

FROM NEWTON'S MOON TO EINSTEIN'S MOON

Continuing the lunar orbit's 300-year role as gravity's testing ground, laser ranging to the Moon precisely confirms the foundations and structure of general relativity.

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Three centuries ago Isaac Newton and his contemporaries sought to understand the details of lunar motion in terms of Newton's theory of gravity. Today, our most precise confirmations of key aspects of Albert Einstein's relativistic theory of gravity are achieved by measuring features of the Moon's orbit to 1-centimeter precision using a technique known as lunar laser ranging (LLR). These present-day measurements address some of the same questions posed by Newton. Do Earth and the Moon (figure 1) fall toward the Sun at the same rate? What, in detail, produces the precession of the lunar orbit's major axis?^{1,2}

The lunar orbit's large size, noncircularity and inclination from the plane of Earth's orbit allow the solar tidal acceleration of the Moon (relative to Earth) to produce a rich assortment of what may be termed lunar irregularities that modify simple Keplerian motion. Three centuries ago, the possibility of finding a quantitative explanation for these irregularities by using Newton's universal law of gravity provided a challenge to and eventual triumph for the then-new dynamical theory, and in the process promoted development of powerful new perturbation techniques for the calculus.

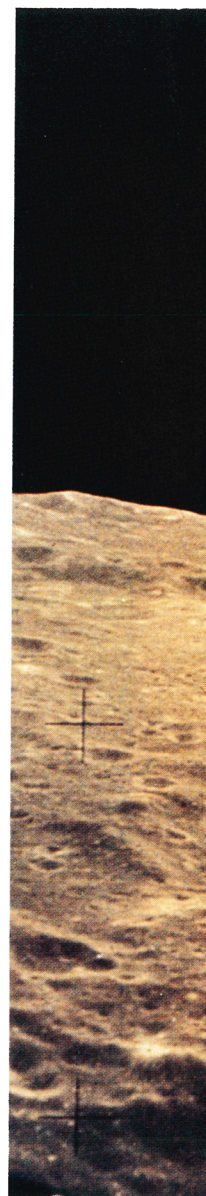
Between 1969 and 1971, astronauts of the Apollo 11, 14 and 15 missions placed passive laser reflectors on the Moon, and the Soviet Lunakhod II spacecraft positioned a fourth in 1973. Since then, round-trip travel times have been measured for laser-produced photons sent from American and French observatories. The photons are received about 2.6 seconds after they are sent to a selected lunar corner-cube reflector array, one of which is shown in figure 2. Accuracies of 2–3 centimeters are now achieved for an observing session's range measurement (called a normal point), which is obtained by averaging tens of minutes of photon returns.

The LLR facility at the McDonald Observatory in Texas sends ten 200-picosecond laser pulses per second, each 6-centimeter-long pulse containing about 10^{18} photons. Several pulses must be sent per reflected photon detected, because of the distance to the Moon, the angular

spreads of the original and reflected pulses, the area of the reflector (about 1000 cm²) and the size of the receiving telescope (76-cm diameter). Filters with narrow acceptance windows for photon frequency, direction and time of reception reduce background noise levels. The laser station at the Centre d'Etudes et de Recherche en Géodynamique et Astronomie, in Grasse, France, uses a larger telescope (160-cm diameter) and a more powerful laser. Figure 3 shows the progress made in LLR accuracy through the years, and further improvement is expected. More than ten thousand such normal point range measurements have been accumulated and archived over the past quarter of a century, and, what is very important, the experiment remains active.¹

The archived range data, available to analysis groups throughout the world, must be fit by a physical model. The basic model used by several American and European groups has at its core the gravitational N -body equations of motion for Solar System bodies, clock rate adjustments and the gravitationally altered global speed of light, all derived from Einstein's general relativity theory. A complete model must also include many other parameterized features. These features represent: masses; selected multipole moments; moment of inertia tensors; initial conditions for positions, velocities and rotation variables for bodies; and laser observatory and reflector coordinates, including periodic tidal displacements. To test the validity of the underlying theory—general relativity—additional theory parameters that characterize the ways in which alternative theories may differ from general relativity can be added to the model. More than a hundred model parameters are optimally adjusted by performing a least-squares-type fit of the observed photon flight times to the model-calculated times.

