How Would a Physicist Design a Tennis Racket?

Physics, anyone? Contemplating the analytical mechanics of tennis rackets may improve your game—but it's no substitute for practice.

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Tennis players dream of finding the perfect racket that will immediately transform them into champions. While that may be wishful thinking, it is generally agreed that today's rackets are much better than those of 20 years ago. Though they may not turn you into an instant Wimbledon winner (after all, your opponent has one too), they will clearly improve your game. There is still hope among inventors, racket manufacturers and players that a perfect racket will come along someday. If and when such a racket is developed, what will its properties be and how will it affect the game of tennis?

As a tennis player for more than half a century, and a physicist for not quite that long, I never thought about these problems until the oversize rackets came along in the late 1970s. None of the tennis professionals I asked were able to explain the new rackets, and there was almost nothing in the technical literature. So I set up a small lab to do simple experiments and I began to see tennis in a different light. There I have accumulated a lot of interesting data, resulting in a number of papers, a book and even a commercial videotape.

The official rules of tennis give the player a great deal of latitude in what he or she may use to hit the ball. The rules specify a maximum total length for the racket frame and maximum length and width for the head. For a long time the rules allowed the frame to be "of any material, weight, size or shape." But the proliferation of oversize rackets in the late 1970s made it clear that some limits were needed. The present rules also require that the crossed string pattern be flat and that the shape or weight distribution of the racket not change in the course of playing a single point.

There still are no restrictions concerning the type of material used in the frame or strings; the number of strings; the racket shape, weight or weight distribution; or even the thickness of the frame. Head shapes have ranged from round or oval to square, diamond or hexagonal. Heads have been joined at various angles to the shaft, which itself is allowed to be bent. Split shafts are quite common. Though eight-sided racket handles are standard, handles have been made with hexagonal and even rectangular cross sections. One need only to look in the files of US Patent Office to appreciate the ingenuity of racket inventors.

The rules concerning the ball, on the other hand, are very stringent about size, weight, color, deformation under load and rebound. To be sanctioned for official tournament play, a tennis ball may rebound no more than 58 inches and no less than 53 inches when it's dropped onto a concrete surface from a height of 100 inches. This requirement, that the ball lose almost half its energy in such a rebound, significantly influences the design of the racket.

What would a perfect racket be like? It would clearly be very light and have minimal air resistance, so that it could be swung with little effort for hours on end; and yet the player's hand or arm would feel no unpleasant shock, jar or vibration when the ball is hit. The racket would be "powerful"; that is to say, high speed could be imparted to the ball with moderate racket head speed. Furthermore, the racket's response would be uniform and completely predictable.

The strings

The strings of a tennis racket act as a medium that absorbs much of the incoming ball's kinetic energy and then returns some fraction of that energy back to the ball. Strings are used instead of an elastic membrane because the air resistance of a membrane would be too great. You can demonstrate that by swinging a racket with and without a piece of paper covering the strings. The effect of the paper is quite pronounced.

Tennis players usually specify the tension they want in their strings, but the absolute value is not very meaningful, because the spacing between strings and the size

JIMMY CONNORS prepares to hit a two-handed backhand. Connors uses less wrist or forearm rotation here than most other professionals, so his two-handed backhand has an unusually long radius of swing. FIGURE 1

of the head vary from racket to racket. A more meaningful measure of how the strings will perform is how much the strings deform under a given load. One can think of the string plane as if it were a spring; then the plane deformation determines an effective spring constant k. If k and the ball mass (58 grams) are known, one can estimate the dwell time of the ball on the strings: The dwell time should be about 5 milliseconds, half the period of the corresponding simple harmonic motion. Such estimates are in good agreement with measurements of the actual duration of contact between racket and ball. But as the ball speed increases, the dwell time becomes shorter.[1] That's because the string plane deformation becomes nonlinear with increasing force. The harder you hit the ball, the stiffer, in effect, are the strings.

In the ball–racket interaction it is advantageous to have most of the energy stored in the strings, which can give back as much as 95% of it, rather than in the ball, which is designed to dissipate energy. Having the strings deform more, and therefore store a larger fraction of the energy, results in more kinetic energy being returned to the ball. Direct measurements of ball rebound speeds for different string tensions tend to agree with this line of argument.

