The Pool-Table Analogy with Axion Physics

A tilted room houses a mysteriously horizontal pool table, sending an imaginary character named TSP on an intellectual journey that parallels that of physicists interested in the strong *CP* problem and axion physics.

Pierre Sikivie

Elementary particle physicists enjoy talking about particles for which there is no experimental evidence, and of these particles the axion seems one of the strangest and least accessible. The mass of the axion is expected to be roughly 10^{-5} eV, a factor of a million below current limits on the neutrino mass, and the axion's couplings are suppressed by a factor of 10^{-12} relative to those of pions and other familiar particles. Yet there are serious claims that axions make up most of the mass of the universe and equally serious experiments to demonstrate the presence of these tenuous particles. Why should one believe in the axion? I attempt to answer this question by drawing an analogy with the physics of a pool table.

Pool-table analogy

Consider the physics involved in playing snooker. The rules of the game require that the pool table be horizontal. If the table is not horizontal, a certain symmetry is broken. Let us call that symmetry S. If S is broken, the balls tend to roll to one side, which is not the way the game should be played. The rules of pool-table physics require that S be a good symmetry.

Similarly, the rules followed by the strong interactions obey discrete symmetries P and CP. P is parity and CP is the product of parity and charge conjugation. These symmetries of the strong interactions have been known for a long time. In fact, the discovery that the weak interactions violate P and CP was a big surprise because physicists, used to seeing the strong interactions and also the electromagnetic interactions obey P and CP, had a hard time conceiving that these symmetries could be violated at all.

We may imagine that the people playing snooker have done so for a very long time. Let us even imagine that they have always lived on the pool table. They have a hard time conceiving that the symmetry S could be broken. However, some day they discover the great wide world. They jump off the table and find themselves on the floor of the playroom. Now, to continue our analogy with the Standard Model of particle physics, we will assume that the playroom floor is not horizontal. The snooker players are astonished to discover that the wider world does not respect the symmetry they had become so used to. The playroom is askew somehow, which is very disconcerting. But after a while the snooker players become accustomed to it. They abandon the prejudice that S should be a good symmetry.

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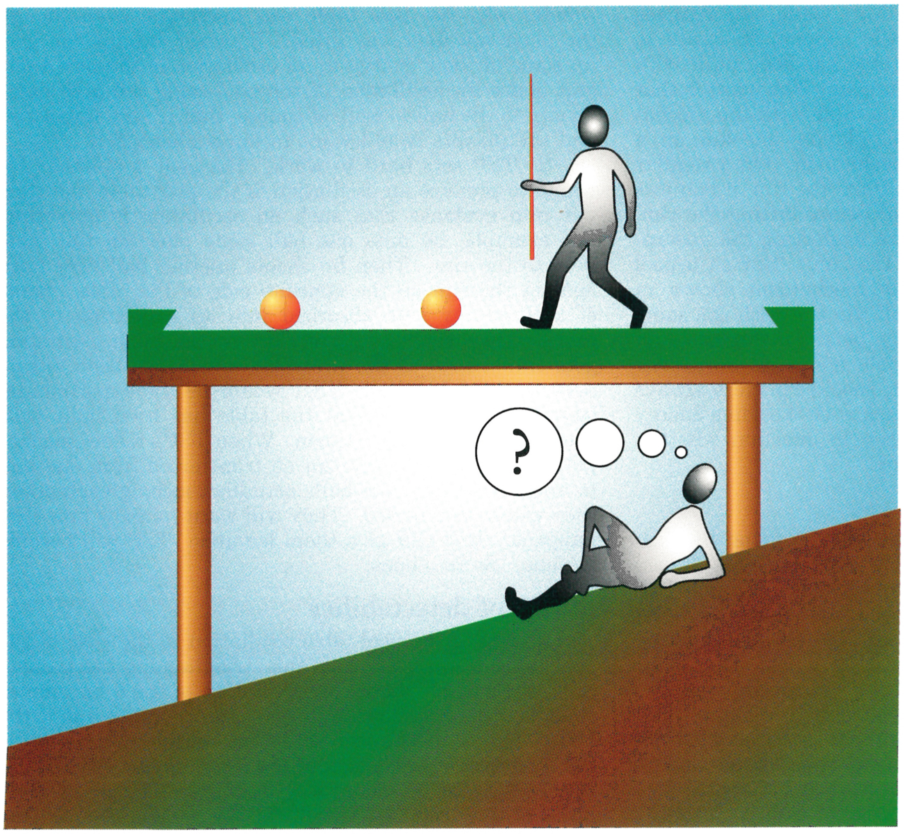
The players have become comfortable with the fact that the symmetry is broken. However, one of them—whom the players call TSP (which could be short for Thinking Snooker Player)—is deeply troubled. TSP realizes there is something wrong with the world he is living in. The playroom floor is not horizontal, because the symmetry is broken. That's fine. But why is the pool table horizontal?