The main features of lunar motion, including both the historic perturbations and the relativistic corrections of modern interest, derive from the truncated equation of motion shown in box 1. In addition to the mutual Newtonian acceleration between the Moon and Earth, the equation includes the Sun's tidal acceleration of the Moon relative to Earth, a difference between the acceleration of



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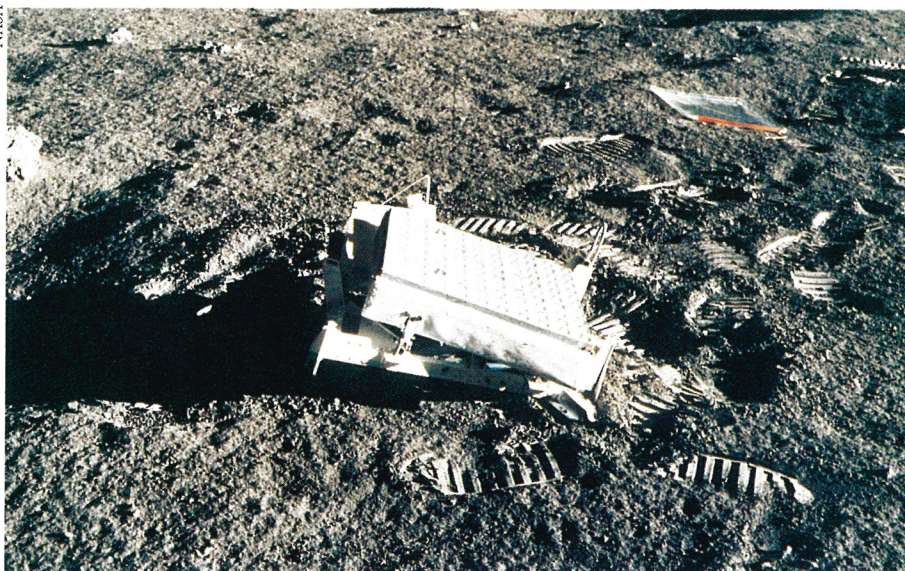
EARTHRISE OVER THE MOON'S HORIZON. Theories of gravitation are being tested by careful monitoring of the Moon's motion. This photograph was taken in 1969 by Apollo 12 astronauts during the second lunar landing mission. (NASA photograph.)
FIGURE 1

Earth and of the Moon by the Sun in the case of unequal ratios of gravitational mass to inertial mass, and a Coriolis-like acceleration term that reflects the rotation of the "local" inertial space that accompanies the Earth-Moon system's motion through the Sun's gravitational field.

The Moon's largest oscillatory deviations from circular motion are illustrated in box 2, which gives their characteristics and historic names. LLR measures each of these radial amplitudes to a precision of about 1 cm! The lunar variation shown in the box is produced by the Sun's tidal acceleration and consists of an elongation of the orbit perpendicular to the direction of the Sun and an equal contraction of the orbit parallel to the solar direction. Although the tidal acceleration pulls the Moon away from Earth at the new and full moon positions (marked N and F, respectively) and toward Earth at the quarter phases (Q), the Moon's motional response is out of phase with this periodic disturbance of twice the orbit's natural frequency, as is the general rule for forced oscillations. New-

ton explained and estimated this orbital irregularity in his *Principia*. The variation does not affect the time of occurrence of lunar or solar eclipses and was therefore not discovered by the astronomers of antiquity. It remained to be found by Tycho Brahe with his more accurate observations in the late 16th century.

