Supersymmetric QCD Sheds Light on Quark Confinement and the Topology of 4-Manifolds

A new analytic computational technique has been developed in a wide class of supersymmetric Yang–Mills models. In physics the technique provides some of the first general exact results in four-dimensional quantum field theory, including a quantitative description of confinement via magnetic monopole condensation. In mathematics the topological study of smooth four-dimensional manifolds has been revolutionized.

Perturbation theory has long been one of the most successful tools in quantum field theory. The technique produces a series of terms, and ideally most of the "physics" is captured by the more readily computed early terms in the expansion, with later terms supplying progressively more negligible corrections. In contrast to such approximations, exact computations in four-dimensional quantum field theory have been few and far between. As Stephen Shenker (Rutgers University) puts it, "The conventional wisdom was that exact results in four-dimensional quantum field theory were essentially unobtainable." This lack of analytic results has been particularly problematic for the theory of quark confinement, which involves the strong force in a thoroughly nonperturbative regime.

In the past 18 months, however, a number of problems have been solved, exactly and in four dimensions, for numerous strongly interacting non-Abelian Yang–Mills models. In particular the solved models exhibit a specific mechanism of quark confinement, induced by a condensation of magnetic monopoles that is analogous to the condensation of Cooper pairs in superconductors.

These developments began with work by Nathan Seiberg[1] (Rutgers; currently on sabbatical at the Institute for Advanced Study, Princeton, New Jersey) and coworkers at Rutgers, who took what is in retrospect a simple insight and used it to remarkable effect. Shenker calls Seiberg's technique "the first general calculus for obtaining exact results in four-dimensional quantum field theory." Seiberg then worked with Edward Witten (Institute for Advanced Study) to apply the new methods to more complicated theories, for which they obtained their first proofs of confinement.[2]

The only catch is that the exactly solved models are supersymmetric, and thus far nature seems very well described by the (nonsupersymmetric) standard model. Nevertheless both Seiberg and Witten believe that some of their results (they are not sure which) will be true for nonsupersymmetric models. In this view, supersymmetry is not a prerequisite, but is simply a tool that has made it easier to prove the results and discover the structures that are at work in strongly interacting non-Abelian gauge theories such as QCD. Others, however, are more skeptical. Gerard 't Hooft (University of Utrecht, the Netherlands) points out that supersymmetric models are very special, containing massless fermions that interact strongly. "Therefore one must be very careful before concluding anything about real quarks and gluons," he warns.

In addition to providing clues about the physical universe, the work has also sparked what topologist John Morgan (Columbia University) calls "a complete revolution" in the study of smooth four-dimensional manifolds by mathematicians. Witten showed how to use the exactly solved physical models to compute topological characteristics of 4-manifolds.[3] Such links between Yang–Mills models and topology have been known for about a decade, but in recent years "the technical difficulties of pushing the results further were becoming overwhelming," Morgan told us. The results of Seiberg and Witten provided, he said, "a new and sharper tool. For a six-week period a new result was announced about every week. No one dreamed it could possibly be this simple."

The power of holomorphy

Numerous models have now been studied by Seiberg and his collaborators. The models can be distinguished by their gauge groups and by which multiplets of particles, such as "quarks," are present as fundamental fields that appear in the Lagrangian. QCD, for example, has the gauge group SU(3), and each flavor of quark is represented by a field with three color components. The models are also classified by the degree of supersymmetry present. The most general class is $N=1$ models, which have at least one supersymmetry. Other classes of interest in four dimensions are $N=2$ and $N=4$ models, which satisfy progressively more restrictive supersymmetry conditions. Ordinary QCD could be called "$N=0$," indicating the absence of super-symmetry altogether. (The "N" indicating degree of supersymmetry is completely distinct from the "n" of an SU(n) gauge group.)

Seiberg began in the fall of 1993 by studying specific $N=1$ models, and through the following year he worked on increasingly complicated examples. One of his results was to prove the nonrenormalization theorem exactly.[4] This theorem in supersymmetry states that renormalization does not change terms appearing in the action, and is usually proven in perturbation theory. Although the proof works to all orders in perturbation theory, nonperturbative effects may invalidate it for strong coupling. The key to Seiberg's work was the observation that in supersymmetric theories an important part of the action is holomorphic in coupling constants, a mathematical property that greatly restricts how the theories may vary as the coupling is changed.[4] In particular this property allows the theories to be studied in the weak-coupling regime (where perturbation theory is valid), and many results extend to strong coupling. This technique allows exact studies of spontaneous supersymmetry breaking—a process of great importance for developing a supersymmetry theory that can reduce to the standard model at low energy.