There is similarly something wrong with the Standard Model. It is called the strong CP problem. The Standard Model violates P and CP. How can the strong interactions, which are part of the Standard Model, conserve those symmetries? Within the Standard Model, it is as surprising to have the strong interactions conserve P and CP as it is surprising to find a horizontal pool table in a playroom that is itself not horizontal. Users of the Standard Model take just pride in being able to explain the violation of CP in an economical and natural way, by allowing the elements of the quark mass matrix to have arbitrary complex phases. This virtue is often emphasized. However, if the matrix elements have arbitrary complex phases, then the θ angle of quantum chromodynamics has an arbitrary value as well—let us say any number between zero and 2π, in which case the strong interactions violate P and CP in blatant fashion. That is contrary to observation. To be explicit, the upper limit on the neutron's electric dipole moment, which provides the most sensitive test of P and CP violation by the strong interactions, requires that θ of QCD be less than 10^{-9}.

His curiosity piqued, TSP sets out to check whether the pool table is as horizontal as it appears. (To be sure, TSP is no casual observer; he's got the soul of a physicist.) He finds that the table is as horizontal as he can make out, and, after much work, having pushed to the limit the measurement technology available to him, he concludes that any deviation of the table from perfect horizontality must be characterized by an angle of less than 10^{-9}. TSP knows that one part in a billion is easier said than done. He is astounded. His stomach tightens with the fear induced by the discovery of a fact at once bizarre and unexplained. "Someone is playing a trick on us, that's for sure," he thinks to himself.

TSP figures the person who made the pool table compensated for the slant of the playroom floor by adjust-



HORIZONTAL POOL TABLE on a slanted floor. Adjusting the lengths of the pool-table legs compensates for the tilt of the room. FIGURE 1

ing the lengths of the pool-table legs, as illustrated in figure 1. To do that, the pool-table maker measured the angle between the vertical and the playroom floor. Vertical direction is determined by gravity and can be made manifest by the plumb line, a well-known and wonderful tool. After taking his measurements, he designed the table's legs accordingly, with a precision of one part per billion. TSP muses that if the table maker has many customers, he must spend a lot of effort adjusting his pool tables to the various angles between the vertical and the floors of his customers' playrooms. Each pool table has to be individually built to ensure the symmetry that the customers demand for their snooker playing, to the tune of one part per billion, apparently.

Automatic zeroing

Some time passes. One day, as TSP is sitting around thinking about the life of the pool-table maker, an idea occurs to him. If he himself were in the pool-table making business, what he would do is build each pool table on a post that would pivot on an axle. At the end of the post opposite the pool table would be a big weight. The axle would be mounted on a tripod. TSP's imagined contraption is illustrated in figure 2. The point is that gravity would automatically pull the weight down, so the post would be vertical and the pool table horizontal. *Et voila!* All pool tables can now be made the same, with tremendous savings in effort and production costs.