A century after Newton's time, Pierre Simon, marquis de Laplace, more exactly analyzed the consequences of the Sun's tidal acceleration. He found a next-order correction to the variation that slightly displaces the orbit toward the Sun, moving the Earth center from E to E' in the figure in box 2, so that the distance E'N is greater than the distance E'F. This displacement is proportional to the ratio of lunar distance to solar distance ($r/R \sim 1/400$), and it acquired the name "parallactic inequality" because its measurement was viewed as a possible method for determining that ratio. Like the variation, this perturbation does not affect eclipse predictions. It produces a longitudinal displacement of the Moon's position of only about 2



RETROREFLECTOR on the Moon. This array of a hundred 3.8-cm corner-cube reflectors was deployed during the 1971 Apollo 14 mission. Three other functioning reflector arrays have also been placed at different locations on the Moon. **FIGURE 2**

arc minutes at the quarters. Precision measurement of the radial amplitude associated with the parallax inequality is today at center stage in testing general relativity, since this amplitude is altered if there is a difference between the Earth's ratio of gravitational mass to inertial mass and the Moon's (as further discussed below).

One of the great steps forward in analytic treatment of the lunar motion problem was made a century ago by American mathematician and celestial mechanician George W. Hill.³ He realized that a special-case distorted orbit resulting from the Sun's tidal acceleration could be computed as a separate and preliminary dynamical problem—and that this tidally affected orbit was a superior starting point from which to carry out more efficient perturbative calculations for the complete lunar motion. The high point of precomputer, analytically based representations of the Moon's orbit, produced by mathematician Ernest W. Brown in the first part of this century, was made possible largely by Hill's calculational innovations.

Free-fall and equivalence

In developing his lunar theory, Newton assumed that Earth and the Moon accelerate at the same rate toward the Sun if identically located, and therefore that the Sun's tidal acceleration is the dominant perturbing force of lunar motion. Like Galileo decades earlier, Newton had studied laboratory pendulums constructed of various materials, and had concluded from a comparison of their oscillation frequencies that the gravitational acceleration rates were equal to one part in a thousand. Newton was also conceptually inclined, and to a significant extent obliged by observation, to assume that this property of gravity was universal.

Newton, however, did explore the consequences of planets and their satellites being accelerated toward the Sun differently. But curiously, his conclusions on this point are in substantial quantitative error and rest on unpublished, undiscovered calculations. In book III, proposition VI of the *Principia*, he stated that

If, at equal distances from the sun, any satellite, in proportion to the quantity of its matter [inertial mass], did gravitate toward the sun with a force greater than Jupiter in proportion to his [inertial mass], according to any given proportion . . . then the distance between the centers

of the sun and of the satellite's orbit would be always greater than the distance between the centers of the sun and of Jupiter, nearly as the square root of that proportion: as by some computations I have found. . . . And by the same argument . . . the weights [gravitational masses] of the moon and of the earth towards the sun are . . . accurately proportional to the [inertial] masses of matter which they contain.⁴

Newton recognized that a satellite's orbit would be displaced along the direction of the Sun if it accelerated differently than its parent planet, but both the sign and magnitude of his estimate of this effect disagree with modern calculation, as outlined in box 3.

When Laplace later calculated the polarization of the lunar orbit called parallax inequality, he noticed the strong similarity between the responsible part of the solar tidal acceleration and the perturbing acceleration given by the third line of the first equation in box 1. Since there was no significant discrepancy in the size of the observed parallax inequality, Laplace could conclude that Earth and the Moon fell toward the Sun at equal rates to an accuracy of better than one part in a million.⁵

By Einstein's time, laboratory experiments had confirmed the universality of gravitational free-fall rates to yet higher precision. In his early efforts to incorporate gravity into the theory of relativity, Einstein elevated this observed property of gravity into a grand hypothesis—the equivalence principle—that gravity is locally equivalent to an accelerated, noninertial frame of reference, this equivalence extending to all aspects of local physical law. From this principle, Einstein was able to predict gravitational deflection of light and the universal slowing of clock rates by nearby matter. The equivalence principle served Einstein well, providing him with important clues in his subsequent development of a complete theory of gravity, which further revolutionized our concepts of space, time and cosmology and their relationships to mass-energy. Einstein's original formulation of this principle has since been repackaged and altered by others in several ways, and a wide variety of experiments have been performed to test the various versions.

