

In a Two-Dimensional Electron System, the Skyrmion's the Limit

Skyrmions—which can be thought of, loosely, as topological twists or kinks in a spin space—are one of those concepts that seem to jump restlessly from field to field. Technically, in a non-linear field theory a Skyrmion is defined as a soliton with spin and statistics different from those of the underlying fields. Introduced by Tony H. R. Skyrme in 1958 as a way of representing the nucleon (a fermion) as a topological soliton of bosonic pion fields, skyrmions have sojourned in nuclear physics, particle physics and condensed matter physics.

Condensed matter theorists have looked for skyrmions in magnetic systems, but because of unfavorable energetics the skyrmions were too small to appear as anything but a flipped spin in the magnet. Other theoretical studies¹⁻³ have suggested that very cold two-dimensional electron systems subjected to a strong magnetic field—the same systems that exhibit the integral and fractional quantum Hall effects⁴—may also be promising places to hunt for skyrmions. (See the figure below.) Now a sensitive nuclear magnetic resonance experiment⁵ has directly measured the spin polarization of a two-dimensional electron system and seems to have found evidence for finite-size skyrmions.

The nmr results are significant for another reason as well. Almost everything we know about two-dimensional electron systems has been learned from charge-transport experiments, which are sensitive to spins in the system only when they affect its energetics. The introduction of a powerful experimental technique that can directly measure the spins promises to further elucidate the dynamics of these fascinating systems.

Two-dimensional electron systems

Assume a very cold two-dimensional electron system is trapped at a semiconductor interface with its electrons constrained to move in the x - y plane. The system is placed in a magnetic field $\mathbf{B}$, which has a component B_z perpendicular to the electron system. The Lorentz force on a current I flowing in, say, the y direction through a resistivity ρ will generate a transverse Hall voltage V_H , from which we can define the transverse Hall resistivity $\rho_H = V_H/I$. Ignoring the electrons' spins, one can think of the electrons in this system as oscillators executing cyclotron orbits. The electrons in the N th energy, or "Landau," level have an energy of $(N + \frac{1}{2})\hbar\omega_c$, where ω_c is the cyclotron frequency. Each

A sensitive nmr technique has found evidence for skyrmions in a two-dimensional electron system and challenged our understanding of the quantum Hall effects.

Landau level contains $n_B = eB_z/hc$ states per unit area, or one state for each magnetic flux quantum $\Phi = hc/e$. A system with an electron density of n_e will have a Landau-level "filling factor" ν of n_e/n_B .

The integral quantum Hall effect—in which the system's longitudinal resistivity ρ nearly vanishes, while the Hall resistivity ρ_H exhibits broad plateaus as a function of ν —occurs when ν is an integer $i = 1, 2, 3, 4, \dots$, that is, when the i th Landau level is just full. The integral quantum Hall effect is explained in terms of the sequential filling of Landau levels by *noninteracting* electrons and the effects of impurities.

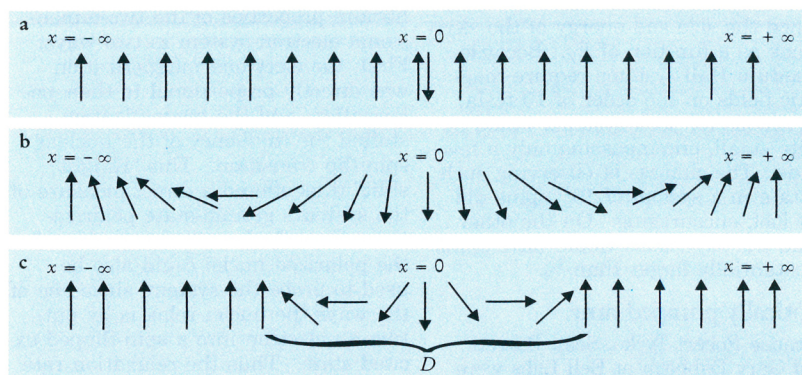
By contrast, electron-electron interactions are essential for explaining the fractional quantum Hall effect, which involves similar plateaus in ρ_H and minima in ρ , but at certain fractional values of ν . (See PHYSICS TODAY, January 1988, page 17.) In the prevailing theory the electron-electron interactions result in a hierarchy of fractionally charged quasiparticles, and the fractional effect is explained in terms of the filling of Landau levels by these quasiparticles. (See PHYS-

ICS TODAY, July 1993, page 17.)

