

References

1. See C. Fronsdal, *Phys. Rev. D* **18**, 3624 (1978) for integer spin, and J. Fang, C. Fronsdal, *Phys. Rev. D*, **18**, 3630 (1978) for half-integer spin.
2. R. M. Wald *Phys. Rev. D*, **33**, 3613 (1986), and J. Fang, W. J. Christensen Jr., M. M. Nakashima, *Lett. Math. Phys.* **38**, 213 (1996).

JOHN FANG

*California State Polytechnic University
Pomona, California*

Schawlow Thinking Disputed about Diatomic Molecules

In his very readable article "Chemistry: Blithe Sibling of Physics" (PHYSICS TODAY, April, page 11), Dudley Herschbach quotes the well-known remark attributed to Art Schawlow, "A diatomic molecule has one atom too many."

My love affair with H_3^+ over many years,¹ has led me to believe that some beautiful subtleties of physics do not appear until one faces a three-particle system. I suspect quantum chromodynamicists agree with this.

As for me, I say that a diatomic molecule has one atom too few.

Reference

1. T. Oka, *Rev. Mod. Phys.* **64**, 1141 (1992).

TAKESHI OKA

*(t-oka@uchicago.edu)
University of Chicago
Chicago, Illinois ■*