In 1968, I suggested that the forthcoming LLR capability could test, at an interesting scientific level, whether Earth and the Moon accelerate equally toward the Sun.⁶

Box 1. Moon's Equation of Motion

The Moon's equation of motion relative to Earth, derived from the Sun-Earth-Moon general relativistic three-body problem, is shown here approximated, but in sufficient detail to produce the main features of the motion.

$$\begin{aligned} \frac{d^2 \mathbf{r}}{dt^2} = & -G M(G)_{e+m} \frac{\mathbf{r}}{r^3} && \left[\begin{array}{l} \text{Earth-Moon} \\ \text{acceleration} \end{array} \right] \\ & + G M(G)_s \left(\frac{\mathbf{R} - \mathbf{r}}{|\mathbf{R} - \mathbf{r}|^3} - \frac{\mathbf{R}}{R^3} \right) && \left[\begin{array}{l} \text{solar tidal} \\ \text{acceleration} \end{array} \right] \\ & + \left[\left(\frac{M(G)}{M(I)} \right)_m - \left(\frac{M(G)}{M(I)} \right)_e \right] G M(G)_s \frac{\mathbf{R}}{R^3} && \left[\begin{array}{l} \text{inequality} \\ \text{of free-fall} \end{array} \right] \\ & + 2 \boldsymbol{\Omega}_{\text{ds}} \times \frac{d\mathbf{r}}{dt} + \dots && \left[\begin{array}{l} \text{de Sitter's geodetic} \\ \text{precession} \end{array} \right] \end{aligned}$$

where e, m and s indicate Earth, the Moon at $\mathbf{r}$ and the Sun at $\mathbf{R}$, respectively. $M(G)$ and $M(I)$ indicate gravitational and inertial mass, respectively, as they appear, for example, in a statement of Newton's gravitational law of motion:

$$M(I)_i \frac{d^2 \mathbf{R}_i}{dt^2} = G M(G)_i M(G)_j \frac{\mathbf{R}_j - \mathbf{R}_i}{|\mathbf{R}_j - \mathbf{R}_i|^3}$$

The third line of the first equation above is absent in both Newton's and Einstein's theories, where all bodies have identical ratios of gravitational mass to inertial mass. This mass ratio is changed in most alternative theories of gravity, in proportion to a body's fractional gravitational binding energy:⁶

$$\frac{M(G)}{M(I)} \approx 1 - \eta \frac{G}{2Mc^2} \int \frac{\rho(\mathbf{r}) \rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}'$$

where $\rho(\mathbf{r})$ is the body's mass density. The theory-dependent dimensionless parameter η in this equation quantifies differences between general relativity (in which $\eta = 0$) and alternative metric theories of gravity such as "scalar-tensor" theories.⁸ The final Coriolis-like acceleration term in the first equation was found by Willem de Sitter in 1916 within months of the publication of Einstein's theory.¹⁶ De Sitter's rotation rate vector is given by

$$\boldsymbol{\Omega}_{\text{ds}} = \frac{3}{2} \frac{GM(G)_s}{c^2 R^3} \mathbf{R} \times \mathbf{V} \approx 19.2 \text{ milliarcseconds per year}$$

This term indicates rotation of local inertial space with respect to distant inertial space because of (and with the same sense as) the orbital motion of the Earth-Moon system through the gravitational field of the Sun.

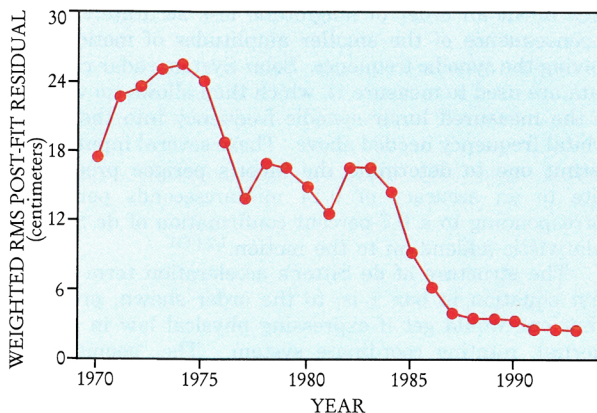
Today, after more than a quarter century of LLR data of ever-improving quality, the key parallactic inequality amplitude is measured¹ to a precision of 1.3 cm. (The error from random measurement noise is only two or three millimeters!) This amplitude is unusually sensitive to any Earth-Moon acceleration difference, so these observations are now the most accurate confirmation^{1,2} of the universality of free-fall for two different bodies, any difference being fractionally less than 5×10^{-13} . Laboratory experiments are slightly less precise and have a narrower scope for theoretical interpretation, but they do have the versatility of comparing acceleration rates among a variety of objects.⁷