Including spins

Including the electron spins in the description of the two-dimensional electron system complicates the situation in several ways. First, because each state can accommodate two electrons with opposite spins, one effectively doubles the number of states in each Landau level. Second, the interaction of the electron spin with the magnetic field $\mathbf{B}$ shifts the energy of the spin-up state (in which the spin is parallel to $\mathbf{B}$) down by $E_Z/2$, where $E_Z = g^* \mu_B B$ is the Zeeman energy, g^* being the effective g factor for the semiconductor and μ_B the Bohr magneton. The energy of the spin-down state shifts upward by the same amount. Thus, ignoring the interactions between electrons, one would expect that all systems with $\nu < 1$ would be fully polarized, with all the spins pointing up. If one added an electron to a system with $\nu = 1$, it would have to go into a down state and would decrease the system's total spin by one electron spin. (See part a of the figure below.)

The first indication that the effects of electron spins might not be so simple came when calculations by Edward Rezayi (California State University at Los Angeles) showed that ignoring the Zeeman energy and adding an electron to a $\nu = 1$ state caused half of the system's spins to flip, de-



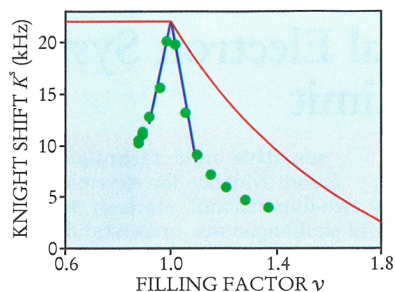
SPINS on the x axis of a two-dimensional electron system illustrate the behavior of the entire system because the system is cylindrically symmetric about the z axis. The full system can be visualized by rotating the slice about the z axis. These drawings show what happens when one adds an electron to a spin-polarized $\nu = 1$ state. a: If the electrons do not interact, the added electron will occupy a spin-down state, leaving all other spins unchanged. b: Electron interactions try to align nearest-neighbor spins, resulting in a skyrmion. c: Turning on the Zeeman energy makes it energetically unfavorable for spins to point down, and the competition between electron interactions and the Zeeman energy results in a finite-size skyrmion of diameter D .

stroying the polarization of the system. These observations remained an intriguing puzzle until Shivaji Sondhi (University of Illinois), Steve Kivelson (University of California at Los Angeles), Anders Karlhede (University of Stockholm) and Rezayi¹ used a formalism developed by Dung-Hai Lee and Charles Kane² of IBM's T. J. Watson Research Center to investigate the effects of electron interactions. They found that the true ground state for $\nu = 1$ is indeed spin polarized. However, for the ground states with ν slightly greater than 1, placing the added down spin at one position, say, the origin, was energetically unfavorable. Rather those ground states had a spin field that pointed down at the origin and then rotated outward smoothly as one moved radially outward until all the spins pointed up at a radius of $D/2$. Such a structure is a skyrmion of the spin field. (See parts b and c of the figure on page 19.) Likewise, if ν is slightly less than 1, the ground state is an antiskyrmion, in which the z and y components of the spin field are the same as for the skyrmion, but the x component is reversed. The diameter D of the skyrmion depends on a competition between the electron-electron interactions—which try to maximize D —and the Zeeman energies—which try to minimize it. If $E_z = 0$, or equivalently $g^* = 0$, the skyrmion becomes infinitely large.

Subsequent calculations by Herbert Fertig (University of Kentucky), Allan MacDonald (Indiana University), René Côté (Université de Sherbrooke in Quebec) and Luis Brey (Universidad Autónoma in Madrid)³ determined the size and energy of the skyrmions as a function of E_z . Because quantum Hall systems require magnetic fields on the order of 10 tesla, skyrmions for such systems would be quite small, encompassing only a few spins. The chances of observing such a state in a system of 10^{11} spins did not look encouraging. On the other hand the skyrmions would have spins substantially larger than $1/2$.