TSP gets excited at the idea of the fortune he could make in the pool-table manufacturing business. His tables would adjust themselves automatically in any playroom, just under the influence of gravity. The beauty of the scheme is that it is gravity that decides what's vertical and what's not. So gravity can do the job by itself.

What TSP has just discovered is the analog of the Peccei–Quinn solution to the strong *CP* problem of the Standard Model of particle physics. Roberto D. Peccei and Helen R. Quinn slightly modified the Standard Model in such a way as to make the θ angle of QCD a dynamical variable. There are nonperturbative effects that produce *P* and *CP* violation in QCD if the θ angle differs from zero or π. The analog of QCD is the physics on the pool table; the analog of the θ angle is the misalignment of the table from the horizontal; the analog of the nonperturbative effects that make QCD physics depend upon the θ angle is gravity, which makes pool-table physics sensitive to a lack of horizontalness of the table; the analog of *P* and *CP* symmetry in QCD is *S* symmetry in the pool-table physics; and so on. In the Peccei–Quinn mechanism, the nonperturbative effects that make QCD physics depend upon θ pull θ to zero once the model has been arranged so that θ becomes a dynamical variable. In TSP's contraption, gravity, which makes pool-table physics sensitive to a slant of the pool table, removes any such slant once an axle is introduced to enable the pool table to pivot.

TSP is pleased with himself, although it turns out he cannot make a fortune based on his insight. For some reason, he is confined to the playroom and that keeps him from going into the pool-table manufacturing business. More time passes. One day, in a more humble mood than the one he got into following his theoretical discovery of the mechanism that can straighten out pool tables, a fresh idea occurs to him. It might be that the pool-table maker who made the table where TSP lives also discovered the mechanism for straightening pool tables and that he incorporated it into the pool table in TSP's room. TSP becomes curious about this possibility. Unfortunately, all around the pool table hangs a dark cloth that hides from view whatever supports it. But after a while, TSP realizes that it is not necessary to see the support structure to deduce whether or not the pool table has been built with the pool-table straightening mechanism. The point is that the physics of playing snooker on a table with the mechanism differs from the physics of playing snooker on a regular pool table without the mechanism. On a regular table, as in figure 1, when the ball hits the rim, it bounces back with the same energy it had before hitting the rim. (For the sake of argument, we are neglecting the absorption of energy by the rubber on the rim.) But on a pool table that has the straightening mechanism shown in figure 2, a ball does not bounce off the rim with the same energy, because some of its energy gets transferred to an overall oscillation of the pool table about its horizontal equilibrium position. In the past, snooker players always perceived that the ball bounced back with the same energy but, of course, they had no reason to question whether that is true with infinite precision.