I have determined a lower limit on the fraction of incident energy returned by the strings to a ball by dropping a wooden bocce ball (which absorbs much less deformation energy than a tennis ball) onto a horizontal, clamped racket head and measuring the ratio of rebound height to drop height.[2] That ratio ran from 0.93 to 0.95, which shows that the strings dissipate very little of the energy they absorb. It follows that tighter strings, allowing less string-plane deformation, generate lower ball speeds in play.

There is, of course, a point of diminishing returns. After all, you can't play tennis with a butterfly net. When the strings start moving and rubbing within the string plane, you begin to get serious energy loss.

The conventional wisdom nowadays is that tighter strings allow the player more control over the ball. Because control is not a well-defined term, I know of no definitive experiment that has proved or refuted this claim. Twenty years ago the conventional wisdom was that *loose* strings gave control and tight strings gave power. The argument went: "Most top professionals string their rackets at high tension, and they hit the ball very hard. Therefore tight strings give power."

The elasticity of the strings is a very important factor in their ability to store and return energy. Because thinner strings are more elastic, manufacturers have been developing thinner strings to improve performance. Gut (made from beef intestines) is the preferred material for strings, since it retains its elasticity at high tensions. Many synthetic materials lose some elasticity as the string tension increases, and that leads to a stiff, or "boardy," feeling when shots are hit hard.

The string manufacturer seeks to optimize strength, elasticity and durability. A typical tennis string is made of a core (or multicore), one or more twisted wrap layers and possibly a coating. Decisions about how to construct the string and what material to use are based on measuring the dynamic tangent modulus of elasticity at high tension, the tensile strength, the abrasion resistance (most strings break because of abrasive weakening) and the relaxation properties (strings eventually lose tension). Stringing at higher tension will often prolong string life, because it inhibits in-plane movement and the resultant abrasion.

Center of percussion

If the racket vibrates or smarts against your hand when you hit the ball, you will be unhappy. But the shock or jar is minimized if the ball strikes the head of the racket at a certain location. That location (actually a pair of conjugate points) is called the center of percussion. When a ball hits the center of mass of a free racket initially at rest, the racket will recoil to conserve momentum. If the ball hits in a region near the geometric center of the head (well beyond the center of mass), the racket will still recoil to conserve linear momentum, but it will also pivot about the center of mass to conserve angular momentum. Most of the racket will then be moving in the original ball direction, but the handle end will be moving in the direction of the recoiling ball.

It is possible then for the two motions (translation and rotation) to cancel out at one location on the handle. If you were holding the racket at exactly that location, your hand would feel a minimum of shock when the ball hit. For every point on the handle, there is a conjugate point of ball impact (the center of percussion) that will yield this desirable cancellation. For the normal one-handed end-of-handle grip, the racket designer strives to have the center of percussion close to the center of the strung region of the head.

One can determine the distance from the hand to the center of percussion by turning the racket into a physical pendulum pivoted on an axis through the grip and parallel to the string plane. One then measures this pendulum's oscillation period T.

If a struck ball undergoes a momentum change Δp, a racket of mass M will have a recoil velocity $V = \Delta p / M$. If the ball hits at a distance b beyond the center of mass, the racket will rotate with an angular velocity $\omega = b\Delta p / I$, where I is the racket's moment of inertia about its center of mass. If a is the distance from the center of mass to the hand, then the condition for cancellation of motion at the hand is that V must equal ωa. Thus we conclude that the moment of inertia about an axis in the string plane and through the racket's center of mass should equal abM. Measuring the period T gives the racket's moment of inertia about the pendulum pivot. Then one can use the parallel-axis theorem of mechanics to conclude that the distance $a + b$ between the hand and the desired impact point is $gT^2/4\pi^2$.

For the classic wooden rackets of bygone days, the center of percussion was usually in the throat area of the head. In figure 1 tennis star Jimmy Connors wields a typical modern racket, with a larger head and lighter frame than the wooden rackets had. These rackets have the center of percussion close to the center of the head. Figure 2 shows me testing such a racket in my lab at the University of Pennsylvania. In some modern rackets the center of percussion is actually closer to the tip of the head than to the throat.