Optically pumped nmr

Because Robert Tycko, Sean Barrett and Gary Dabbagh at Bell Labs were unaware of the theoretical studies on skyrmions, they did not start out looking for them in their studies of two-dimensional electron systems using optically pumped nuclear magnetic resonance.⁵ Using an optical pumping technique based on a 1968 discovery by Georges Lampel they had found that they could enhance the nmr signals from GaAs quantum wells by roughly two orders of magnitude, thus allowing



UNEXPECTED BEHAVIOR. The Knight shift K^s of a nuclear magnetic resonance is directly proportional to a two-dimensional electron system's magnetization, or equivalently its polarization. The data of Sean Barrett and colleagues⁷ for K^s around filling factor $\nu = 1$ (green) are consistent with the hypothesis that the system's quasiparticles are skyrmions (purple curve) and inconsistent with the individual-electron hypothesis (red curve).

direct measurement of the spin dynamics and polarization of two-dimensional electron systems.

The optical pumping technique used circularly polarized laser light to polarize the electrons in the quantum wells. While the laser was on, hyperfine interactions between the electrons and the ^{71}Ga nuclei strongly polarized the nuclear spins, resulting in a strong nmr enhancement. When the laser was turned off, the electron system equilibrated rapidly, while the nuclei remained polarized for much longer times. (See PHYSICS TODAY, June, page 17.) Barrett, Tycko and Dabbagh used this highly polarized sample of nuclei to monitor the equilibrium properties of the two-dimensional electron system in two ways. First, the electrons' magnetization was directly proportional to their polarization, and the magnetization shifted the frequency of the nuclear spin-flip transition. This "Knight shift" thus offered a direct measure of the system's ground-state polarization. Second, the relaxation rate of the polarized nuclei could also be used to probe the system, since one of the ways the nuclei relax is by putting an electron into a spin-flipped excited state. Thus the relaxation rate of the nuclear polarization was proportional to the number of spin-flipped excited states that lay near the system's ground state.

Meanwhile their collaborators Loren Pfeiffer and Kenneth West, also at Bell Labs, were using their molecular beam epitaxy machine to its full capacity to grow an unusually large GaAs-GaAlAs multiple-quantum-well structure (containing 40 wells) for the group's opti-

cally pumped nmr studies of filling factors near $\nu = 1$. The group placed this sample in a magnetic field and realized the $\nu = 1$ state. By tilting the sample with respect to the magnetic field to change B_z , the experimenters could realize states with ν slightly greater than 1—corresponding to a fully polarized state plus a quasiparticle—and ν slightly less than 1—corresponding to a fully polarized state plus a quasihole. If the quasiparticles for the $\nu = 1$ state were spin- $1/2$ quasielectrons or quasiholes, one would expect the polarization to be constant for $\nu < 1$ and to decrease gradually for $\nu > 1$. Instead Barrett and company saw the polarization drop steeply and symmetrically on either side of $\nu = 1$. (See the figure on this page.)

Barrett, Tycko and Dabbagh were also obtaining interesting results with the relaxation data, which showed very slow, temperature-dependent decay rates at $\nu = 1$ and $\nu = 2/3$, where quantum Hall effects are observed, and much more rapid, temperature-independent rates for $\nu = 0.88$, where no quantum Hall effect is seen. These results indicated that the system had energy gaps between the ground state and spin-flipped excited states only at the values where quantum Hall effects were evident.

At this point the experimenters knew they had some puzzling results. Their data for $\nu = 1$, which should be in the integral quantum Hall regime, indicated that the relevant quasiparticles were not individual electrons. Jim Eisenstein of Bell Labs had the preprint of Fertig's paper on his desk when Barrett showed him the results, and so was able to play matchmaker between the experimental results and the skyrmion theory.

Anyons, anyone?

The match has so far been a happy one. Kivelson thinks the observations are particularly important because they challenge our prejudices about the quantum Hall effects: "Skyrmions are a qualitative effect of interactions. Hence the discovery of skyrmions at $\nu = 1$ overturns the conventional wisdom that the integer quantum Hall effect, in contrast to the fractional effect, is not fundamentally affected by interactions."

