

THE FLUID PHYSICS OF PARACHUTE INFLATION

Cloth packed to the density of oak emerges from its container in a second, changes its shape from cylindrical log to 'cloth streamers' to hemispherical cup in another 2 seconds, and decelerates a one-ton payload from the speed of sound to highway speed limits.

Carl W. Peterson

Parachutes have a fascinating history and a promising future. They were born long before airplanes, in ancient China. Leonardo da Vinci made sketches of a parachute-like device at the end of the 15th century, and Fauste Veranzio parachuted from a Venetian tower in 1617. By the time airplanes arrived on the scene, parachutes were older than Methuselah.

Today, parachutes are busier than ever. They have become even more essential as people and their precious cargo spend more and more time in the air and in space. Because they are by far the lightest, least expensive method for decelerating objects in the atmosphere, parachutes are used extensively for recovering valuable airborne objects, enabling crew escape and rescue, and delivering military weapons and payloads.

Examples of payloads recovered using parachutes already include expensive atmospheric research instrumentation and the 175 000-pound solid-fuel rocket boosters that help launch the space shuttle. Future soft-landing and payload recovery missions include landing instrumentation on Mars and returning materials manufactured in space to the parking lot of the laboratory where they will be processed.

Rescue missions today include escape from high-performance aircraft and in a few years will include return of injured astronauts from spacecraft. The US Army would like to deliver 60 000-lb tanks behind enemy lines from cargo aircraft flying only 300 feet above the ground to escape detection during covert operations. Simultane-

ously, parachutes will be used to deliver the squadrons of troops to drive the tanks and participate in the mission. Each of these uses demands parachutes that are as technically complex as the vehicles that lifted them into the air in the first place.

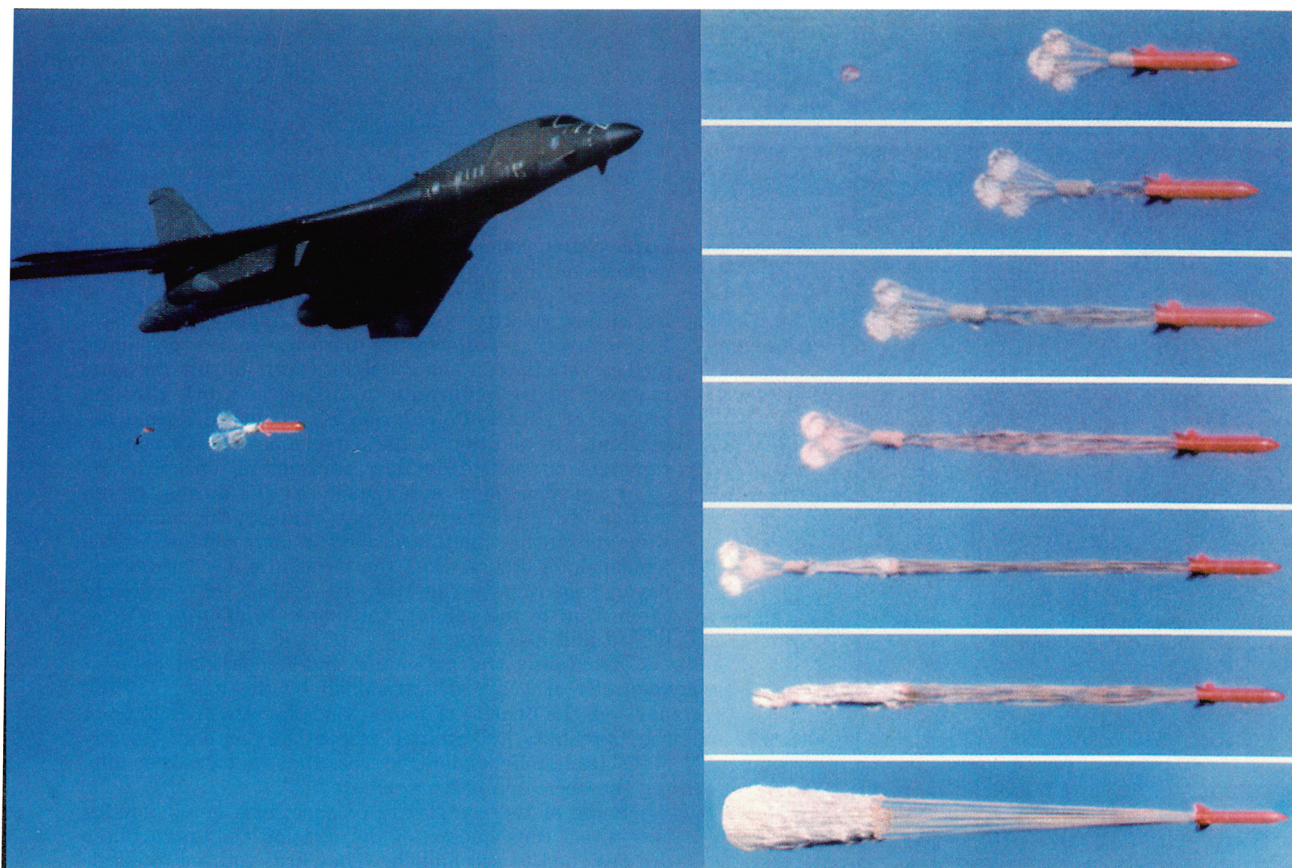
This article describes the physics of the motion of the parachute and the surrounding air as the parachute emerges from the payload, inflates, and decelerates the payload. Such a physical understanding is a prerequisite for developing computational tools for designing parachutes. The design tools must be predicated upon analytical and numerical models of the unsteady motion of the air coupled to the equations of motion of the parachute.