Racket vibrations

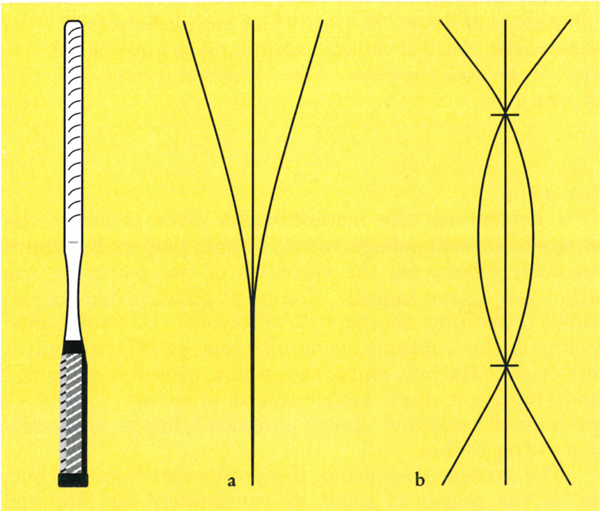
When a tennis racket handle is clamped in a vise, the frame can oscillate in the manner shown in figure 3a. A freely suspended racket would exhibit the oscillation mode shown in figure 3b. To determine the frequencies and nodes of these two modes, I taped a small piece of Kynar, a thin piezoelectric film, to the throat of a racket and observed its output when the racket was struck in various locations. For a typical racket with its handle firmly clamped, the fundamental frequency was about 25–30 hertz, while a free racket had a lowest frequency in the range of 100–200 Hz. The precise eigenfrequencies depend on the frame's mass and stiffness. For a given racket weight, the measured resonant frequencies are good stiffness indicators.

To determine the behavior of a free (as distinguished from constrained) racket, I suspend the racket from strings attached along the nodal line in the handle in order to minimize external damping. Because there has been some question as to whether a clamped or free racket is a better laboratory model for a handheld racket, I measured the vibration frequencies of rackets held in the hand. Finding no sign of the lower frequencies when the rackets were struck near the throat or far end, even when the hand grip was very tight, I concluded that a freely suspended racket is the better laboratory approximation to what happens on the court.[3]

IN HIS LABORATORY at the University of Pennsylvania, Brody examines whether the impact position of a tennis ball can be determined from the magnitude and phase of the resulting racket vibration. FIGURE 2

The amplitude of the frame's oscillation depends on the relative velocity between racket and ball, the frame's stiffness and the distance from the impact point to the node in the head. You can determine the location of that node without special equipment, simply by holding the frame at the top of the grip (the other node) with two fingers and striking the strings at various places along the midline. You will be able to feel the amplitude of the resultant oscillation and it will have a minimum when the racket is struck at the node. Figure 4 illustrates the dependence of the oscillation amplitude on the point of impact. Looking at the leading edges of the signals, it is clear that the oscillation phase changes as the impact point moves from below to above the node.

Many manufacturers advertise that their frames damp out vibrations quickly and therefore feel better and do less harm to your arm. There is no clinical evidence that racket vibrations are the cause of the notorious "tennis elbow" ailment that plagues so many players. The very best device for quickly reducing the amplitude of vibrations is the human hand, and not some special material built into the racket frame.[4] Figure 5 shows the oscillation of a handheld racket (top trace) and of the same frame freely suspended. Some designers have tried using encapsulated granules, lead shot or other very small objects that can move around inside the



VIBRATION MODES of a tennis racket (a) with the handle clamped and (b) with both ends free. FIGURE 3

frame. Such schemes can do an effective job of absorbing some of the vibrational energy left in the racket after the ball has departed.

One often sees the strings near the throat festooned with small rubber beans, plastic worms or other objects put there to damp out vibrations. They do indeed reduce string vibration near 500 Hz, but they do essentially nothing to damp out frame vibrations.[5] The effect on play is purely psychological; they just make the hit sound cleaner. If such small ornaments attached to the strings were really absorbing most of the vibrational energy of the much heavier racket, they would get pretty hot after a few hundred hits.

I know of very few serious attempts to find out where on the racket the average tennis player hits the ball on a ground stroke. There is, however, a published report by Herbert Hatze[6] (University of Vienna) which contends that the node, rather than the center of percussion or the geometric center of the strung area, is the place where most players hit the ball.

