In a Quantum Hall System, Is the Insulator Really a Conductor in Vortex Clothing?

Dan Shahar, Daniel Tsui and Mansour Shayegan at Princeton University were studying the liquid-to-insulator transitions in a quantum Hall system recently when they noticed a surprising symmetry in the plot of longitudinal voltage versus current: For each point close to the phase transition in the liquid phase, they could find a corresponding point in the insulating phase such that the (normalized) *I–V* curve for one fell precisely on the *V–I* curve of the other—that is, the curve with the voltage and current coordinates switched. Such behavior suggests that the underlying dynamics of the liquid and insulator phases must be intimately related, despite their overtly different transport characteristics.

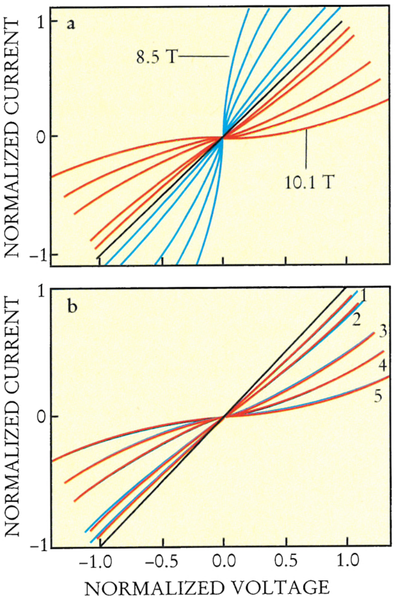
Puzzled by what they were seeing, the experimenters enlisted the help of theorists Shivaji Sondhi, now at Princeton, and Efrat Shimshoni of the University of Illinois at Urbana-Champaign. In their subsequent analysis,[1] the five researchers tentatively concluded that the experiment might be evidence for the existence of a duality transformation relating the insulating and quantum Hall phases. That duality would relate the conductor, when viewed in terms of certain charged particles, to the insulator, viewed in terms of vortices. But further studies are required to confirm this hypothesis, and to date, the additional data have only added to the intrigue.

> Experimenters know something fascinating is going on when the *I–V* curve for a conducting phase—a property of the dynamics—falls right on the *V–I* curve for an insulating phase.

Duality

Interest in the experiment was sparked just by the possibility of seeing experimental signs of duality, which has been primarily a theoretical tool. First used in statistical mechanics, the technique has spread to other fields, including superstring theory. Theorists use duality to transform a problem expressed in one set of variables into an alternate formulation in terms of other variables. In rare cases, a system may be self-dual—that is, the transformation of variables may map the problem back onto itself. One example of self-duality is the transformation in electromagnetism that exchanges the roles of electric and magnetic fields and of electrical charges and magnetic monopoles.

The formulation in the dual variables can be more tractable or more illuminating than the original problem. Hendrik Kramers and Gregory Wannier introduced a duality transformation in 1941 and showed that the Ising model of a two-dimensional array of interacting magnetic spins is self-dual: The properties of the system at high temperatures mapped onto those of the dual system at the low temperatures. Because the system was self-dual, the critical temperature had to be the same in each; the resulting equality enabled Kramers and Wannier to calculate the critical temperature before Lars Onsager solved the problem exactly.



CURRENT VS. LONGITUDINAL VOLTAGE V_L in a quantum Hall system. **a.** I–V_L curves at 11 different values of magnetic field, from 8.5 to 10.1 tesla. The transition from the $\nu = 1/3$ quantum Hall liquid (blue curves) to the insulator (red curves) occurs at 9.1 T (black line). **b.** Same data but with *I*, *V* coordinates reversed for points in the quantum Hall state. Corresponding curves are labeled by pair number. (Adapted from ref. 1.)

