Remembering Wigner's Contributions—and Tip Tiff with NYC Cabbie

The scientific thinking, philosophical ideas and personal kindness of Eugene P. Wigner played such a decisive role all through the creative and active phase of my life that I would like to comment on the Wigner-related letters from James B. Lee and Arthur A. Broyles that appeared in your September 1996 issue (page 11).

As one who enjoys classical music just as much as mathematical physics (after all, both have their spring in divine grace), I was touched by Lee's enthusiastic report on how the "Wigner distribution" has helped him in trying to improve concert hall acoustics. Lee's account illustrates how an abstract mathematical structure envisaged and developed in a particular area can become useful in a completely different field of study. The roots of this miracle lie in the fact that mathematics searches, not for computational results, but for general structures of inspired thought that underlie people's creative thinking.

Lee is also emotionally impressed by the truly surprising observation that "some of the stuff we do actually works." Indeed, that was also one of the "miracles" that, besides the role of mathematics, Wigner contemplated. He talked about the two related miracles: that nature has any laws at all, and that the human mind can grasp (to some extent) these laws. I agree with both Wigner and Lee that these are truly mysterious facts of life.

However, I must politely disagree with Lee's assertion that "in the long view [Wigner] may be recognized more for the mathematics of time frequency distributions than for any of his other contributions." Given Wigner's incredible breadth of work in pure and applied mathematics, in basic quantum theory, in atomic, molecular and nuclear physics, in fields and particles, in reactor engineering, in civil defense and in epistemology, I think it is clear that no one of his great insights has priority over the others.

Broyles is unhappy about a specific point in David Gross's article on Wigner in your December 1995 issue concerning Wigner's definition of what we mean by an "elementary particle" (that is, an irreducible continuous unitary representation of the Poincaré group). Broyles believes that Wigner later changed his view because, on one occasion, Wigner told him emphatically that a particle is a point object that moves on a world line. Broyles does not reveal the context in which Wigner made that statement, but it is surely obvious that a particle's (and not only an "elementary" particle's) motion is described properly by its world line. (Devotees of string theory may differ here too, and prefer to talk of a world tube.) However, the intrinsic properties of an (elementary) object are not described at all by its world line but rather by its spacetime and "internal" quantum numbers. Of course, Wigner's 1939 statement in which he referred to the Poincaré group as the basic systematizing agency should be extended nowadays to some as-yet-unknown wider group (or product of groups). Wigner would be the first to agree with such a generalization; nevertheless, from the heuristic point of view, the group representation approach is still valid.

Broyles correctly points out that the best framework for discussing "particles" is quantum field theory. Wigner, in fact, was one of the first proponents of this view (see his 1930s works on quantization of Fermi–Dirac fields, or the much later Bargman–Wigner field equations). Here one must realize that what we perceive as (asymptotically free) "elementary particles" are in fact quanta of relevant fields and, as such, are characterized by quantum numbers that are associated with the group theoretical representations that the fields carry. Thus, we are back to Wigner's first insights on this question. Nevertheless, it is true that "particle theory" is a somewhat outdated and obscure term and, though less specific, "high-energy physics" is more adequate. Still, there are other approaches to "elementary particles," like C^* or von Neumann algebras (in which, long ago, Wigner also had a hand) that should be recognized by practitioners in the field.

Like Broyles, I would like to conclude with a reminiscence about Wigner. In the 1970s, an anecdote about Wigner was making the rounds to the effect that, on getting into an argument about a tip with a New York City cab driver, Wigner lost his patience, stamped his foot and said, "Oh, go to hell, *please!*" In 1981, I had an opportunity to ask Wigner if there was any truth in the story. He got rather agitated and answered (in Hungarian, our common native language), "Of course the story is true. But Paul, you must understand: That driver was *very impolite!*"

Three centuries ago the French scientist and philosopher Blaise Pascal wrote: "One should not be able to say: 'He [this distinguished person] is a mathematician, a preacher, an elo-

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quent speaker.' One should rather be able to say: 'He is a gentleman!' " Eugene Wigner was a gentleman in every sense of the word.

PAUL ROMAN
Ludenhausen, Germany

I would like to comment on Eugene Wigner's definition of a particle, which identifies an elementary particle as a unitary irreducible representation (UIR) of the Poincaré group, as discussed in Arthur Broyles's letter to the editor (September 1996, page 13) in response to David Gross's article on Wigner's legacy (December 1995, page 46). I believe there has been some misunderstanding about this definition that has a profound implication. I also am interested to see that Wigner's definition is equivalent to, albeit not in a literal sense, Broyles's preferred definition that "a particle is a point object that moves on a world line" (as Wigner emphatically told Broyles).

Wigner's definition not only is a correct concept but mathematically provides a unified approach to construct relativistic field equations for all integer and half-integer spins. Unfortunately, most books on field theory devote a lot of effort to providing a detailed classification of UIRs of the Poincaré group but neither give a simple physical explanation (except for referring to them as a representation that cannot be further subdivided) nor proceed to develop the relativistic field equations for all spins from the point of view based on Wigner's classification. Instead, they simply follow the historical development to introduce relativistic field equations (for example, Maxwell–Dirac equations) as if the development of UIRs were irrelevant to the construction of field equations.