Power from the racket

Players look for the frame that will give them the highest ball speed for the least effort. Two-body kinematics tells us that for a given head velocity a heavier racket will impart more speed to the ball. But the head speed is of course not entirely independent of the racket mass. Unlike a baseball player, who may swing 10–15 times in an entire game, a tennis player swings hundreds of times during a match. I have seen no data on swing speed versus racket weight or moment of inertia. Nor have I seen any studies of fatigue versus racket weight.

The recent trend, however, has been toward lighter rackets. The classic wooden rackets of 20 years ago weighed 14 to 15 ounces, and they were of a neutral balance; that is to say, the center of mass coincided with the geometric center. Some of the newest frames on the market weigh only 9 or 10 ounces, and they are head-heavy. This very considerable weight reduction has been accomplished by using modern composite materials (usually graphite fibers) and removing as much weight from the handle and shaft as possible. Even though a 30–40% overall reduction has been achieved in the weight of the racket, the "swing weight," as tennis players call the moment of inertia about the butt end, has not been reduced in the same proportion. The result is a racket that is significantly lighter but still packs almost the same punch as a heavier racket would.

In the tennis literature there are often statements to the effect that a very firm grip on the racket will add the arm's weight (inertial mass) to the racket's weight, therefore giving more "power." By performing a simple experiment I have been able to prove that assertion wrong. I measured the resonant frequencies of both a free racket and one that was being gripped very tightly. The gripped racket had a lower frequency, as one would have expected, because of damping by the hand. Ignoring the damping, we can assume that the entire lowering of the resonant frequency is due to some fraction of the mass of the hand and arm being added to the racket. I determined experimentally that adding 40 grams to the handle end of the free racket produces the same frequency shift as a tight grip. A scant ounce and a half is not enough additional mass to increase ball speed appreciably.

The speed imparted to a ball depends on where it strikes the racket head. If the ball is hit precisely at the racket's center of mass, no energy goes into racket rotation. With increasing distance from the center of mass, more and more energy is diverted into racket rotation. By moving mass from the handle to the head, the designer moves the center of mass up toward the center of the head, thus increasing the speed imparted to the ball. Sometimes, however, the ball strikes the racket off center, or even off axis. To minimize racket rotation when this happens, one wants to maximize the moments of inertia about the two principal axes in the plane of the frame.

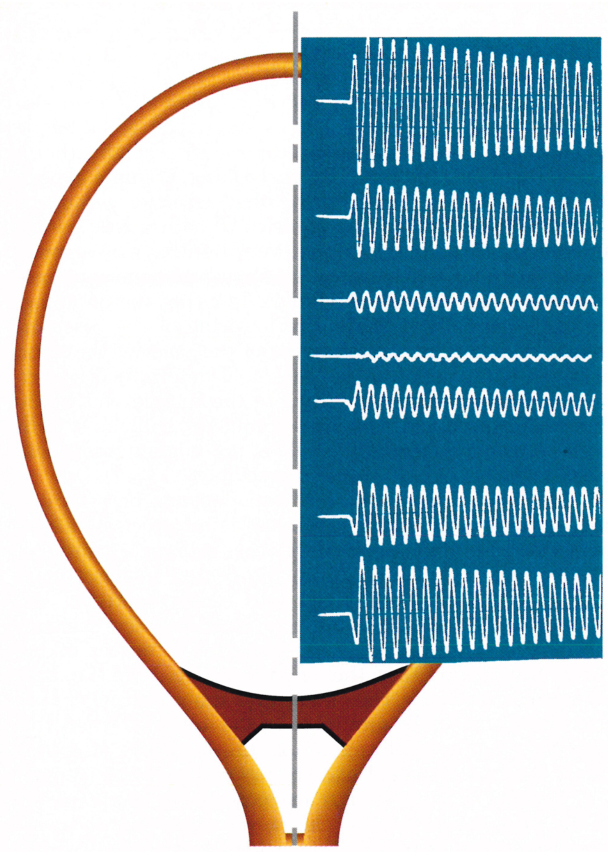
The fact that the strings are more elastic in the middle of the head tends to shift the power point slightly away from the center of mass. The racket is additionally deadened near the tip because that's where the frame is most flexible and flexing takes energy away from the ball. The newest (inappropriately called "wide-body") rackets overcome the latter problem by increasing the thickness of the frame near the tip.