Precise calculation of parachute deployment, inflation and payload deceleration requires solution of the nonlinear, time-dependent equations of motion for turbulent, separated airflow around and through the parachute coupled to the equations of motion of the parachute. Because of the difficulty of this problem, it is not surprising that parachute designers have for many years relied on empirical methods rather than on analysis. Once I show what physical phenomena are involved, I will describe attempts to model the important events numerically so that parachutes can someday be designed on computers instead of by trial-and-error flight tests.

Parachute aerodynamics

Parachute deployment and inflation is arguably the most complex physical event in aerodynamics. One source of complexity is the requirement that the parachute provide drag only after the payload has been allowed to fly by itself while accomplishing its mission. Before the parachute is needed, the payload must retain its own aero-

Carl Peterson is manager of the defense and space programs department at Sandia National Laboratories.



Deployment of a parachute. After the payload is released from the aircraft, a cluster of three pilot parachutes pulls the deployment bag away from the payload. The suspension lines are pulled out of the bag first, and then the main parachute canopy. The plane, an Air Force B-1B bomber, dropped the 2400-lb instrumentation package at a speed of about 500 miles per hour. **Figure 1**

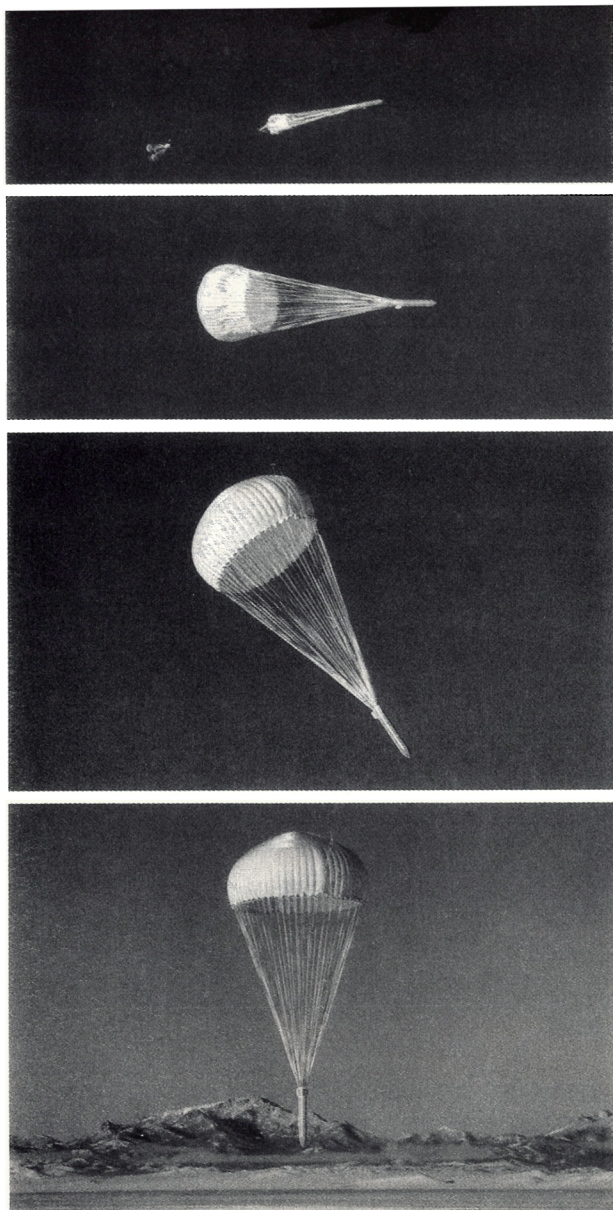
dynamic characteristics, not those of the parachute. This means that the parachute must be stored out of the airstream. Space limitations on the payload invariably dictate that the parachute's stored shape be much different from its inflated shape. When it is time for the parachute to be used, it must be transformed into its high-drag shape as quickly as possible; this is why parachutes are made of cloth. The unfolding and aerodynamic loading of the cloth contribute to the complexity of parachute deployment and inflation.