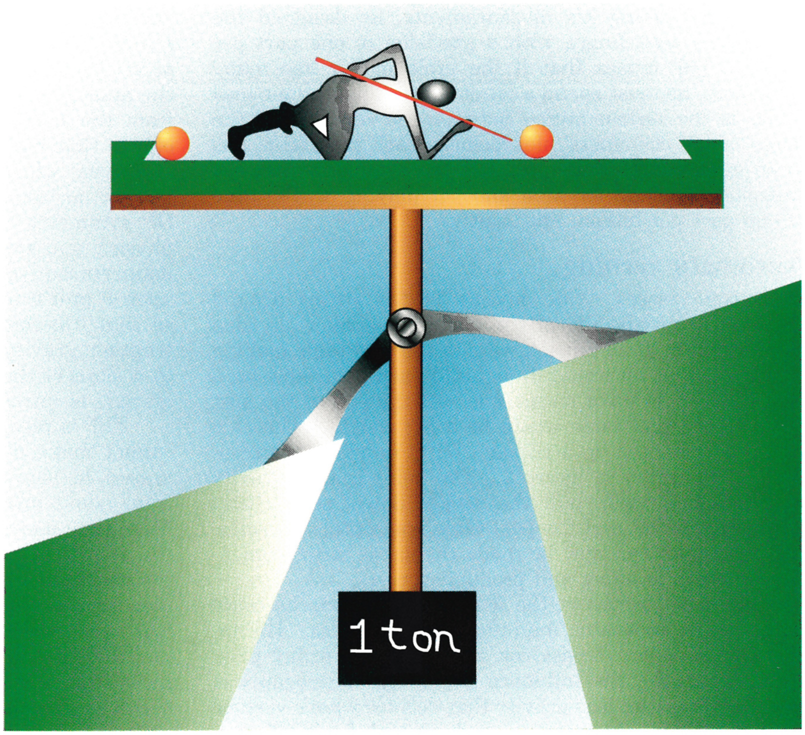
Theorist's motivation

Let me digress briefly to explain what is going through TSP's mind at this moment. TSP's fellow snooker players have always thought him a bit odd because, although TSP was recognized from early on to be quite smart, he didn't achieve much in real life. TSP just sits around thinking about this and that, but he doesn't do much. His fellow players thought TSP was acting very strangely when he had insisted that there was something "terribly wrong" about a horizontal pool table in a room that was itself not horizontal. "What's so wrong about that?" they said to each other, "It's actually good to have a horizontal pool table to play snooker." Their viewpoint is just so totally different from TSP's. Of course, TSP enjoys thinking, and that's why he does that rather than anything else. So, contrary to what his fellow players think, TSP has a happy life. Still, he would like it better if he were more appreciated. Now, with his theoretical discovery of the pool-table straightening mechanism, TSP sees an opportunity to impress the other players. If he can show that the rules of snooker are not quite what they appear to be and hence that there are new possibilities in the game, that is something his fellow players would appreciate. They did not care to wonder why the pool table was horizontal even though the playroom floor was tilted. However, if balls can give up some of their energy to an oscillation of the pool table and hence an oscillation of the pool table can give extra energy to the balls—well, of course, that is very important and the players would want to know about that.

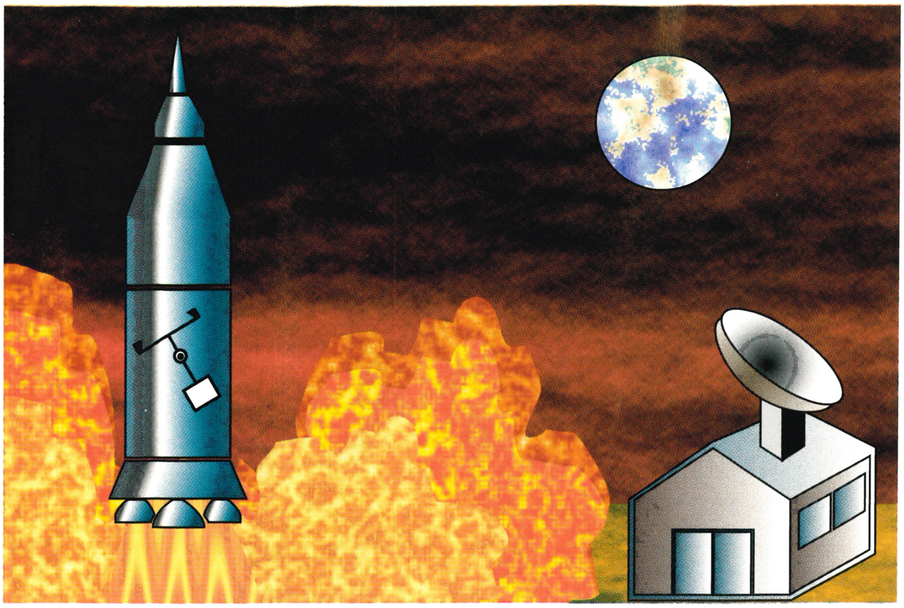
So TSP sets hard to work. His goal is simple. He wants to produce an oscillation of the pool table and then put into evidence that such an oscillation is occurring. For example, he puts one ball some place on the table next to the rim. Then he shoots another ball very hard against the rim on the opposite side of the table. Some of that energy gets absorbed into an oscillation of the table. Then some of the energy in the table oscillation gets transferred to the first ball, which has been sitting next to the table rim. That would be the experimental signature of the fact that the table has been built with the straightening mechanism. When TSP's fellow snooker players see that energy can be transferred from one ball to another without the balls actually touching each other, they will be astounded. They will want to know how that happens. TSP will give them lectures. TSP will become famous. So he hopes.

