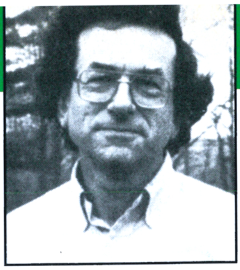

WHAT'S WRONG WITH THIS TEMPTATION?

N. David Mermin

Once upon a time everybody knew why measurements in quantum mechanics don't reveal preexisting properties. It was because the act of acquiring knowledge unavoidably messes up the object being studied. What you learn is not intrinsic to the object, but a joint manifestation of the object and how you probe it to get your knowledge.

In 1935 this state of happy innocence was forever dispelled by Einstein, who with Boris Podolsky and Nathan Rosen discovered how to learn about an object by messing up only some stuff it left behind in a faraway place. They concluded that knowledge acquired in this way was indeed about preexisting properties of the object, revealed—not created—by the act of probing the stuff left behind. Bohr, however, insisted their conclusion was unjustified, and 30 years later John Bell *proved* that no assignment of such preexisting properties could agree with the quantitative predictions of quantum mechanics.

A couple of years ago Lucien Hardy[1] gave this tale an unexpected twist, by finding a charming variation of the Bell–EPR argument. Hardy's theorem is even simpler than the argument of Daniel Greenberger, Michael Horne and Anton Zeilinger that I enthused about in this column four years ago (June 1990, page 9). The reason he was able to pull the the trick off, and the reason, I suspect, nobody had noticed so neat an argument for so long, is that Hardy's analysis applies to data that are not correlated strongly enough to support the argument of EPR. But they do give rise to an argument every bit as seductive, which Hardy is then able to demolish with surprising ease. Parts of the formulation I give here of Hardy's *gedankenexperiment* are similar to those of Henry Stapp[2] and Sheldon Goldstein.[3] (Thomas F. Jordan brought Hardy's work vividly to my attention in articles submitted to *Physical Review A* and the *American Journal of Physics*.)

We consider two particles that originate from a common source and fly apart to stations at the left and right ends of a long laboratory. At the left station we can experimentally determine the answer to one of two yes–no questions, A or B. There is a choice of two other yes–no questions, M or N, to be answered by experiment on the right. Hardy provides questions A, B, M and N, and a two-particle state $|\Psi\rangle$ for which the answers to the questions have the following features:

(i) If the questions are B and N, the answers are sometimes both yes.

(ii) If the questions are either B and M or A and N, the answers are never both yes.

(iii) If the questions are A and M, the answers are never both no.

Though two correlated particles subject to local probes in two faraway places also appear in an EPR experiment, Hardy's experiment is interestingly different. In an EPR experiment correlations in the data make it possible to predict the answer to whichever question you ask at one end of the laboratory by asking a suitable question at the other end. In Hardy's experiment you cannot perform this trick. If, for example, you want to learn the answer to A without messing up the particle on the left, you can try getting the answer to M on the right. If that answer is no, then (iii) does indeed guarantee that the answer to A will be yes. But if the answer to M is yes, you cannot predict the answer to A. If you try instead to measure N on the right, you run up against the same problem: You can predict with certainty the result of measuring A on the left in only an unpredictable and uncontrollable fraction of the runs. Similar difficulties arise if you try to learn the answer to B from measurements on the right or the answer to M or to N from measurements on the left. We have here what one might call a semi-EPR situation.

But that semi-EPR situation leads one into temptation just as irresistibly as the full-blown variety. The temptation emerges when you imagine a series of runs in which one chooses the question at each end of the laboratory by tossing a coin at that end after the particles have left their common source but before they arrive at the ends to be tested.

We all agree—even Bohr might agree—that something in the common origin of the two particles must underlie the correlations described in (i)–(iii). Since the questions to be asked are not picked until after the particles have left their source, the features of the particles responsible for those correlations cannot depend in any given run on what happens when the coins are tossed. Furthermore, since each question probes only one of the particles, the answer to a question at one end of the laboratory can be influenced only by features residing in the particle at that end and not by features residing in the faraway particle at the other end.

If you accept those last two sentences, then you are in trouble. According to (i), in some of those runs in which the questions end up being B and N, the answer to both is yes.

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In these particular runs the particle on the left is indisputably of a type that allows the answer yes to question B, and the particle on the right is of a type that allows the answer yes to question N. But in any of those particular runs the tosses of the coins could have resulted in questions B and M being asked instead. Since the particle on the left is of a type that allows the answer yes to B, the particle on the right must be of a variety that prohibits the answer yes to M. Otherwise if the coins had come up differently, it would be possible to get answers yes to both questions B and M, which (ii) forbids. By the same token, since the tosses of the coin could have resulted in A and N being asked, and the particle on the right allows the answer yes to N, the particle on the left must prohibit yes to A. But the tosses of the coin in any of those particular runs could also have resulted in A and M being asked. Since each particle in those runs is of a type that prohibits the answer yes to its question, such a run would have to give the answer no to both A and M. But that is precisely what (iii) forbids.

People who find Bell–EPR profoundly mysterious ought to find this state of affairs equally bizarre. Those immune to the charms of EPR will have stopped reading after my second paragraph. So since you, faithful reader, are eager to know what underlies this astonishing trick, let me tell you one way to do it.