There are two ways to interpret these LLR results. Each challenges general relativity in a fundamental way. The Earth's average composition is substantially affected by its iron-nickel core, while the less-dense Moon is believed to consist primarily of silicates. The LLR observations therefore confirm very precise equality in the acceleration rates of two different materials. This equality provides strong foundational evidence that gravity is predominantly a metric field phenomenon in which the carrier of the gravitational interaction is a dynamical second-rank tensor field $g_{\mu\nu}(\mathbf{r}, t)$ (perhaps supplemented by a scalar field) with a universal form of coupling to all the other particles and fields that compose matter. General relativity is certainly the premier metric theory, but there are alternatives.⁸

But unlike laboratory bodies, Earth contains an appreciable fraction (about 5×10^{-10}) of gravitational binding energy, which also contributes to both Earth's inertial and gravitational masses. If such contributions are not equal, the crucial mass ratio is altered⁶ in the manner indicated by the third equation in box 1. The theory-dependent parameter η appearing in this equation is therefore measured by LLR to be less than 10^{-3} . This measurement is now the most precise test of relativistic gravity's first level of structure beyond the Newtonian, and it supports general relativity theory, in which $\eta = 0$. This result constrains any alternative metric theory of gravity, if so chosen by nature, such that it has to be very "close" in this designated sense to general relativity. Gravity (in this case the Sun's) pulls on gravitational energy (Earth's binding energy) just as it does on all other forms of mass-energy, confirming the specific nonlinear structure of general relativity.

Perigee precession

Although Newton's use of the Sun's tidal disturbance to explain the irregularities in the Moon's motion was generally successful, he experienced, in his own phrase, a persistent "headache" in attempting to understand the



IMPROVING ACCURACY of lunar laser ranging. The graph shows the weighted root-mean-square differences between ranging observations and a standard model fit to the data, over the 25-year history of measurements. Note the significant improvement in accuracy since the mid-1980s. (From ref. 1.) **FIGURE 3**

most obvious lunar motion irregularity—the advancement of the major axis of its eccentric orbit by over 40 degrees of arc per year.⁹ In first approximation, Newton found that the solar tidal acceleration produced both a rotation of the orbit's major axis (perigee precession) and a precession of the inclined plane of the lunar orbit (nodal precession). He obtained identical magnitudes but opposite signs for these rates. Although his estimate agreed well enough with the observed retrograde rotational period of 18.61 years for the node (intersection of the lunar orbital plane with the Earth–Sun orbital plane), it accounted for only about half of the observed precession of perigee, which takes only about 8.85 years to make a complete revolution. (See box 2.)

For many decades neither Newton nor his contemporaries understood how this large discrepancy could be eliminated by treating the very small solar tidal disturbance to the next order of approximation. (Comments made by Newton in later life suggest he may have solved this problem, but the details have not been found.) Indeed, as late as 1747, French mathematical physicist Alexis-Claude Clairaut concluded that this unexplained rate of precession of the Moon's major axis would require modification of the basic inverse square force law of gravitation. Fortunately, Clairaut tackled the problem once more, this time taking into account all the motions—eccentric, evective and variational—and their mutual interactions. His more complete analysis revealed a surprisingly enhanced next-order correction to the perigee precession rate, which reached good agreement with observation.