Limits of detectability

The analog of the pool-table oscillation in the case of the pool table being built with the straightening mechanism of figure 2 is, of course, an oscillation of the θ parameter of QCD if the Standard Model has incorporated into it the Peccei–Quinn mechanism described earlier. The axion is the quantum of oscillation of the θ parameter of QCD. It is a particle in the same way that the quantum of oscillation of the electromagnetic field, the photon, is a particle. To discover whether the Peccei–Quinn mechanism has been incorporated into the Standard Model, one searches for the axion. To search for the axion, one tries to produce a few axions and then detect them. It is necessary to produce them first because they are unstable and hence cannot be around for a long time. (This last statement



POOL-TABLE STRAIGHTENING MECHANISM. This pool table can accommodate any tilt of the room.
FIGURE 2



Pool table in spaceship about to land on Mars. Only when the rockets are fired does the straightening mechanism go into action. Figure 3

is not always true, but let's accept it for the moment; we will return to this point later.) To produce axions, one may take a beam of protons and dump it into a block of material. The axions produced may then be put into evidence by a detector that converts their energy back into more immediately visible forms of energy such as photons. This experiment and many others aimed at producing and detecting axions were carried out in the late 1970s and early 1980s, but no axions were found. The experiments represent an analog of the experiment TSP proposes to carry out to put into evidence the pool-table straightening mechanism.

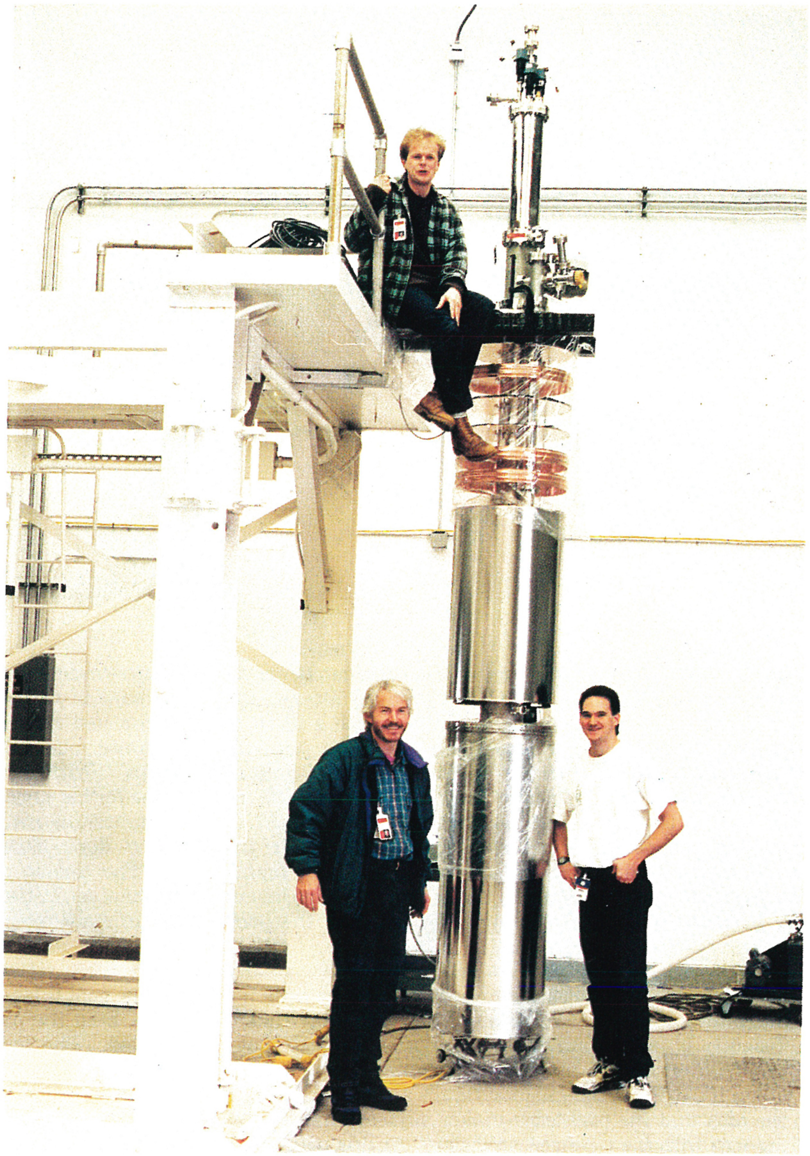
As it turns out, TSP's hopes are dashed, totally, mercilessly. . . . No matter how hard he tries, he does not manage to produce an oscillation of the pool table that is sufficiently large for him to detect. What is he to make of that? What makes the table horizontal to one part in a billion if not the mechanism of figure 2? Must he return to the idea that the pool-table maker adjusted the lengths of the legs with the required precision? At this point, TSP realizes that his ability to put into evidence oscillations of the table depends upon the length l of the lever arm between the axle and the big weight. If l is very large, it becomes very difficult to produce pool-table oscillations by hitting balls against the rim. TSP also notices that when l is very large, the oscillation frequency of the table is very low. TSP now carries out detailed calculations. He finds that if l is more than about 3 meters, his attempts to produce and detect pool-table oscillations must fail even if the table is constructed with the straightening mechanism. Thus, his experiments rule out the mechanism only if l is less than 3 meters or, equivalently, if the oscillation frequency of the pool table is more than 0.18 cycles per second, which is the oscillation frequency of a pendulum of length 3 meters on Mars, where TSP and his fellow snooker players happen to be living.