Figure 1 shows the deployment from a 2200-lb, 1.5-ft-diameter payload of a 46.3-ft-diameter "ribbon" main parachute. (The canopy—the part of the chute that creates drag—is not solid cloth as one might expect, but is constructed of strips of material, called ribbons, which are laid out with gaps between them.) A cluster of three 3.8-ft-diameter ribbon pilot parachutes was developed at Sandia to pull the main parachute deployment bag out of the payload and then pull the main parachute out of its deployment bag. The main parachute is positioned in the deployment bag so that the suspension lines of the parachute are pulled out of the bag first as the deployment bag is pulled away from the payload. After all the suspension lines are out of the bag, the parachute canopy emerges. It must be pulled completely out of the bag, without damage, before it inflates and decelerates the payload.

The sequence of events that occurs during deploy-

ment of the 46.3-ft-diameter parachute from its four-panel bag includes the cutting or breaking of the cloth "ties" that secure the parachute suspension lines and canopy to the panels of the bag. Successful deployment requires obtaining enough drag from the pilot parachute to pull the main parachute out of its bag, as well as careful selection and location of line and canopy ties, cutting knives, the canopy's retainer and other parts of the deployment bag. The entire sequence of events, from ejection of the pilot parachutes to stretching out all 65 feet of the main parachute behind the payload, takes place in less than 2 action-packed seconds. The physics of the deployment process involves the dynamics of the payload (which is coupled to the dynamics of the cloth structures), the aerodynamics of these strange shapes and the response of the textile materials to the loads imposed upon them.

Inflation of the canopy begins as soon as the parachute is pulled free from the deployment bag. Figure 2 illustrates the inflation process, and figure 3 shows the structural elements of the parachute canopy. Air flows into the canopy through the inlet opening at the bottom of the canopy (the skirt) and out through the porous canopy fabric, the vent and the gaps between ribbons. The porosity of the canopy material and configuration is designed so that air is retained in the canopy. This "captured" air causes the pressure inside the canopy to increase above the pressure outside of the canopy. The



Inflation and descent of a parachute. With the parachute completely out of the bag (at lower left of top photo), the canopy inflates, slows the payload and allows it to descend to the ground. **Figure 2**

radial component of force generated by the pressure differential across the canopy accelerates the canopy outward and causes it to inflate. Inflation continues as long as the integrated outward radial pressure forces remain greater than the integrated radial tension and so long as the canopy encounters no structural constraints such as circumferential reefing (which limits the diameter of the canopy skirt).

No other aerodynamic structure undergoes such an enormous change in shape during the course of performing its aerodynamic mission. And the motion of no other aerodynamic structure is coupled as strongly to the motion of the air: The shape of the parachute depends on the aerodynamic forces acting on the canopy, but the

airflow that generates the aerodynamic forces in turn depends on the shape of the parachute canopy.

As the parachute decelerates the payload, the oncoming airflow velocity (relative to the parachute) decreases. For many high-performance parachutes, the oncoming airflow velocity can change significantly during the time required for the parachute to inflate, in which case the process of parachute inflation is intrinsically time dependent. In addition to the dependence of inflation parameters on nonsteady flow parameters, one observes other time-dependent aerodynamic events during the operation of high-performance parachutes. An example is wake recontact, sometimes called "canopy collapse." This phenomenon occurs when the parachute decelerates the payload so rapidly that the air behind the parachute catches up to the canopy, causing it to deform, or "collapse," and lose drag.

Eventually the parachute and its payload descend vertically at a speed determined by the weight of the payload, the density of the air and the size and shape of the parachute. Terminal descent is the only "steady state" (time independent) process that we encounter during the operation of a parachute.

Some of the complexity of parachute aerodynamics is a direct consequence of the mission that parachutes are designed to accomplish. They are required to decelerate payloads that have insufficient drag to decelerate quickly enough on their own. Hence parachutes must have much more drag than the objects that they decelerate. Whereas most of the aerodynamics community is concerned with optimization of the aerodynamic efficiency of streamlined, low-drag shapes, parachute designers seek to create the maximum amount of disturbance to the oncoming airflow—that is, the maximum drag.