Seiberg further exploited the holomorphic property to determine the exact quantum ground states, or vacua, of the $N=1$ gauge models that he studied. It is a general feature of these models that they have a space of degenerate vacua, somewhat like the space of equivalent vacua in electroweak theory before spontaneous symmetry breaking selects a single vacuum. An important difference, however, is that the supersymmetric vacua are not equivalent; different vacua have different spectra of excitations.

In the spring and summer of 1994 Seiberg worked with Witten on $N=2$ models, and it was with these models that they first demonstrated results relating to confinement and chiral symmetry breaking. The papers by Seiberg and Witten present the calculations in detail for the gauge group SU(2). Subsequent work by Philip C. Argyres and Alon E. Faraggi at the Institute for Advanced Study and Albrecht Klemm, Wolfgang Lerche, Stefan Thiesen and Shimon Yankielowicz at CERN has extended some of the results to a general SU(n) gauge group. After the $N=2$ work with Witten, Seiberg worked with Rutgers postdoc Kenneth Intriligator to further extend the results to more general $N=1$ models.[5]

When asked about the prospects for extending the results to models akin to the SU(3) × SU(2) × U(1) standard model, Seiberg told us: "Changing the symmetry group and particle representations, or doing it without $N=2$ symmetry but in the more generic $N=1$ models, is relatively easy. The real challenge is to do it without supersymmetry at all."

Monopoles and confinement

In all the models an important role is played by magnetic monopoles. (The "magnetism" is related to the charge of the elementary quarks in the same way that ordinary magnetism is related to electric charge.) The monopoles are not elementary particles added to the theory by hand, but instead they arise naturally as collective excitations (solitons) of the true elementary fields. For some of the models, such as the $N=2$ cases, these monopoles are present for all the vacua. For other models they are known to be present only for special vacua and it's unknown if they are present for others.

Similar monopoles were studied in the 1970s by 't Hooft and by Alexander Polyakov (then at the Landau Institute, Moscow) and Lev Okun (Institute for Theoretical and Experimental Physics, Moscow), but those monopoles were very massive objects. In the supersymmetric models, Seiberg explained, "as you move around the space of vacua the monopole masses change, and for certain vacua one of the monopoles becomes massless. Now if we turn on an appropriate perturbation in the theory, these massless monopoles condense the same way that Cooper pairs condense in a superconductor." In the $N=2$ theories the perturbation is the generic interaction that gives a mass to certain fields and weakly breaks the $N=2$ symmetry down to an $N=1$ symmetry; in the $N=1$ theories the interaction is similarly generic.

In a superconductor, the condensation of electrically charged pairs causes the Meissner effect, the exclusion of the magnetic field from the superconductor. Conversely, in the dual process, condensation of magnetically charged particles would cause the "exclusion" of the electric field from the vacuum. When the "electromagnetism" is that of quarks and SU(3), this is precisely the confinement of quarks. In type-II superconductors, tubes of magnetic flux can thread through the superconducting material; in QCD such vortices of "electric" flux correspond to the interiors of hadrons and mesons.

In supersymmetric models that have enough flavors of quarks, Seiberg and Witten also see chiral symmetry breaking via the monopole condensation mechanism. (Chirality relates left- and right-handed massless fields.)

An important approach to confinement in standard (nonsupersymmetric) QCD is with lattice gauge models, work originated by Kenneth Wilson (then at Cornell University). "The numerical evidence is extremely convincing," David Gross (Princeton University) told us. "But it's not analytic," he added. In the quest for an analytic description in the 1970s theorists tried various mechanisms, including monopole condensation. (For a review see reference 6.) "Perhaps the most ambitious attempt was that of 't Hooft," Gross said, "but you couldn't really do calculations. The work by Seiberg and Witten provides some evidence that a picture involving monopole condensation is correct." 't Hooft, who also points to related work by Stanley Mandelstam (University of California, Berkeley), says that in the 1970s qualitative understanding of confinement was sought—and achieved. The chief advance of the new work is the exactness, he says.