TSP believes he understands everything now. The reason the pool table is horizontal is the mechanism of figure 2. The reason he cannot put into evidence pool-table oscillations is that the length l of the lever arm is more than 3 meters. TSP thinks he ought to be pleased with his insight, but instead he feels frustrated. He understands why the pool table is horizontal but he cannot produce the pool-table oscillations that would confirm his understanding and surprise his fellow players. If the mechanism is implemented with a very long lever arm, there's just no way anyone will ever be able to put into evidence pool-table oscillations. Yet, the mechanism works! With some bitterness, he mutters to himself a name for his invention. He calls it the "invisible pool-table straightening mechanism," because it works yet cannot be visibly demonstrated.

The analog of the invisible pool-table straightening mechanism in the world of particle physics is the Peccei–Quinn mechanism with an "invisible" axion. The properties of the axion depend upon a parameter f, called the axion decay constant, which is analogous to the length l in the pool-table straightening mechanism. If f is very large, then the axion becomes very light and very weakly coupled. The axion mass m is related to the minimum oscillation frequency ν of the θ parameter of QCD by the famous relation $mc^2 = h\nu$, where h is Planck's constant. So, the small mass of the axion if f is large is analogous to the low pool-table frequency if l is large. Also, the fact that the axion is weakly coupled is analogous to the fact that it is difficult to produce pool-table oscillations. If f is large, the axion production and detection rates in the axion search experiments described earlier are so low that the experiments cannot find axions even if axions exist. But the Peccei–Quinn mechanism still works!

Relic oscillations

TSP ponders his fate. What is the worth of theoretical insight without experimental confirmation? Einstein discovered general relativity and very soon afterward his theory was confirmed by the measurement of the deflection of starlight by the Sun. Democritus discovered (correctly guessed?; what is the difference between a theoretical discovery and a good guess?) that matter is made of atoms. At the time, there were no experiments that could put atoms directly into evidence; those experiments came 23 centuries later. . . . As he walks around the playroom, pondering this and other questions, TSP glances for the umpteenth time at a copper plate that is affixed to the side of the pool table. It reads "Made in Minneapolis, Minnesota, USA, home of 'Minnesota Fats.' " TSP always wondered what is "Minnesota Fats". . . . But he does know about the USA. The USA is a country on Earth.