Barrett, now at Yale University, plans to extend his measurements to lower temperatures and higher magnetic fields. Another promising line of inquiry is studying two-dimensional electron systems with significantly lower electron densities. These systems, which exhibit quantum Hall effects at much lower magnetic fields, would have much lower Zeeman shifts and so might be expected to

have larger skyrmions.

Andi Schmeller, also at Bell Labs, and Eisenstein have been performing charge-transport experiments on such a system, using the quasiparticles' Zeeman energies to determine their spins. Preliminary results suggest that near $\nu = 1$ the system's spin changes by the equivalent of 5 to 7 electron spin flips for every skyrmion-antiskyrmion pair created. Schmeller and Eisenstein also see a qualitative difference between the behavior of the system at $\nu = 1$ and that at $\nu = 3$ or 5, where skyrmions might also be expected to occur. This observation would seem to confirm predictions by Jainendra Jain and Xiao-Guang Wu of the State University of New York at Stony Brook, and by Wu and Sondhi,⁶ that skyrmions will not be the relevant quasiparticles for $\nu = 3, 5, \dots$

While the calculations of Sondhi, Jain and Wu and the experimental results of Schmeller and Eisenstein seem to rule out skyrmions for $\nu = 3, 5, 7, \dots$, the fractional regime still appears to be a promising hunting ground. Sondhi is particularly enthusiastic about the prospects of skyrmions around $\nu = 1/3$: "At $\nu = 1$ skyrmions are fermions, but at $\nu = 1/3$ they would be anyons, obeying fractional statistics—they would be a particularly elegant illustration of Skyrme's deep ideas." Could skyrmionic anyons be observed in a two-dimensional electron system? Direct experimental observation would be difficult, according to Frank Wilczek of the Institute for Advanced Study, who helped to introduce the concept of anyons. (See PHYSICS TODAY, November 1989, page 17.) However, he adds, "Fortunately, fractional statistics is deeply connected to fractional spin, and the recent developments make me quite optimistic that *this* will soon be observed clearly and directly."

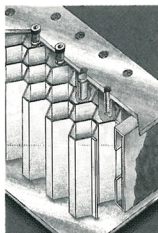
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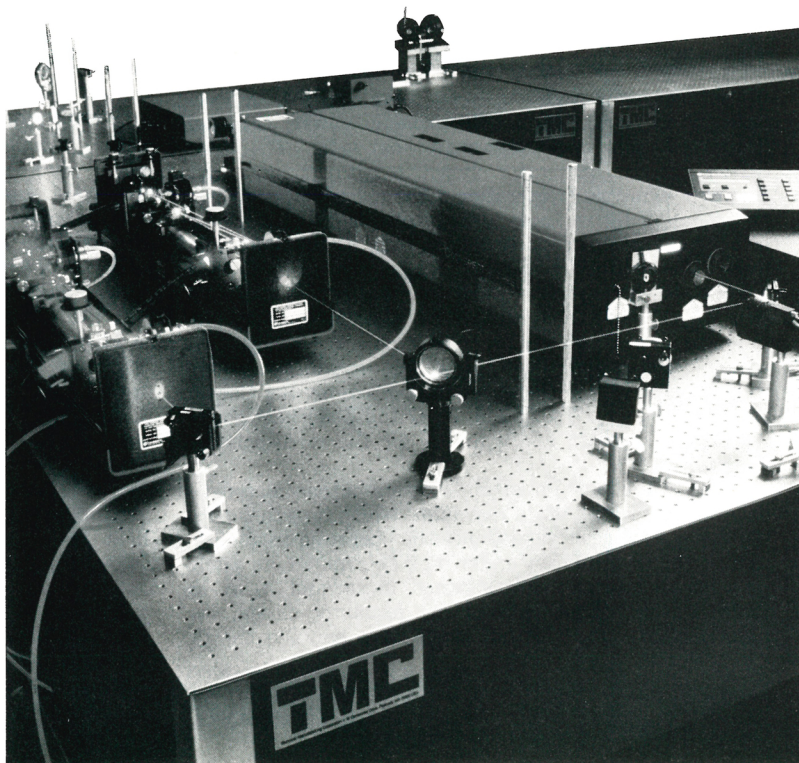
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