

REFERENCE FRAME

A Model of Turbulence

Leo P. Kadanoff

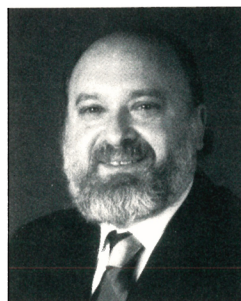
Turbulence is one of *those* problems. Interesting. Vexing. Long-standing. Unsolved.

The basic physics is easy to describe: Imagine a fluid that initially has some sort of simple flow with a nonuniform velocity. As time goes on, the nonlinear interactions among the fluid elements will tend to produce more and more complex structures, with more and more fine details. This complexity will continue to grow until, at a very fine scale, it is limited by viscous damping, which tends to smooth everything out. Note, for example, the extremely complex small structures in figure 1.

The parameter for describing the degree of complexity is the Reynolds number $Re = LU/\nu$, where U is the typical nonuniformity in velocity, L is the size of a region in which the nonuniformity occurs and ν is the kinematic viscosity of the fluid in question. In everyday systems, fine scales are produced precisely because this viscosity is extremely small—of order 10^{-6} m²/s for water.

Figure 2 (on page 13) shows a schematic of these turbulent processes, using concepts introduced over 70 years ago by L. F. Richardson. The dynamics is best described in Fourier-transform language in which the wavevector $\mathbf{k}$ serves to describe the inverse of the different spatial scales. Some large-scale process, perhaps stirring, is used to feed energy into the fluid at small $\mathbf{k}$. The range of scales at which this addition of energy occurs is called the integral range. Then the nonlinear coupling of velocities causes the energy to cascade toward larger wavevectors. The range of scales dominated by the cascade is called the inertial range. This range can be quite large, since it is proportional to a power of the Reynolds number. For even larger $\mathbf{k}$'s, one passes into the viscous range. Here, viscous processes dissipate the energy into heat and smooth out all details. The problem of turbulence is to predict and understand what happens when the inertial range is large.

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A quantitative analysis of energy conservation in turbulent flow was introduced by Andrei N. Kolmogorov in two papers written in 1941 and 1962 that are now generally called K41 and K62. The key idea is that all the energy that is added on the largest scale flows unchanged through the inertial range toward higher $\mathbf{k}$. The

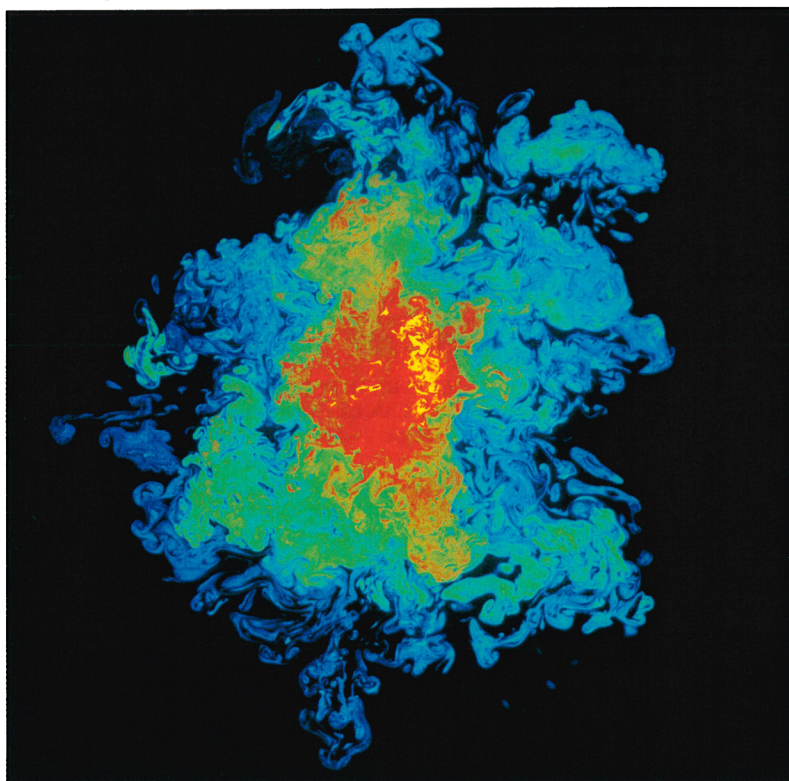
flow is described by a current. By dimensional analysis Kolmogorov found that the current of energy is proportional to $\mathbf{k}[\mathbf{u}(\mathbf{k})]^3$, where $\mathbf{u}(\mathbf{k})$ is the spatial Fourier transform of the velocity. The conservation of energy implies that in the inertial range this current must be independent of $\mathbf{k}$. The 1941 paper thus gives an order of magnitude statement