The dominant factor in a racket's response to a particular hit is the distance of the impact from the two in-plane principal axes. A ball hitting just a few inches away from the long (polar) axis will rebound with low velocity, because the moment about this axis is about one-tenth of the moment about the other in-plane axis. One can increase the polar moment either by adding weight at the periphery of the frame or by making the racket wider. Because the moment of inertia goes as mr^2, a wider racket head is the preferred solution. Measuring the polar moments of a number of rackets with a thin-wire torsional pendulum, I have found that the moments scale well with mass times the square of the head's width.[7] That's because most of the popular rackets, even the oversize ones, have essentially the same head shape. So it's size that determines the moment of inertia.

Racket manufacturers test their products by firing tennis balls at a stationary racket that is either freely suspended or held in a very flexible mount. They record the ratio of the ball rebound speed to incident speed for impacts at many locations on the head. That ratio, designated as e, is often mistakenly described as the coefficient of restitution. But it is not, because it neglects the recoil velocity of the racket. Typical values of e run from about 0.5 for on-axis impacts near the center of mass to 0.2 for impacts well off axis or near the tip.

Ball–racket interaction

Unlike the laboratory situation, on the tennis court most shots are hit with the racket moving at a speed comparable with that of the incoming ball. Therefore to apply the



VIBRATION AMPLITUDE TRACES for a freely suspended tennis racket struck at various locations on its longitudinal axis. At the node the phase of the oscillation is reversed. FIGURE 4

laboratory data to actual play one must transform the interaction into a reference frame where the racket is at rest, solve the problem and then transform back into the court (moving-racket) frame. For most players a nonrelativistic treatment should suffice.

Doing this simple exercise, one finds that v', the ball's rebound velocity on the court, is given by

$$v' = -ev + (1+e)V \qquad (1)$$

where v is the ball's incident velocity on the court and V is the velocity of the racket's impact point. (Both terms are positive, because v is negative relative to the other velocities and e is positive by definition.)

Because players *swing* rackets rather than simply translating them forward, the velocity of the impact point depends on the effective pivot point of the swing at the moment of impact and on the distance of the impact from that pivot.

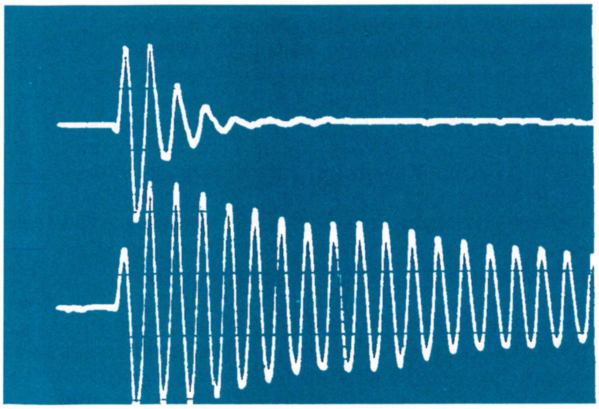
I determined the instantaneous pivot points of the swings of several college varsity tennis players by simultaneously measuring the velocities of two points on the racket just before impact. The pivot point varied considerably depending on the type of stroke. A two-handed backhand, for example, generally has an effective pivot very close to the butt end of the racket, while a one-handed forehand has a much longer radius of swing. Connors's two-handed backhand, shown in figure 1, has an unusually long swing radius.

The average distance of the pivot point from the butt end of the racket is about 20 centimeters for forehand ground strokes. With that information and the e map of the racket head, one can determine the location on the racket that will produce maximum ball speed, either for a specific player and racket or for an average swing radius and a mathematical model of rackets of various mass distributions and string properties. In the older rackets, the maximum-power location was close to the throat, but in the latest generation the power point is near the center of the head and close to the node and the center of percussion.