As a result, parachute aerodynamics is irrevocably associated with the airflow around blunt-ended, or "bluff," bodies, which encompasses several of the yet unsolved problems in fluid dynamics. For example, the air flowing around the parachute separates from a location on the canopy that is *a priori* unknown. The shedding of vortices from the bluff canopy shape may affect canopy stability and cause a periodic motion of the parachute and payload.

These bluff-body aerodynamic phenomena are further complicated by the presence of the payload just ahead of the parachute. The turbulent wake generated by the payload flows into the parachute, reducing the parachute's drag and causing instabilities in the parachute's inflated shape. This means that the performance of the parachute may depend on the payload's physical characteristics as well as on the parachute's own characteristics.

Parachute deployment simulation

Now that we have some physical feel for what happens during the flight of a parachute, I shall review how aerodynamicists have approximated the relevant physics in numerical models of parachute inflation, beginning with parachute deployment. James Purvis of Sandia National Laboratories has developed a computer code called Linesail¹ to provide guidance for the design of the

pilot parachute and deployment system for parachutes like the 46.3-ft-diameter parachute in figures 1 and 2. Because different parts of the parachute exhibit independent motions during inflation, one must model the parachute suspension lines and canopy as a collection of flexible distributed-mass structures connected to a finite-mass forebody. The payload, suspension lines, canopy, pilot parachute and deployment bag for the main parachute are each modeled as a series of elastically connected mass nodes, as shown in figure 4. The motion of each mass node is determined by the tensile and aerodynamic forces acting on it. The forebody and the pilot parachute and deployment bag are separate special nodes; all undeployed suspension line and canopy mass nodes are lumped in the pilot parachute and deployment bag node. The forebody node includes forebody drag, and the deployment bag node includes pilot parachute drag.

At time $t = 0$, the deployment process begins and the drag of the pilot parachute begins to pull suspension line nodes out of the bag, one at a time. The equations of motion for the i th deployed mass node are

$$m_i \ddot{x}_i = T_i \cos \theta_i - T_{i-1} \cos \theta_{i-1} + X_i$$

in the direction of flight (x) and

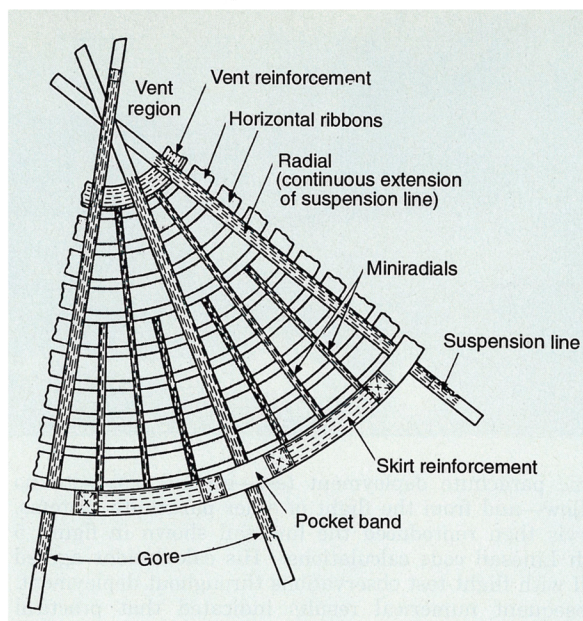
$$m_i \ddot{r}_i = T_i \sin \theta_i - T_{i-1} \sin \theta_{i-1} + R_i$$

in the direction perpendicular to the flight path (r).

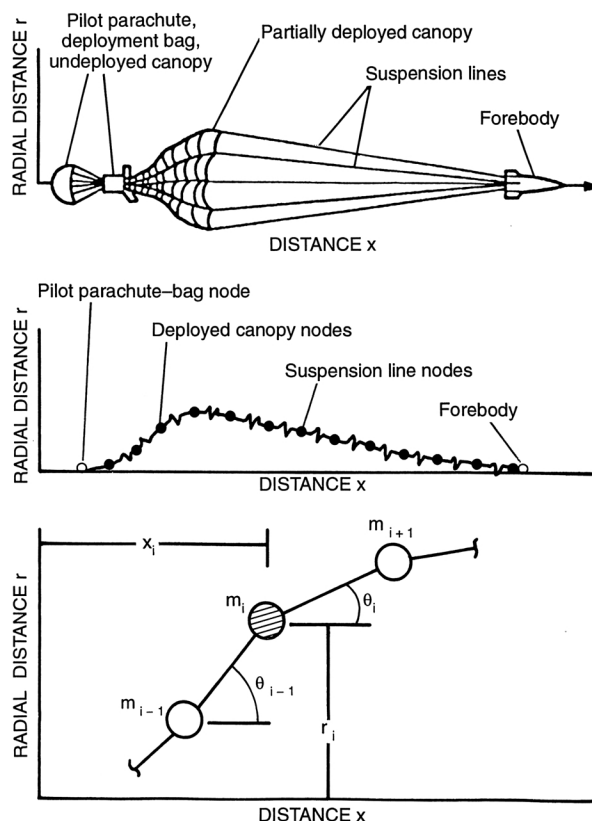
These equations are solved at successive times until the main parachute has been pulled completely free from its deployment bag. The distribution of mass is specified from the payload through the apex of the canopy. The factor m_i is the mass of the i th node; $\ddot{x}_i$ and $\ddot{r}_i$ are the accelerations of the i th mass nodes in the x and r directions, respectively. T_i is the tension between nodes i and $i + 1$. The tension is modeled as a combination of a linear stress-strain rate for the actual textile material being used and a semiempirical damping term. The terms X_i and R_i are the axial and radial components of the aerodynamic forces acting on the segment of suspension line or canopy; they are based on familiar formulations for the cross flow and drag of a cylinder.