$$u(\mathbf{k}) \approx \left(\frac{\varepsilon}{k}\right)^{1/3}$$

where ε is the energy current.

There are two crucial assumptions in K41:

- ▷ The current ε is assumed to be roughly constant and thus essentially equal to the energy input per unit time.
- ▷ Further, since the dissipation never directly appears in the argument, the energy flow is assumed to depend

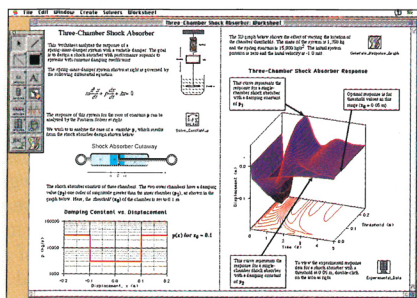


JET FLUID CONCENTRATION in a plane normal to the jet axis, 300 diameters downstream of the jet nozzle, measured by laser-induced fluorescence. Black indicates the reservoir fluid into which the jet is discharging. Pale yellow, red, green and blue indicate successively lower levels of jet fluid concentration. (Courtesy of Harry Catrakis and Paul Dimotakis, Caltech.) FIGURE 1

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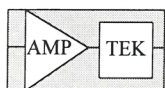


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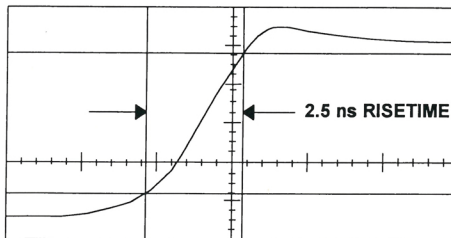
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only on what happens in the inertial and integral scales. Thus, for example, the value of ϵ should depend only on the nature of the external forcing and should be independent of the Reynolds number.

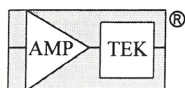
This argument, and many subsequent arguments, sees the energy flow as a kind of pipeline. (See again figure 2.) Energy is added to the pipe at the large scales, flows through the very long pipe and then flows out via dissipation at the far end. The 1941 analysis implies that the flow through the pipe at a given point depends only on processes upstream of that point. Thus, for example, inertial-range behavior would not be influenced by the value of the viscosity, since the viscosity appears only in the dissipative range, far downstream.

But the pipeline analogy casts doubt upon the assumption that no effects flow upstream. In a pipeline, a choking of the outlet would tend to diminish the flow.

Initially the field focused on the other basic assumption of the K41 theory, that of the constancy of energy dissipation. Partially in response to a criticism by Lev Landau, Kolmogorov changed his own point of view. In K62 he argued that because of fluctuations, the energy input should be a function of space and time that can vary over many, many orders of magnitude. These huge fluctuations are termed *intermittency*. While still assuming that effects only flow downstream, Benoit Mandelbrot and others developed richer cascade models that showed how a highly fluctuating behavior might arise and be reflected in a complex fractal structure of $u(k)$.

Theory cannot for the moment distinguish between the weak fluctuations of K41 and the intermittency of K62. To settle questions of this kind we should look to experiment. The experimental evidence suggests the existence of intermittency but is not yet conclusive. We might also look to simulations. Turbulence problems can easily be set up on the computer, since a turbulent system may be correctly described by a well-known partial differential equation, the Navier-Stokes equation. Unfortunately computers are too slow to allow simulations to cover a sufficient range of scales. How can we learn more? One way is to set up models that correctly realize in specific form the previous theoretical ideas and then see how consistent these ideas might be.