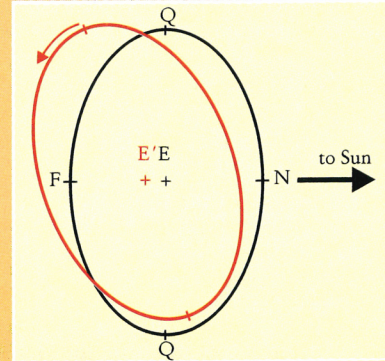
Willem de Sitter's relativistic Coriolis-like term in the first equation in box 1 adds an additional contribution to the Moon's perigee precession. So the modern series expression for the total precession rate in general relativity theory takes the following form:

$$\omega - \omega_0 \approx \frac{3}{4} \frac{\Omega^2}{\omega} + \frac{225}{32} \frac{\Omega^3}{\omega^2} + \dots + \frac{3}{2} \frac{GM(G)_s}{c^2 R} \Omega + \dots$$

where Ω is Earth's orbital frequency around the Sun, ω is the lunar orbital frequency and ω_0 is the (anomalous) frequency of perigee occurrence. The equation shows Newton's leading-order term, Clairaut's surprising next-order contribution (not only containing the large factor 225/32 but also being of order Ω^3 rather than a “more

Box 2. Moon's Orbital Behavior

The lunar orbit's variation and parallactic inequality result from the Sun's tidal acceleration of the Moon relative to Earth. The variation orbit is compressed at new (N) and full (F) moon, and equally elongated at the quarter (Q) phases, as first estimated by Newton. Laplace later found Newton's distorted orbit to be slightly polarized toward the Sun (parallactic inequality), with Earth's center shifted (E to E') toward the full moon (F) position. Additional polarization of the orbit results if Earth and the Moon accelerate at different rates toward the Sun (see boxes 1 and 3), forming the basis for the present-day lunar laser ranging test of relativistic gravity. The unforced oscillations in the Earth–Moon distance are symbolized by the red orbit and consist primarily of the eccentricity plus its surprisingly large modification—the evection—produced by the solar tide. The eccentricity defines the orbit's major axis, which turns 40 degrees of arc per year. For illustrative purposes, these orbit distortions are shown here greatly exaggerated, the actual amplitudes given in the table being small compared to the Moon's mean distance of about 385 000 km.



Amplitude (km)	Historic Name	Period (days)
-20 905	(Eccentricity)	$\frac{2\pi}{\omega_0} = 27.554$
-3699	(Evection)	$\frac{2\pi}{2(\omega - \Omega) - \omega_0} = 31.812$
-2956	(Variation)	$\frac{2\pi}{2(\omega - \Omega)} = 14.765$
+110	(Parallactic inequality)	$\frac{2\pi}{\omega - \Omega} = 29.531$

ω = Moon's orbital frequency.
 Ω = Earth–Moon system's orbital frequency around the Sun.
 ω_0 = Moon's (anomalous) frequency of perigee occurrence.
 $\omega - \omega_0$ = precession rate of lunar orbit's major axis.

expected” order Ω^4) and de Sitter's general relativistic term.

Among the lunar motion frequencies, LLR best measures ω_0 , to a fractional accuracy of about 10^{-12} . The (synodic) frequency of full moon occurrences $\omega - \Omega$ is measured about an order of magnitude less accurately, being a consequence of the smaller amplitudes of motions involving the synodic frequency. Solar System radar ranging data are used to measure Ω , which then allows conversion of the measured lunar synodic frequency into the lunar orbital frequency needed above. These several inputs then permit one to determine the Moon's perigee precession rate to an accuracy of 0.14 milliarcs per year, corresponding to a 0.7-percent confirmation of de Sitter's relativistic addendum to the motion.^{1,2,10,11}

The structure of de Sitter's acceleration term in the first equation in box 1 is, to the order shown, precisely what one would get if expressing physical law in a non-inertial, rotating coordinate system. The “geometrical” interpretation of de Sitter's effect is based on just that: By reason of being in orbital motion around the Sun, the

inertial frame near Earth, which eliminates the Sun's "local gravity," must not only be free-falling toward the Sun, but must also be rotated at the rate Ω_{ds} relative to the inertial space established asymptotically far from the Solar System. And this "geodetic" rotation of local inertial space carries the lunar perigee along with it.