On a serve, as distinguished from a ground stroke, the effective pivot point is very close to the butt end of the racket, because the serve involves considerable pronation, or axial rotation, of the forearm. Therefore the racket tip is moving much faster than the throat. Furthermore the new head-heavy rackets have the center of mass closer to the tip than did the old neutral-balance frames. Also, the extra stiffness of the newest racket heads cuts down energy dissipation due to frame deformation near the tip. These three effects result in a displacement of the maximum-power point up the racket from where it is for ground strokes. All other considerations aside, that upward displacement makes the latest rackets better for serving simply because, in effect, they make the server taller. Figure 6 shows these locations for a flexible, neutral-balance, oversize racket (the old Prince Classic) and for a more modern, quite stiff, head-heavy frame.

Designing a better racket

No tennis racket will correct for your mistakes if you hit the ball in the wrong direction or much too hard. But a good racket provides a uniformity of response when the ball hits at different locations on its face. That lets you miss the "sweet spot" (or whatever spot on the strings you are aiming for) and still have the ball end up where it would have gone if it had hit the optimal location on the strings. Some of the ways one might achieve this desired uniformity have been outlawed by the International Tennis Federation: for example, a curved string surface to "focus" off-axis hits or a stabilizing gyroscope built into the frame. There are, however, other ways to get a more



DAMPING BY HAND-HOLDING is evident in the vibration trace (top) of a hand-gripped tennis racket struck at its head end. For comparison, the bottom trace records the vibration of a freely suspended racket struck in the same way. FIGURE 5

POWER SPOT FOR SERVING a tennis ball is higher on the racket than for a ground stroke, because the effective pivot point of the service motion is very close to the butt end of the racket. For a very stiff, head-heavy racket (right) the power point is higher than it is for a more flexible, neutral-balance racket (left). Successive contours going outward from the optimal power point (red) indicate loci for 99%, 95% and 90% of maximum ball speed. FIGURE 6

uniform response over the face of the racket. We've already discussed some of these: stiffer frame, larger moment of inertia and center of mass higher on the racket head. Minimizing the moment of inertia about the butt end while maximizing the moment about the center of mass yields a powerful yet maneuverable racket.

Another way to compensate for off-axis hits is to design an asymmetrical racket. If the long axis of a symmetric racket is horizontal at impact, a ball striking above the axis will twist the racket so that the ball rebounds at a large upward angle with little speed. Thus the ball is likely to clear the net and land in the court. But if the ball strikes below the axis, its diminished speed accompanies a more downward trajectory, so that the ball usually goes into the net. That would be an argument for designing an asymmetric racket with the handle displaced to one side in the string plane. Players would then aim to hit the ball above the axis.

So much for off-axis hits. What about the location of the hit along the racket's long axis? It would be nice if the response of the racket to the swing of a typical player could be made independent of the longitudinal position of the hit. For a steady rally, the incident velocity v of equation 1 is typically half of v', the ball's velocity just after it's hit. Therefore

$$v' = \frac{1+e}{1-e/2}V \qquad (2)$$

The racket speed V at the point of impact equals ωr, the product of the racket's angular velocity and the distance from the effective pivot to the impact point.

If one wants v' to be independent of r, then the quantity $r(1+e)/(1-e/2)$ should be a constant k, independent of where along the axis the ball hits the racket. Solving that condition for e, one gets $e = (k-r)/(r+k/2)$. The constant k is determined once the effective pivot distance is known. For typical values of e that can be achieved with present racket technology, k ranges between 1 and 1.4 meters. Manufacturers have not yet attempted to implement this desired variation of e.

Buying a racket

If you're not going to design and make your own racket, you'll have to buy a frame off the shelf. Some words of advice: Don't automatically pick the one that Andre Agassi or Steffi Graf uses. The manufacturers give the stars a lot of money to use specific rackets with prominent logos. The professional player selects a frame that suits his or her style of play. It may not be the best frame for you.

As a general rule, a bigger racket head gives the average amateur more stability, more power and fewer mis-hits. A lighter frame with a midsize head is best for the serve-and-volley player or the very competitive ground-stroker who swings with abandon. The more conservative recreational player should use an oversize (or "super-oversize") racket, not strung too tightly, of a weight and inertial moment that make the hits feel solid.

Knowing the physics of the game may improve your pleasure. But it's not enough: I have never won a tennis tournament.

Equipment used in some of the test measurements was provided by grants from the United States Tennis Association and from the University of Pennsylvania Research Foundation. Tennis rackets used for testing were supplied by Prince Mfg.

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