Several one-dimensional models have been developed that include in a very rough way cascades from shell to shell in momentum space. One such model, which has received con-

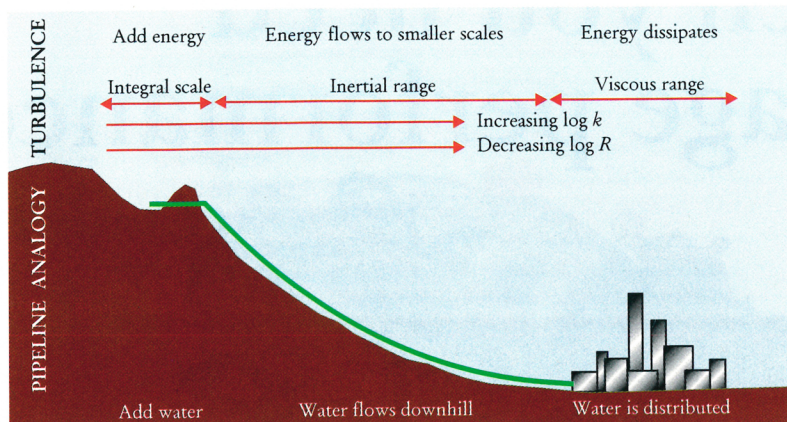


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TURBULENCE AS A DOWNHILL FLOW. In this physical model energy is added on the integral scale and flows "downhill" toward shorter scales, R , and higher values of the wavevector k . The process can be viewed as a kind of pipeline in which the energy flows downhill through a long inertial range and finally is dissipated in a viscously determined range of scales. **FIGURE 2**

siderable attention lately, is connected with the names of E. B. Gledzer, K. Okitimi and M. Yamada (inelegantly referred to as GOY). This is the simplest model that can correctly realize a cascade through orders of magnitude of wavevector k . The system is described by the Fourier coefficients $u(k)$ of the velocity. Imagine a cascade that goes from a k of order the inverse size of the system, L^{-1} , to one in a shell that contains k 's a factor of λ larger, and then to a higher shell with k 's of order $\lambda^2 L$ and so forth until finally we reach $\lambda^N L$, a wavevector that sits far into the dissipative range. The real system involves many, many Fourier coefficients $u(k)$, with the number per shell increasing sharply with the shell number. This model tries to describe the flow by keeping precisely one Fourier coefficient for each shell. The equations of motion are picked to mimic Navier-Stokes behavior, with nonlinearity, viscous damping and energy conservation all included. The model involves two parameters that have no direct counterpart in Navier-Stokes: the shell width λ and a parameter c . The latter determines the ratio of the energy flux toward higher wavenumber to the flux in the opposite sense.

Because the equations of this model involve many fewer variables than do the Navier-Stokes equations, its qualitative properties can be established with the aid of simulations. We can thus ask, Do the size of the fluctuations in the model agree with K41 or with K62? The question is sharply posed, and it may be sharply answered. The answer is yes. Yes! In one range of parameters (c close to unity) the system shows small fluctuations and K41 behavior, while in another

range (smaller c) it shows very strong fluctuations and something more like K62 behavior. For this domain, the model permits independent fluctuations in the velocity ratio between neighboring shells. When one multiplies out a set of many independent random multipliers, the result can have truly huge fluctuations. Thus the model serves as a partial justification for both K41 and the later, more fluctuating theories.

However, K41 and some later theories say that the inertial-range behavior should be independent of v . In response to some prompting from Zhen Su She (University of Arizona), Norbert Schörghofer, Jane Wang, Detlef Lohse, Roberto Benzi and I looked at the GOY model's inertial range. In one situation Schörghofer's numerics showed that these theoretical guesses were wrong: In the static solution, the energy flux down the pipe certainly does depend on v , even in the limit of high Reynolds number. In contrast to the early theories (both K41 and K62), here the flow down the energy pipeline depends on an interaction between inlet and outlet conditions, induced by correlations carried up and down the pipe. (The detailed manifestation of the correlation in the GOY model depends on the shell thickness and is thus unphysical.) This correlation result points us toward the possibility that both inlet and outlet might play a role in determining the flow in real turbulence. If so, there will be some Reynolds-number dependence of inertial-range behavior. Thus a vastly oversimplified model led to insights that might well play back into a deeper understanding of a truly complex phenomenon.

Models are fun, and sometimes even instructive. ■

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