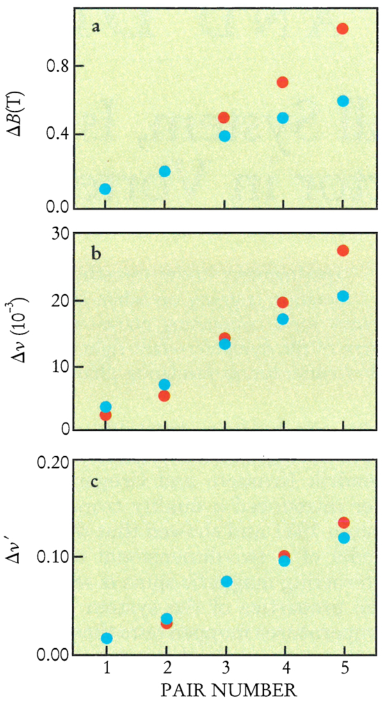
In the case of the quantum Hall transition, the Princeton–Illinois group has proposed that their experiments provide evidence for self-duality, in which one exchanges the roles of charges and magnetic vortices. The charges they have in mind are not the simple electrons of the physical system but hypothetical bosonic composite particles consisting of an electron associated with an odd number of magnetic flux quanta. Perhaps, say Shahar and his colleagues, the quantum Hall liquid described in terms of the bosons maps precisely onto an insulating state, as described in terms of vortices of the boson field. Such a transformation is analogous to one for two-dimensional superconducting films:[2] When the charged bosons (the electron pairs) are in a superconducting state, the associated vortices are in an insulating state, and vice versa. A number of experiments on transitions in superconducting thin films have been interpreted in terms of duality. And in 1992, a group from the University of Delft noticed a symmetry in the *I–V* curves for an array of Josephson junctions.[3]

The evidence

The recent Princeton experiment was performed on a quantum Hall system—that is, on a two-dimensional array of electrons formed at the interface between two semiconductors and subjected to a strong magnetic field. In such systems the Hall resistance—the resistance measured perpendicular to the applied current—is quantized, having values of $h/\nu e^2$, where ν can be an integer or a fraction. The factor ν is the so-called filling factor, or the ratio of the number of electrons to the number of magnetic flux lines.

The Princeton experimenters set out to study the transition from a $\nu = 1/3$ quantum Hall state, which has a finite Hall conductivity, to an insulating state with zero conductivity. They measured the curves of the current I versus the longitudinal voltage V_L at values of the magnetic field close to the transition point (both variables were normalized so that the slope of the curve is one at the critical point). They noticed that the I–V_L curves (blue curves in the upper panel of the figure on page 17) for magnetic field values just below the transition had similar shapes to the curves just above the transition (red). One could imagine reflecting each of the blue curves—for example, the one at 8.5 teslas—through the straight line (which corresponds to the exact transition point) onto one of the red curves—such as the curve at 10.1 T. To further explore this apparent symmetry, the researchers reversed the I and V_L coordinates for the curves in the liquid phase and plotted them on the same I–V_L graph as the curves for the insulator phase. The curves mapped closely onto one another in pairs (see the lower half of the figure, where each pair is identified by a number).

In the figure on page 17, each curve is a trace taken at a given value of the magnetic field. One might have expected the mapping to relate points that were equidistant from the transition point, in terms of the difference ΔB in the magnetic field, but that was not the case. (See the plots of ΔB versus pair number in the uppermost part of the figure on this page.) Nor were the pairs the same distance from the transition point when measured in terms of the deviation, $\Delta\nu$, of their filling factors from the exact value at the transition, as seen in the middle part of the figure. (The filling factor is inversely proportional to the magnetic field.) But the Princeton–Illinois team found a closer fit (see the bottom panel) when they replotted in terms of an effective filling factor, which flips the ratio of bosons to vortices in the two phases. The success of this last change of variables implies that the symmetry is an underlying property of the composite particle picture, not of the electron system itself.



Distance from the transition point of points in a quantum Hall conductor or insulator measured in various ways. **a.** Distance in magnetic field. Pair numbers are those assigned in the figure on page 17. Blue (red) points are in the quantum Hall conducting (insulating) phase. **b.** Distance in filling factor. **c.** Distance in an effective filling factor that takes into account the lower field seen by composite particles. (Adapted from ref. 1.)

Composite particle pictures

There are two alternate views of the quantum Hall state in terms of composite particles, depending on whether the electron is paired with an even number of flux quanta (to make composite fermions) or with an odd number (composite bosons). In a 1989 paper, Jainendra Jain of the State University of New York at Stony Brook used the composite fermion picture to treat the integer and fractional quantum Hall effects on the same footing.[4] (See PHYSICS TODAY, July 1993, page 17.) In particular, the $\nu = \frac{1}{3}$ to insulator transition of electrons is equivalent to the $\nu = 1$ to insulator transition of composite fermions.