Seiberg and Witten believe it is likely that the supersymmetric models that exhibit confinement are "continuously connected, without a phase transition, to ordinary QCD." Such an interpolation, if it can be proved, would provide a concrete realization of 't Hooft's proposals from the 1970s.

The secrets of duality

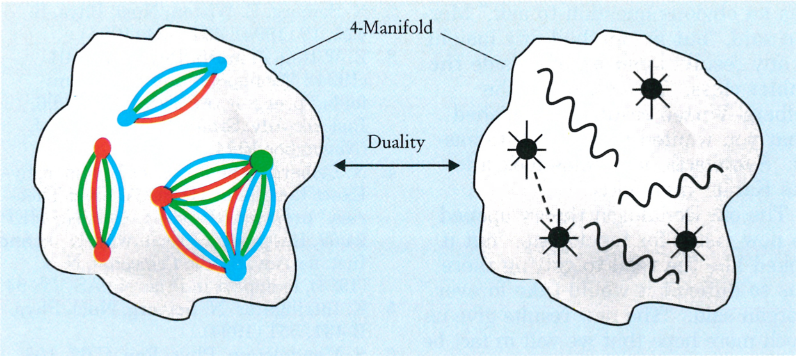
What lies behind the remarkable results other than the powerful consequences of holomorphy? The answer seems to be an as-yet poorly understood duality. "I think the duality business is very important for physics and will lead to a lot more discoveries," Witten said. "It has a lot of secrets. We know one of its mathematical secrets—the key to understanding invariants of 4-manifolds—but I think it also has a lot of secrets in physics. My interest is mainly in understanding the dualities better."

Duality is essentially the ability to interchange electric and magnetic aspects of a theory. Electromagnetism with the addition of magnetic monopoles is a dual theory; the equations for charges and monopoles have the same form. $N=4$ non-Abelian gauge theories are also apparently dual, as was first conjectured by Claus Montonen and David Olive in the 1970s. The theories can be formulated with electrically charged fundamental fields, and the magnetic monopoles appear as collective excitations. Alternatively, one can regard the magnetic variables as fundamental, and then the electric variables appear as collective excitations. Evidence for the duality of $N=4$ models has been accumulating for the past year or so: Ashok Sen (Tata Institute, India) showed that the spectrum of stable particles is dual, while Cumrun Vafa (Harvard University) and Witten demonstrated that certain partition functions were exactly dual. Seiberg and Witten's work established the duality for the vacuum structure.

Seiberg told us that duality is used "a little bit" in the proofs of confinement and so on, but like Witten he thinks the duality "is at the heart of the whole story." Unlike the $N=4$ models, for the generic $N=1$ and $N=2$ models the whole theory is not invariant under duality. "Nevertheless," Seiberg explains, "near the point where the monopoles become massless, the more appropriate description of the theory is in terms of monopoles and photons, instead of the underlying electric variables."

From this comes much of the power of the techniques: The electric variables at such points are strongly interacting, non-Abelian fields, as in the intractable infrared regime of QCD. The monopoles and photons, on the other hand, are weakly interacting and Abelian, and thus are as calculable as QED. Remarkably, both describe the same physical system.

Recently Seiberg showed[7] that the generic $N=1$ theory with n_f quark flavors has an unusual new kind of duality: The strongly coupled theory with gauge group SU(n_c) is related to a weakly coupled theory with SU($n_f - n_c$). (That is, a theory in which quarks have n_c colors is related to one where they have $n_f - n_c$ colors!) Daniel Friedan (Rutgers) considers this the most exciting development of the



DUALITY IS ONE OF THE KEYS to the new results. In general, duality maps between "electric" and "magnetic" variables. In the recent work duality maps from a strongly interacting regime of a non-Abelian theory with "electric" fundamental particles (left), in which calculations are intractable, to an equivalent system in which elementary magnetic monopoles interact weakly with Abelian photons (right) and calculations are simple.

work to date. He told us that the situation is reminiscent of two-dimensional conformal field theories, where many different Lagrangians may describe the same physical theory.

Manifolds and mathematics

While the implications of Seiberg and Witten's work for understanding nonsupersymmetric physical reality are still largely conjectural, profound new results in mathematics have already been proved with the aid of their N=2 Yang–Mills models.