Figure 5 shows the severe "line sail"—the premature removal of the suspension lines from the deployment bag by the oncoming wind blast—that was observed during deployment of the 46.3-ft-diameter parachute using a single 5-ft-diameter pilot parachute while the payload was traveling at Mach 1.28 after release from an F-111 aircraft. The 5-ft-diameter ribbon pilot parachute was not able to remain inflated in the flow field of the wake behind the payload and in the shock waves created by the F-111. The collapsed pilot parachute could not provide enough drag to avoid severe suspension line sail, which resulted in unacceptable damage to the canopy.

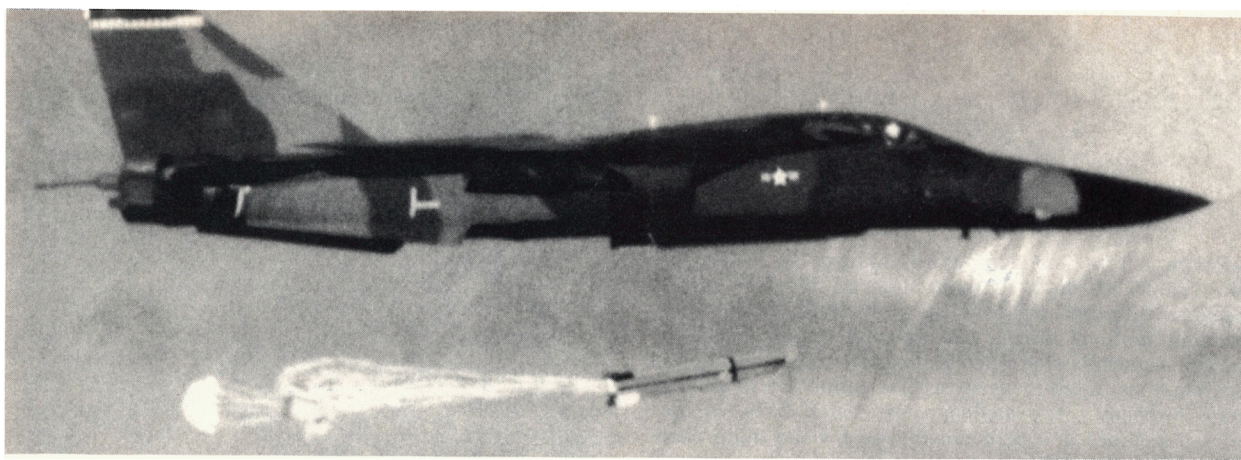
Purvis used the Linesail code to guide the redesign of the deployment system for the 46.3-ft-diameter parachute to overcome the line sail problems shown in figure 5. Simple engineering approximations were used to model breakage of the line ties, bag friction and other details of deployment. The dynamics equations and engineering approximations were verified with data from a



Structural components of a ribbon parachute. Figure 3



Computer model of a parachute system during deployment. Top: Physical system. Middle: Mass-node model. Bottom: Mass-node geometric parameters. Figure 4

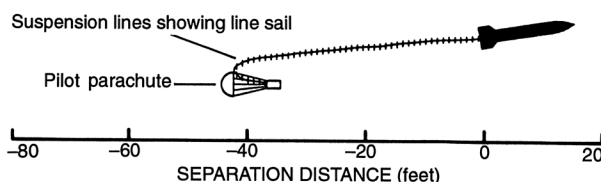


static parachute deployment test—ground test with no airflow—and from the flight of other parachute systems. Purvis then reproduced the line sail shown in figure 5 with Linesail code calculations. His calculations agreed well with flight test observations throughout deployment. Subsequent numerical results indicated that practical changes in line tie strengths would not control the line sail, but that increasing the pilot parachute drag would be a very effective method of controlling it. The pilot parachute drag area was increased from about 10 ft² to 17 ft² by replacing the 5-ft-diameter parachute with the cluster of three 3.8-ft-diameter parachutes shown in figure 1.