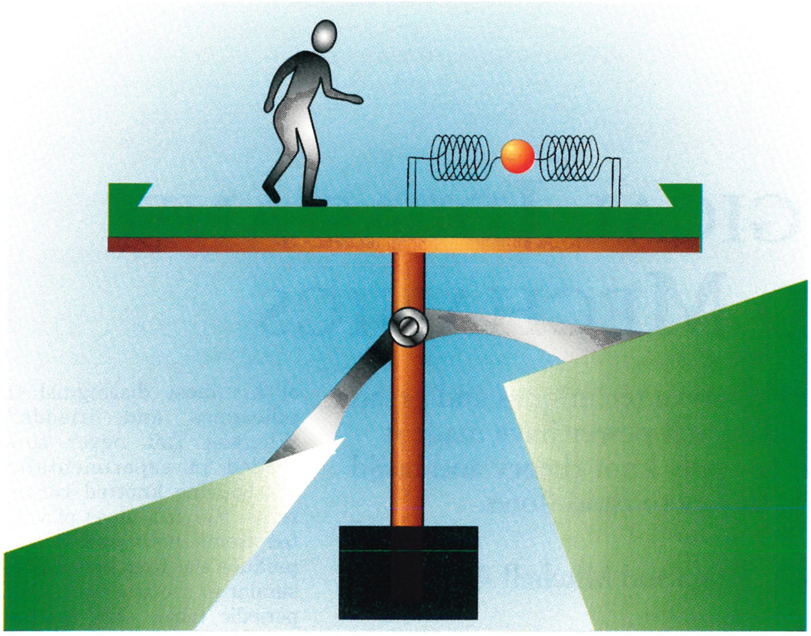


CRYOGENIC INSERT for the axion search experiment now taking data at Lawrence Livermore National Laboratory. The stainless steel dewar containing the cavity is at the bottom. (Photo courtesy of Karl van Bibber, LLNL.) FIGURE 4

TSP imagines how the pool table was brought to Mars from Earth on a spaceship. That's actually pretty interesting because during such a trip, when the spaceship is just coasting along, there is "no gravity." In that situation, the pool table is not oriented in any particular direction. It would seem impossible to play snooker then. But TSP is now thinking about something else altogether. What strikes him is that when the spaceship approaches Mars and prepares for landing by firing its retrorockets, the pool table is not initially horizontal with respect to the direction of gravity at the place on Mars where the spaceship is going to land. The landing on Mars is illustrated in figure 3. Only when the rockets are fired does the big weight of the pool-table straightening mechanism begin to feel Mars's gravity. It then begins to pull the table horizontal with respect to the direction of gravity on Mars, but it overshoots! The table does not get pulled nicely to a horizontal position. Instead, because there is no damping mechanism, it oscillates about the horizontal. Once the spaceship has landed, the pool table will oscillate about the horizontal with constant amplitude indefinitely because it turns out that, if l is longer than 3 meters, the pool-table oscillations are so weakly coupled that they continue for very long times, much longer than the present age of the Solar System. (We are assuming for the analogy's sake that there is no friction on the axle about which the table pivots and that the only way pool-table oscillations get damped is by giving off energy to the large collection of balls on the table.)