The results relate to the topology of smooth four-dimensional manifolds. (Nonsmooth manifolds have a different story and we don't discuss them here.) Even if we did not live in a universe with three space dimensions and one time, mathematicians would find 3- and 4-manifolds the most interesting cases. Lower dimensional cases, 1- and 2-manifolds, have comparatively little structure and are understood in great detail; they are "too small" to be very complicated and are readily visualized embedded in three dimensions. Surprisingly, manifolds of five or more dimensions have also been well understood for 15 to 20 years. Many theorems have been proven for such manifolds as a class. Speaking figuratively, in four dimensions there is just enough "room" for highly complicated "tangles" to develop, while in five or more dimensions there is space enough to straighten the tangles out. "All I know," jokes Witten, "is that there aren't any quantum field theories up there, so people never ask about interesting things that have to do with quantum field theory."

Perhaps mathematicians are lucky that we *do* live in four dimensions. "It turns out," Witten told us, "that the most interesting results in three and four dimensions come from things connected with physics. In 1982 Simon Donaldson, then a student of Michael Atiyah's at Oxford, introduced the use of self-dual Yang–Mills equations to study topological invariants of 4-manifolds. Ever since then, most of the interesting results about smooth 4-manifolds have come from the study of self-dual Yang–Mills theory."

Topological invariants are properties of manifolds that do not change when the manifold is stretched and deformed. Understanding the complete set of invariants amounts to understanding the manifolds themselves. (Invariants of 3-manifolds also involve a very rich story; Witten derived one class of important invariants from non-Abelian quantum gauge theories—Chern–Simons theories—about seven years ago.)

The Donaldson invariants, as they are known, became one of the foundations of studies of 4-manifold topology. However, computation of the invariants and analysis of their structure remained tremendously complicated. Many papers in the field ran into hundreds of published pages. About seven years ago Witten showed that Donaldson's work was equivalent to computing correlation functions in an N=2 suspersymmetric gauge theory formulated on the 4-manifold that one wished to study. "At the time this wasn't useful," Witten says. In the ultraviolet regime one could use asymptotic freedom, and then one saw the equivalence to Donaldson's theorems. In the infrared the computations were as intractable as the analogous strong-coupling regime of QCD.

But the infrared behavior of the N=2 theory is precisely what Seiberg and Witten recently determined. This enabled a completely new way of computing the Donaldson invariants in the infrared. In either the ultraviolet or infrared regime, the computation amounts to counting classical solutions: Supersymmetry ensures that quantum fluctuations cancel out, because every bosonic contribution is balanced by an equivalent fermionic mode with a negative sign. In the ultraviolet, the classical solutions are instantons in a non-Abelian Yang–Mills theory. In the infrared, using the new results, one counts solutions of the "monopole equations"—the system of elementary monopoles and photons to which duality maps the theory.[3]

Columbia topologist Morgan says that "mathematicians do not understand the physical insight that led Seiberg and Witten to see that the two theories are equivalent. But they've given us a new set of equations that we can study in mathematically rigorous ways. They've given us the goose that's laying the golden egg. We don't know where they found the goose, but this time it doesn't matter."

Among the golden eggs are many results that have long been suspected to be true, based on special cases proven using old Donaldson theory. One example is a theorem on a type of manifold known as Kahler manifolds. These can be viewed either as real 4-manifolds or equivalently as complex 2-manifolds—something that is not possible for a general 4-manifold. However, the complex viewpoint adds additional structure to the surfaces. From the viewpoint of complex geometry, a central feature of such a manifold is its "canonical class": Specify the canonical class and you know a lot about the manifold. The new result, proved in the general case by Peter Kronheimer (Oxford) and Tom Mrowka (Caltech), shows that even if one "forgets about" the complex structure, the canonical class can still be recovered. "Compared to all other dimensions," Morgan says, "that's astounding. Only in dimension 4—and dimension 2, where things are simple—can anything like that possibly be true. We had been working toward that result for ten years, but only very special cases had been proven. There was no real hope of establishing it in general." The canonical classes turn out to be simple in the new approach because they are just the magnetic flux of the monopoles.

Another striking result was the Thom conjecture, which involves the embedding of a Riemann surface (a two-dimensional Kahler manifold) into the simplest Kahler manifold (the complex projective plane). Two groups—Kronheimer and Mrowka and Clifford Taubes (Harvard), Zoltan Szabo (Princeton) and Morgan—proved related results using the Seiberg–Witten techniques, and the Thom conjecture quickly followed.