There are a few more general relativistic corrections to the lunar orbit that can be discerned in the data. These effects are relatively minor, being few-centimeter corrections to the variation, evection and parallactic inequality amplitudes. Because they have minimal usefulness for putting general relativity to additional empirical tests, I have not discussed them, but they cannot be ignored. For example, because the Earth-directed coordinates of the reflector positions on the Moon are poorly known, the mean Earth–Moon distance is best inferred (albeit to precision of only about 80 cm) from the variation amplitude. From this determination of the mean lunar distance and the measured lunar orbital frequency, one determines the gravitational mass of the Earth–Moon system. The Moon's mass, determined from the tracking of lunar-orbiting satellites, can be subtracted from that total. If the general relativistic corrections to the amplitude of variation were not included, a final gravitational mass of Earth determined by LLR would not agree with a value determined independently from the laser tracking of near-Earth satellites.

Time variation of coupling parameter

During much of this century, there has been speculation that the strength of the gravitational interaction might change in time with cosmological evolution.^{8,12} General relativity is rather unique among contemporary theories of gravity in not producing a coupling strength dependent on location in space and time. Alternative theories generally predict time variation of G in some proportion to the universe's Hubble expansion rate, which is about 10^{-10} per year. Both a time evolution of G and the torque resulting from ocean tides on Earth will produce rates of change of the Moon's orbital period and radius, though in ratios that differ between the mechanisms.¹³ The tidal torque fortunately is also independently measured from its effects on near-Earth satellites.

The present fit of the LLR data is consistent with no time variation in the strength of gravity, the experimental bound being^{1,14}

$$\left| \frac{dG}{G dt} \right| < 10^{-11}$$

per year. This constraint is modestly smaller than the observed expansion rate of the universe and is comparable to the constraint that has been obtained by use of interplanetary radar ranging observations.¹⁵ But the prospects are good for improving this measurement from the continuing LLR experiment. The accumulation of an ever-increasing time span of highest-accuracy ranging data will make possible better separation of the tidal effects and consequently a substantially more accurate measurement

Box 3. If Free-Fall Were Not Universal

If Earth and the Moon fell at different rates toward the Sun, then there would be an additional polarization of the lunar orbit, as follows.

▷ According to Newton:

$$\left(\frac{R_S - R_P}{R_P} \right) \approx \frac{1}{2} \left(\frac{a_S - a_P}{a_P} \right)$$

▷ According to contemporary calculation:

$$\left(\frac{R_S - R_P}{R_P} \right) \approx -\frac{3}{2} \left(\frac{a_S - a_P}{a_P} \right) \frac{\omega_P}{\omega_S} \mathcal{F}(\omega_P/\omega_S)$$

where R_S and R_P are the distances of the satellite's orbit and the planet, respectively, from the Sun; a_S and a_P are the accelerations of the satellite and the planet, respectively, by the Sun; ω_S and ω_P are the orbital frequencies of the satellite and the planet, respectively; and $\mathcal{F}(\omega_P/\omega_S)$ is the dynamical amplification factor from the solar tide. $\mathcal{F}(0) = 1$ and $\mathcal{F}(\omega_{\text{Earth}}/\omega_{\text{Moon}}) \approx 1.74$.

of the time evolution of G .

In the coming years, LLR can remain a frontier experiment, testing relativistic gravity in several ways to ever-increasing accuracy. The passive reflectors on the Moon should continue to function indefinitely. As the time span of the data grows longer, measurement accuracy will improve, especially that of the key lunar frequencies and their secular rates of change. Precision of amplitude measurements will also improve, but more modestly, as high-accuracy range measurements accumulate. The experiment has already become limited by systematic error rather than by $N^{-1/2}$ random noise. Hence, more detailed modeling of the geophysics effects at Earth and the Moon has become essential to exploit fully the LLR data, which continue to grow in both quality and quantity.

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