Therefore, if the pool table in the playroom where TSP lives is horizontal because of the so-called invisible pool-table straightening mechanism, then it should be still oscillating now. The oscillation is a relic of the epoch when the pool table was brought to Mars. What is the amplitude of this relic oscillation? TSP realizes that the crucial parameter is the ratio of the pool-table oscillation period to the time scale over which Mars's gravity gets effectively turned on when the spaceship bringing the pool table lands on Mars. Assuming that the initial misalignment angle (which is a random angle between 0 and 180 degrees) does not accidentally lie close to zero, the final amplitude of oscillation will be large if the landing is sudden compared to the oscillation period of the pool table. This possibility is incompatible with the present state of the pool table because the table does not appear to oscillate at all now. If, on the other hand, the landing occurs very slowly so that Mars's apparent gravity is turned on progressively, then the amplitude of oscillation decreases while the landing occurs. The switch-on of gravity is adiabatic in this case, and the oscillation amplitude decreases as the inverse of the square root of the oscillation frequency, and the oscillation frequency increases as the square root of Mars's apparent gravity.

TSP carries out careful observations on the pool table



DETECTOR of relic pool-table oscillations. FIGURE 5

to determine whether it is oscillating at present, for he realizes now that a relic oscillation is the telltale sign of the pool-table straightening mechanism. He does not detect any, and places an upper limit of 10^{-12} on the present oscillation amplitude of the table. That rules out the possibility of making the straightening mechanism invisible at will by lengthening the lever arm, because the longer l, the lower the oscillation frequency of the pool table and, comparatively, the more sudden the switch-on of gravity when the spaceship lands on Mars, and hence the larger the amplitude of relic pool-table oscillations. From a NASA publication that happens to be in the playroom, TSP can deduce the time scale over which the retrorockets are fired for the Mars landing, which is also the time scale over which Mars's gravity gets effectively turned on. From that, he can figure out the amplitude of relic pool-table oscillations as a function of l. He finds that the upper limit of 10^{-12} on relic pool-table oscillations requires that l be smaller than 10 meters. TSP is very excited about this result. On the one hand, l must be larger than 3 meters because he was unable to produce and detect pool-table oscillations. On the other hand, l must be smaller than 10 meters because he was unable to detect relic oscillations. It seems TSP is closing in on a resolution of the horizontal pool-table mystery.

The switch-on of gravity when the spaceship approaches Mars is analogous to the switch-on of nonperturbative QCD effects when the universe is about 10^{-7} seconds old and the temperature is about 1 GeV. The relic pool-table oscillations are analogous to the coherent axion field oscillations that constitute the present cosmological axion energy density if the axion decay constant, f, is large. The requirement that the axion cosmological energy density not overclose the universe puts an upper limit on f and hence a lower limit on the axion mass. Just as TSP found lower and upper limits on the length l of the lever arm in the pool-table straightening mechanism, there are lower and upper limits on the axion decay constant. If the axion mass is near its lower limit, axions may be the dark matter of the universe. Experiments are now under way at Lawrence Livermore National Laboratory and at Kyoto University that attempt to detect the axion field oscillations that constitute the dark matter in our Galaxy. In these experiments, an electromagnetic cavity is placed in a large static magnetic field. The cavity is tunable and its frequency is slowly changed. When the cavity frequency matches the axion mass in natural units ($h\nu = mc^2$), a tiny amount of excess microwave power appears in the cavity, caused by resonant axion-to-photon conversion in the externally applied magnetic field. Figure 4 shows a photograph of a piece of the LLNL apparatus.

TSP has similarly found a means of detecting relic pool-table oscillations if l is in the range of 3 to 10 meters, or equivalently, if the frequency of such oscillations is in the range of 0.18 to 0.097 cycles per second. His device is just a simple high-quality oscillator placed on the pool table. (See figure 5.) The oscillator frequency is tunable by changing the mass at the end of the spring. TSP plans to slowly change the frequency. When it matches that of relic pool-table oscillations, his oscillator will get excited. TSP should be able to see this effect.

Will TSP succeed in his latest venture and solve at last the mystery of the horizontal pool table? We don't know yet, but if he does, there may be a sequel to this story.

I adapted this article from a talk I gave at the 30th Moriond Meeting, at Villars-sur-Ollon, Switzerland, in January 1995. I thank my colleagues on the axion search experiments for their insightful comments and Cynthia Chennault for stylistic suggestions.

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