Take A and M to be any nontrivial questions you like. Pick any four one-particle states lying entirely in their yes and no subspaces. Call them $|Ay\rangle$, $|An\rangle$, $|My\rangle|$ and $|Mn\rangle$. Take the two-particle state $|\Psi\rangle$ to be a superposition of products of these yes and no eigenstates. We guarantee feature (iii) of the data by requiring the no–no state $|An, Mn\rangle$ to be absent from that superposition:

$$|\Psi\rangle = \alpha\,|Ay, Mn\rangle + \beta|An, My\rangle + \gamma|Ay, My\rangle \tag{1}$$

Take the question B to have a single yes eigenstate $|By\rangle$ that is a nontrivial linear combination of $|Ay\rangle$ and $|An\rangle$, and take N to have a single yes eigenstate $|Ny\rangle$ that is another such linear combination of $|My\rangle$ and $|Mn\rangle$. Feature (ii) requires $|\Psi\rangle$ to be orthogonal to $|By, My\rangle$,

$$0 = \langle By, My|\Psi\rangle = \beta\langle By|An\rangle + \gamma\langle By|Ay\rangle, \tag{2}$$

and orthogonal to $|Ay, Ny\rangle$,

$$0 = \langle Ay, Ny|\Psi\rangle = \alpha\langle Ny|Mn\rangle + \gamma\langle Ny|My\rangle. \tag{3}$$

Feature (i) requires $|\Psi\rangle$ *not* to be orthogonal to $|By, Ny\rangle$:

$$\begin{aligned} 0 \neq p = |\langle By, Ny|\Psi\rangle|^2 &= |\alpha\langle By|Ay\rangle\langle Ny|Mn\rangle \\ &+ \beta\langle By|An\rangle\langle Ny|My\rangle \\ &+ \gamma\langle By|Ay\rangle\langle Ny|My\rangle|^2, \end{aligned} \tag{4}$$

which (2) and (3) reduce to

$$0 \neq p = |\gamma|^2\, |\langle By|Ay\rangle|^2\, |\langle Ny|My\rangle|^2. \tag{5}$$

This tells us that the coefficient γ must be nonzero, and (2) and (3) then tell us that β and α can't be zero either. So for any questions A and M we can perform Hardy's magic trick in any two-particle state of the form (1) with three nonzero coefficients.

How big can we make the probability p whose non-vanishing gets us into all this trouble? It follows directly from (2) and (3) and $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$ that

$$p = \frac{p_l(1-p_l)p_r(1-p_r)}{1-p_l p_r}, \tag{6}$$

where $p_l = |\langle By|Ay\rangle|^2$ and $p_r = |\langle Ny|My\rangle|^2$. Maximizing (6) gives uniquely $p_l = p_r = (\sqrt{5}-1)/2 = 1/\tau$, where τ—would you believe it?—is the golden mean. This gives p, the fraction of BN runs in which both answers are yes, the maximum value $1/\tau^5 = 0.09017$. Nine percent of the time something happens that the correlations described in (ii) and (iii) would appear absolutely to prohibit. Sensational!

Some experts might question my enthusiasm, since constructing arguments like Bell's in the absence of perfect EPR correlations is old stuff, originally inspired by the inability of any real experiment to demonstrate that correlations are perfect. The semi-EPR argument in the Hardy state, such experts might maintain, is merely an example of a violation—and not a very strong one—of an inequality[4] that, though very plausible even in the absence of perfect correlations, has already been reported to be violated in many earlier experiments:

$$p(By, Ny) \leq p(By, My) + p(An, Mn) + p(Ay, Ny) \tag{7}$$

To understand what it might mean to violate this inequality, imagine a world in which each particle carried information specifying its answer to either question it might be asked. Call a particle x if its answer for question X is yes. The small side of the inequality is the fraction of particles that are b and n. The first term on the large side is at least as big as the fraction that are b, n and m, while the third is at least as big as the fraction that are b, n and a. So we would have an upper bound if we were to add in the fraction that are b and n but neither a nor m. Since $p(An, Mn)$ is an upper bound for this last fraction, the inequality, must hold in such a world. It is violated, however, in a Hardy state, with 0 on the right and a probability as big as 0.09 on the left.

An experiment to confirm such a violation in the optimal Hardy state will require detectors accurate enough to distinguish a 9% event rate from a rate of 0%. Earlier tests for violations in states inspired by EPR had an easier time of it, using questions that made the probability on the left $\frac{1}{4}(2+\sqrt{2}) = 85\%$ and the sum of probabilities on the right $\frac{3}{4}(2-\sqrt{2}) = 44\%$. Hardy states will not lead to more definitive experiments.

But to rest with that conclusion is to fail to see what makes the Hardy experiment so charming. My quick explanation of the inequality (7) required each particle to carry information specifying its answers to two incompatible questions. This is not only grossly un-quantum mechanical but, in the absence of an EPR argument, not even especially plausible. Much thought has gone into relaxing the assumptions underlying this inequality, and it can be made much more compelling than I have bothered to do here. But the refutation of even those refined assumptions simply doesn't hit you with anything like the impact of Bell's old refutation of the EPR argument or Hardy's new semi-EPR demolition job. So although Hardy's four questions provide a rather weak basis for a laboratory violation of the experimentally relevant inequality, they reign supreme in the *gedanken* realm. There they achieve their effectiveness not by refuting the subtle assumptions behind the inequality, but by leading you down the garden path every bit as enticingly as the full EPR argument does and then turning around and kicking you out of the garden with unprecedented efficiency and force.

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