Canopy inflation models

Numerical methods and computational hardware have matured to the point where it is feasible to solve more of the fluid dynamics of parachute inflation directly on a computer and to rely less on empirical studies. Such an approach was not realistic ten years ago. Even today, numerical simulations of parachute inflation should not be construed as being ready for use in the design of actual parachute systems. They are “computational experiments” to determine which numerical and fluid dynamic approximations to the full equations of motion have the potential to become parachute design tools. These computational fluid dynamics inflation models still contain approximations to the real physics of inflation; they are not “exact” even though their models incorporate no empirical data. The approximations used in computational fluid dynamics simulations lie in the choice of the equations used to describe the fluid motion, in the representation of the canopy (its dimensionality, shape and degrees of freedom) and in the numerical methods used to solve the equations.

The inflation of a parachute like the 46.3-ft-diameter canopy sets in motion all of the molecules in a “tube” of air nearly a mile long and hundreds of feet in diameter. Computer technology is still many, many years away from permitting us to track each molecule as it interacts with the parachute or other molecules. Instead, the air is treated as a continuum of molecules with properties such as viscosity, temperature, pressure and density. The resulting equations of motion for the air as it interacts with any body passing through it are called the Navier-Stokes equations, named after two of the people who derived them between 1823 and 1845. They are nonlinear, coupled equations for the conservation of mass, momentum and energy. The Navier-Stokes equations are written in many different forms, depending on the



Experimental test and computational simulation of parachute deployment. Shock waves from the F-111 aircraft caused the small pilot parachute to collapse and produce insufficient drag. As a result, the supersonic airstream pulled the suspension lines out of the bag prematurely, causing the lines to “sail.” The computer simulation, shown here 0.3 seconds after parachute deployment, reproduced the scenario. **Figure 5**

coordinate system and the particular class of flow problems one wishes to solve. I will write them in words, rather than in mathematical symbols, so that you can see the balances among the diverse physical phenomena that must be taken into account.

Conservation of mass: The time-dependent variation of the density of air at any location in the flow field must be balanced by the convection of density into and out of the same location.

Conservation of momentum (one equation for each of the three spatial directions): The time-dependent variation of the momentum of the fluid at any location must be balanced by the combined forces imposed by pressure gradients, convection of momentum, viscosity (friction forces) and any body forces acting on the fluid at the same location.