In addition to such "expected" results, there have been some unexpected ones. An example relating to symplectic manifolds was proven by Taubes. Symplectic manifolds are more general than Kahler manifolds; physicists know them from classical mechanics with finite degrees of freedom: The phase space of position and momentum variables is a manifold with a symplectic structure. A wide open question in Donaldson theory was whether the invariants of symplectic manifolds were very similar to those of Kahler manifolds. "It was an obvious question to ask," Morgan said, "but no one had any insight or any results to speak of outside the Kahler class. Taubes, using the Seiberg–Witten results, established what you wanted to prove quite easily: Symplectic manifolds look a lot like Kahler manifolds."

The old Donaldson theory opened up new vistas for topologists, "but it looked like the road to getting there was so difficult it would take forever," Morgan said. "The new results give us much more hope that we will in fact be able to establish everything we could ever reasonably expect to know."

▶GRAHAM P. COLLINS

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Applications of High-Temperature Superconductors Approach the Marketplace

Eight years after transition temperatures first exceeded that of liquid nitrogen, high-T_c superconductors are being used in magnetometer sensors, prototype filters for cellular-phone base stations and magnetic resonance applications. Further progress in thin-film technology and electronics could lead to applications for high-T_c materials such as nondestructive testing, medical and geophysical sensors, communications, and multichip modules.

When high-temperature superconductivity reached temperatures above that of liquid nitrogen eight years ago, thousands of researchers jumped in, lots of funding followed, and the most enthusiastic people talked of magnetically levitated trains, computers and motors all soon to be operating above 77 K. When reality set in a couple of years later, the field settled into a large, active one, but with only the simplest of products being proposed for the following few years. (See the June 1991 special issue of PHYSICS TODAY on high-temperature superconductivity.)

Now, after many years of hard work, outstanding progress is being reported in thin film technology and electronics applications using the high-temperature superconductors. Some products are already being sold and others appear imminent. Last month newly developed high-T_c filters were displayed at the Cellular Telecommunications Conference in New Orleans, and other exciting new results were discussed at a meeting in Kobe, Japan, in December (before the devastating earthquake struck), and at the 1994 Applied Superconductivity Conference in Boston in October. (Excellent progress has also been made in high-temperature superconducting wire for magnets and power transmission, but will not be covered in this story.)

The first high-T_c products to be sold in quantity are sensors for magnetometers. Work is well along on sensors for nmr spectrometry, and prototypes are being built for use in cellular-phone base stations and in military systems. At the Kobe meeting John Rowell, chief technical officer of Conductus in Sunnyvale, California, quoted a projection of industry experts that the market for high-T_c electronics and medical applications could reach \$75–\$100 billion per year by the year 2020.

One major advantage of high-T_c devices is that when the coolant is liquid nitrogen instead of liquid helium, the refrigerator and packaging is a lot less cumbersome, the coolant boils away ten times more slowly, and it's a lot cheaper. (Liquid helium costs about \$5 per liter, whereas liquid nitrogen costs about \$0.30 per liter.)

Support

In FY 1994 the US government was spending an estimated \$148 million per year for high-temperature superconductivity R&D (including large-scale applications). The largest chunk, \$70.2 million, was from the Defense Department, followed by \$49.8 million from the Department of Energy and \$23.4 million from NSF. The biggest DOD spender is the Advanced Research Projects Agency, which was spending an estimated \$46.0 million.

Since 1987 the ARPA program on high-temperature superconductivity has spent \$220 million on developing several promising applications: rf and microwave passive components and subsystems for radar, electronic warfare, wireless communications and medical instrumentation; a conductor for power applications; and high-T_c interconnects for multichip modules. ARPA is also supporting development of a variety of low-cost reliable cryocoolers that will "enable" those applications.

ARPA also supports the Consortium for Superconducting Electronics, formed in 1989 to do precompetitive R&D on high-temperature superconductors on a cost-shared basis with several industrial firms. The consortium members are Lincoln Labs, MIT, AT&T, IBM, Conductus and CTI-Cryogenics of Mansfield, Massachusetts. The consortium is now pursuing two system applications—cellular base stations for wireless communications and medical instruments using superconducting quantum interference device (SQUID) magnetometers and gradiometers.