An alternative to the composite fermion picture is the composite boson picture, developed[5] in 1989 by Shou-Cheng Zhang, now at Stanford University, Hans Hansson, now at the University of Stockholm, and Steven Kivelson, now at the University of California, Los Angeles, based on earlier work by Steven Girvin and Allan MacDonald of Indiana University. Nicholas Read at Yale independently developed a similar treatment. In this picture, at the filling factors exactly corresponding to the plateaus in the Hall resistivity, the flux tubes exactly cancel the applied field, so that the composite bosons act as if they are in a zero-field environment. It is well known that at very low temperatures in the absence of a magnetic field, bosons will condense into a superconducting state. In the composite boson picture, then, the quantum Hall effect is manifested as superconductivity. The superconducting phase of the composite bosons is, in turn, related to an insulating phase, when written in terms of vorticies, as discussed in 1992 by Kivelson, Dung-Hai Lee (University of Calfornia, Berkeley) and Zhang, based on earlier work by Lee and Matthew Fisher (University of California, Santa Barbara).[6,7]

The Princeton–Illinois group suggested that the symmetry they see in the I–V_L curves is a manifestation of a charge–vortex self-duality in the composite boson picture. In other words, going from a quantum Hall liquid to an insulator is governed by the transition of the composite bosons from a superconducting to an insulating state. The group also notes that the data could be explained in terms of a particle–hole symmetry.

The charge–flux duality of composite bosons is equivalent to a particle–hole symmetry in terms of composite fermions, which relates the effective filling factor, ν, of composite fermions to the effective filling factor for the holes, $\nu' = 1 - \nu$. Jain notes that the particle–hole symmetry is not exact for composite fermions. Nevertheless, he says, the remarkably good experimental evidence for this symmetry must indicate that the residual interaction between composite fermions is weak.

It's fascinating to think that these gauge-field constructs may be revealing themselves through experiment in even stronger ways than the theorists expected. Kivelson remarks that he and his colleagues noticed that the vortex and composite-boson descriptions were very similar, but they never expected them to be self-dual away from the critical point.

Subsequent experiments

The five coauthors of the paper that reported the symmetry in the I–V curves have since conducted further studies of the transition, joined by another colleague, John Cunningham, now at Bell Laboratories, Lucent Technologies. These researchers have used three samples to measure the longitudinal resistivity near the origin of the I–V curves, where the curves are linear. Despite the different nature of the transitions they studied, the resistivity measured in the quantum Hall liquid was precisely the inverse of the resistivity measured at the corresponding filling factor in the insulating phase. That much is consistent with the symmetry observed in the earlier experiment. But Shahar said he and his coworkers were surprised by the wide range over which this symmetry extended—over more than a factor of 350 in the resistivity, which is wider than the scaling regime around the transition for these samples. The data thus raise the question of whether the symmetry is really related to the critical phenomenon, or whether some other mechanism is at work.

Working with a very different system, Dmitri Simonian, Sergey Kravchenko and Myriam Sarachik of the

City College of the City University of New York have recently found a symmetry between the resistivities measured near the transition between two phases,[8] which is very much like the symmetry seen by the Princeton–Illinois team. In this study, the system was a silicon metal-oxide semiconductor field-effect transistor, in the absence of a magnetic field.

At first blush, this experiment might appear to confirm the Princeton–Illinois result, but, in fact, it raises some questions about the interpretation of the experiments. First, it suggests that the symmetry, at least in the City College experiment, has nothing to do with charge–vortex duality, because of the absence of magnetic field-induced vortices. Second, it makes one wonder whether some other mechanism is at work, if a similar symmetry can arise in such apparently diverse systems.