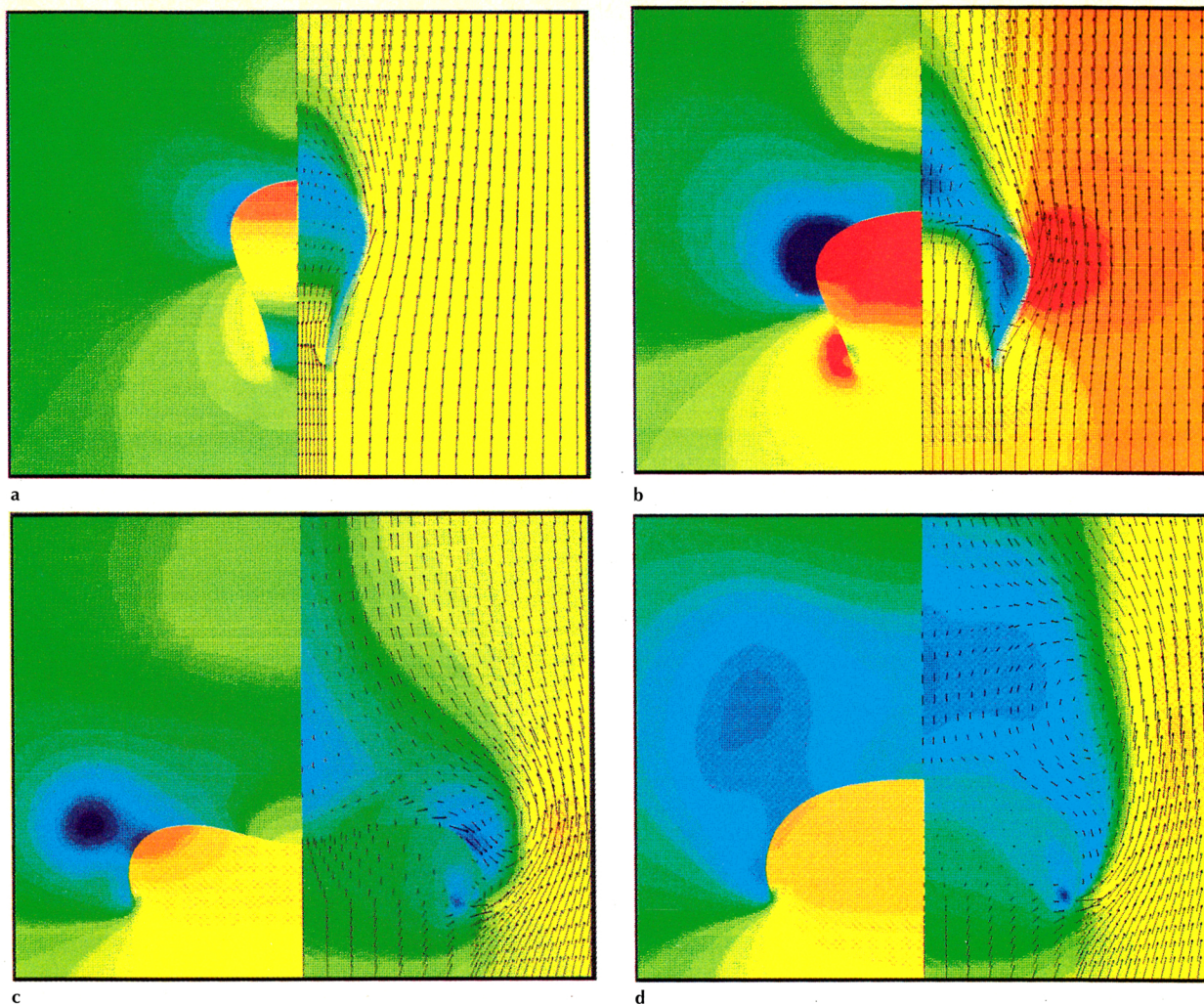
Conservation of energy (needed when considering high-speed flows): The time-dependent variation of total energy (thermal plus kinetic) of the fluid must be balanced by changes in energy due to convection, viscous dissipation, conduction and work done on the fluid.

To complete the set, one needs an equation that describes the relationship between the pressure, density and temperature of the fluid. For parachute inflation problems, the ideal-gas law will suffice:

$$p = \rho RT$$

Here p is the pressure, ρ is the air density, T is the temperature and R is a constant.

Computational fluid dynamicists are experimenting with a variety of algorithms for solving the Navier-Stokes



Computed pressures and flow velocities for an inflating, decelerating parachute. The left side of each frame shows the pressure field; the right side, the velocity field. **a:** Prediction for early in the inflation process, 0.2 seconds after the start of inflation. **b:** Approaching fully inflated diameter; $t = 0.8$ sec. **c:** Canopy "collapse"; $t = 1.4$ sec. **d:** Recovery to steady descent; $t = 2.0$ sec. **Figure 6**

equations and gridding schemes for representing the parachute shape and surrounding volume. Figure 6 shows the results of Navier-Stokes computations made by researchers at the US Army Research, Development and Engineering Center that predict the pressure and velocity fields around a parachute canopy during inflation.² Their time-dependent axisymmetric Navier-Stokes fluid code is coupled to a mass-spring-damper parachute-structure code to enable the parachute shape to "inflate" in response to the computed pressure distribution around the canopy.

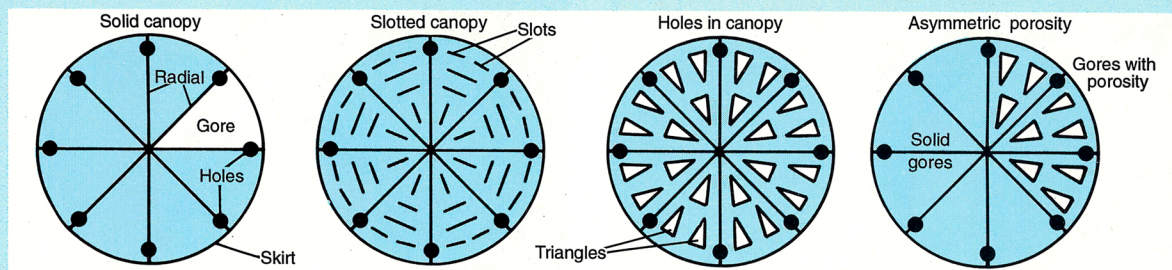
The left side of each frame in figure 6 shows the pressure levels inside and outside the canopy. Blue denotes pressures below ambient (atmospheric) pressure; red denotes pressures above ambient. The canopy is located where the pressure (color) changes discontinuously. The right side of each frame shows the corresponding fluid velocities as viewed from a reference point on the payload. The location of the parachute canopy in the right side of each frame can be inferred from its location on the left; it is simply a mirror image.