Gross's article provides an accurate and a pictorial description of UIRs. However, a more down-to-the-earth explanation may be needed and would help to show that UIRs are equivalent to Broyles's preferred definition. Physically, an elementary particle is regarded as a stable, pointlike, structureless entity (structureless except for having mass, spin and other possible quantum numbers), which, in its free state, moves on a world line with momentum k.

Regardless of whether this notion of an elementary particle will continue to stand the test of time, mathematically it requires that all the solutions for the particle (which obeys the corresponding free-field equation) form an infinite dimensional solution space that is invariant under the action of all elements of the Poincaré group, which consists of the usual Lorentz transformations, as well as translations in space and time (this is exactly what we mean by irreducibility). For example, if $\exp(ikx)$ is the solution for a spin-0 particle, it requires that the state $\exp(ik'x)$, with $k' = \Lambda k$, also be a solution. That is, there must be no other solutions forming an invariant subspace that cannot be reached by the action of the group elements. Thus, the infinite dimensional solution space $\{\exp(ikx), k^2 = m^2\}$ carries the UIR of the Poincaré group. And unitarity is required on the representation simply to preserve the norm (probability) of its inner product.

This requirement, in turn, imposes a profound restriction on the construction of relativistic field equations—that is, the equation should contain no invariant subspace. In principle, we can construct a field equation for spin-s by fixing the mass and the spin, which are the eigenvalues of two Casimir operators, P^2 and S^2, respectively, by imposing field equations $(\Box - m^2)\phi = 0$ and $[S^2_{op} - s(s+1)]\phi = 0$, with ϕ to be some tensor or spinor of rank s that contains spin s and lower spins. Usually, ϕ is transformed according to a finite nonunitary irreducible representation $D(k,1)$ of the Lorentz group. However, Wigner's definition of an elementary particle allows us to replace the relativistic noncovariant second equation with a set of relativistic covariant subsidiary conditions, ϕ being a symmetric, traceless and divergenceless tensor or spinor of rank s. By taking away those relativistic invariant subspaces for lower spins, we guarantee that there are no more distinct invariant subspaces in the solution space of the field equations, as is confirmed by counting the number of independent components in ϕ to be $2s + 1$. By further carrying out the Fierz–Pauli program to construct a single equation that contains both the main equation and the subsidiary conditions (to introduce interaction later in a consistent manner), we will have achieved our final goal of establishing a general free-field equation for arbitrary spin-s.[1] By substituting $s = 0, 1/2, 1, 3/2, 2, \ldots$ in the general equation, we reproduce those familiar free-field equations: Klein–Gordon, Dirac, Maxwell, Rarita–Schwinger, Fierz–Pauli. . . .

Yet Wigner's remarkable definition is not restricted to free-field equations. It also serves as a guiding principle for introducing interactions among various fields. We can impose a mathematically consistent condition on a set of field equations to derive their mutual interactions, as well as self-interactions. This is based on the same notion of Wigner's UIR as our requirement that the number of independent spin components $2s + 1$ for each particle (or two helicity states for the massless particle) remains unchanged even in interaction. This simple requirement leads to the usual minimum coupling for spin (1, 1/2) of the Maxwell–Dirac system, SU(N)-type coupling for gauged spin-l photons and Einstein's equation for the spin-2 system.[2]

This definition of an elementary particle is truly simple and ingenious. It leads directly to relativistic field theory, bypassing the nonrelativistic quantum mechanics. And from the relativistic field equation, we can derive the Feynman diagram rules and calculate scattering amplitudes. The probability interpretation of nonrelativistic quantum mechanics can be understood now from a different perspective by looking at every Feynman diagram as a real physical process—how an electron is scattered depends on whether it encounters a photon. Maybe Einstein was right—God does not play dice with us. Maybe it's a game of marbles instead.

References

1. See C. Fronsdal, Phys. Rev. D **18**, 3624 (1978) for integer spin, and J. Fang, C. Fronsdal, Phys. Rev. D, **18**, 3630 (1978) for half-integer spin.
2. R. M. Wald Phys. Rev. D, **33**, 3613 (1986), and J. Fang, W. J. Christensen Jr, M. M. Nakashima, Lett. Math. Phys. **38**, 213 (1996).

JOHN FANG
California State Polytechnic University
Pomona, California

Schawlow Thinking Disputed about Diatomic Molecules

In his very readable article "Chemistry: Blithe Sibling of Physics" (PHYSICS TODAY, April, page 11), Dudley Herschbach quotes the well-known remark attributed to Art Schawlow, "A diatomic molecule has one atom too many."

My love affair with H_3+ over many years,[1] has led me to believe that some beautiful subtleties of physics do not appear until one faces a three-particle system. I suspect quantum chromodynamicists agree with this.

As for me, I say that a diatomic molecule has one atom too *few*.

Reference

1. T. Oka, Rev. Mod. Phys. **64**, 1141 (1992).

TAKESHI OKA
(t-oka@uchicago.edu)
University of Chicago
Chicago, Illinois ■