Superuniversality

The possible appearance of duality near a phase transition in the quantum Hall state is of great interest to those who have been studying the nature of those phase transitions. There are several, ostensibly different, phase transitions: from a fractional quantum Hall state to an insulating state, from an integer quantum Hall state to an insulator, from an integer quantum Hall state to another integer quantum Hall state, and so forth. But experiments indicate that at least some of these transitions are governed by the same critical exponent and amplitudes.[9]

Several theorists have proposed the concept of "superuniversality," according to which all the quantum Hall transitions are governed by the same critical exponent.[6,10] That is, in a way, a surprising assertion, as one might expect that the different particles and different statistics governing each phase would be quite different. Kivelson feels, however, that the Princeton–Illinois results strengthen the case for superuniversality: If the particle dynamics are fairly independent of the phase of the system, as the Princeton–Illinois experiment indicates, then the phase transition can't depend very strongly on the properties of the individual phases.

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Auger Project Seeks to Study Highest Energy Cosmic Rays

Fifty-nine years ago, Pierre Auger discovered the extensive air showers generated by high-energy cosmic rays when he saw that two Geiger counters several hundred meters apart in the Alps were recording coincident counts. Five years from now, if the funding is forthcoming, a pair of 3000 km^2 air-shower detector arrays bearing his name will begin their concerted assault on the greatest remaining mystery posed by cosmic rays. The Pierre Auger Project, a collaboration of physicists and astronomers from 40 institutions in 19 countries, plans to have one array in Utah monitoring the northern sky while an identical array in the high desert of western Argentina keeps watch over the southern sky.

> We have no idea how protons can get accelerated to energies above 10^{20} eV, but they occasionally do. Existing air-shower arrays have seen less than a dozen in 35 years.

The mystery that this $100 million undertaking seeks to address is the provenance of the highest-energy cosmic rays, with energies of order 10^{20} eV. These are single-particle energies comparable to the kinetic energy of a tennis ball in flight—far beyond anything one can make in a particle accelerator. Fewer than a dozen cosmic-ray events with incident energy above 10^{20} eV have been recorded in the 35 years since the first one was discovered by John Linsley and colleagues at a modest air-shower array near Albuquerque.

The observer's principal problem is the painfully low flux at these energies. The steeply falling high-energy cosmic-ray energy spectrum yields only about one incident event near 10^{20} eV per square kilometer *per century*. Japan's Akeno Giant Air Shower Array, the largest now in existence, covers only 100 km^2. Even with its 3200 water-Čerenkov detector modules arrayed over 6000 km^2, the Auger Project is expected to harvest only about 30 events above 10^{20} eV per year in each hemisphere. The highest energy cosmic-ray event ever seen, with an incident energy of 3×10^{20} eV, was recorded in 1991 by the Fly's Eye atmospheric-fluorescence detector in Utah.

Why bother?

There are several very important reasons to wonder about cosmic-ray particles hitting the Earth's atmosphere with energies exceeding 10^{19} eV. The steep power-law descent of the high-energy cosmic-ray spectrum kinks and flattens somewhat at 10^{19} eV. (See the figure on page 20.) That feature, called "the ankle" because the spectrum beyond the "knee" at 10^{16} eV resembles a dangling leg, suggests the emergence of a population of cosmic rays from outside the Milky Way galaxy. The Galactic magnetic field is too weak to confine protons above 10^{19} eV.

But there seems to be no credible astrophysical mechanism, either inside or outside the Galaxy, for accelerating protons to energies above 10^{19} eV. "If we hadn't actually seen the 10^{20} eV events," says theorist John Bahcall (Institute for Advanced Study), "there'd be many convincing papers proving that such high energies are impossible." So one has to look for exotic new kinds of sources, perhaps even involving new particle-physics phenomena or topological spacetime defects left over from the Big Bang.

Theoretical flights of fantasy, however, are rather constrained by the fact that no charged particle traversing the cosmos can maintain an energy above 10^{20} eV for more than a few hundred million light-years. That's because 5×10^{19} eV is the threshold energy for protons to create pions in collisions with the ubiquitous 2.7-kelvin blackbody photons of the cosmic background radiation. In fact, the record 3×10^{20} eV invader seen by the Fly's Eye in 1991 could not have traveled farther than 150 million light-years, even if it began its journey with an energy of 10^{22} eV. By cosmological standards, that's still in the well-scrutinized local neighborhood.

At ordinary energies, the arrival direction of a cosmic ray tells us noth-