Figure 6a shows the parachute shaped like a light

bulb; the canopy is just beginning to inflate. Pressures are building where the canopy stops the inflowing air. The velocities behind the canopy are very low. In Figure 6b, pressure inside the canopy has increased, causing the diameter to grow. Air flows into the canopy to support this growth. The highest velocities are just outside the location of the canopy's maximum diameter. Air has started to approach the canopy from behind. By 1.4 seconds after the start of inflation (figure 6c), the payload has slowed down. Pressures inside the canopy are lower. The air behind the canopy has caught up to it, causing it to deform, or collapse. The canopy recovers in figure 6d and descends to Earth.

Navier-Stokes computations reproduce the flow-field features observed in experiments, such as a recirculation region with free vortices behind the parachute, but the quantitative accuracy of the computer results has not yet been established. This work stresses that much remains to be learned about the effects of surface and flow-field grid methodology, computational approximations and techniques used to perform Navier-Stokes calculations for parachutes, in addition to what

Do-It-Yourself Parachute Aerodynamics



You can learn a great deal about parachute aerodynamics by doing experiments that you can conduct in an open field using small parachutes that you can make at home. The experiment described below compares the flight characteristics of identically sized parachutes whose canopies have different configurations of slots or holes. Varying the porosity changes the airflow patterns in and around the parachute, which in turn alter the parachute's flight characteristics.

From an old bedsheet, cut out four circles of material approximately 30 inches in diameter. Use a fine marker to segment each circle into eight equal "pie slices"; these are the "gores" of the parachute. Just inside the circumference of the circle of material, punch a small hole along each radial line that divides the canopy into gores. In parachute terminology, the hole is on the canopy "radial" near the "skirt" of the canopy. These holes will anchor the suspension lines, which you will attach after you design the porosity—slices or holes—into each canopy.

The first canopy will have no geometric porosity. This configuration, called a flat circular canopy, is one of the earliest parachute designs. It is depicted in the first sketch above.

In one gore of the second parachute, cut five "slices" as shown in the second sketch. Choose any spacing between the slices that you wish in the first gore, and then duplicate that slice pattern in each of the remaining seven gores. (The sketch shows what my daughter used in her science project.)

In one gore of the third parachute, cut out three triangles as shown in the third sketch. Each triangle should have a height of about 3 inches and a base of about 2 inches. Repeat the pattern of triangular cutouts in the remaining seven gores.

Use the same pattern of triangles in the fourth parachute, but put them only in three adjacent gores, as shown in the fourth sketch; leave the other five gores with no holes or slots in them.

After preparing each canopy, tie light string through the

eight holes in each canopy. Tie the strings together in a knot approximately 35 inches from the skirt of the canopy, and trim off the excess string. The eight strings are the suspension lines that connect the canopy to the payload. The payload can be any *robust* object weighing roughly 5 ounces that can be easily attached to the knot of the suspension lines. My daughter and I used an empty plastic shampoo dispenser, with stones added to bring the weight up to 5 ounces, and looped a string through the pour spout and over the suspension line knot. The same payload can be used with each parachute.

Now comes the flight test phase. Launch the parachutes as high above the ground as possible to maximize the flight time and your opportunity to observe how they fly. Please observe "range safety" rules, however; hanging out of upper-story windows or launching from the top rung of a ladder is not worth the altitude gained! Bundle the parachute with the payload and throw, then observe how each parachute flies. Compare the flight dynamic attributes among the parachutes and see if you can deduce the aerodynamic causes for what you observe. Some questions to ask your research team:

- ▷ How does each of the parachutes move with respect to the payload during descent? Do you observe any dynamic motion of the canopy?
- ▷ Are the trajectories the same for each payload-parachute combination?
- ▷ Do you observe any differences in the time for inflation among the parachutes?
- ▷ Do some parachutes descend more rapidly than others?

Graduate study: Invent your own canopy configurations to optimize the flight characteristics you think would be important for payloads you envision. Make them glide, spin, descend more slowly and so on as you see fit. Perhaps you will invent a new type of parachute that will better suit the applications described at the beginning of this article!

must be learned about the fluid dynamics. But the work also provides strong encouragement that Navier-Stokes calculations of parachute inflation will become the engineering design tools for future high-performance parachutes. The parachute technical community must give high priority to developing Navier-Stokes methods for predicting parachute inflation and to conducting experiments to validate those methods.

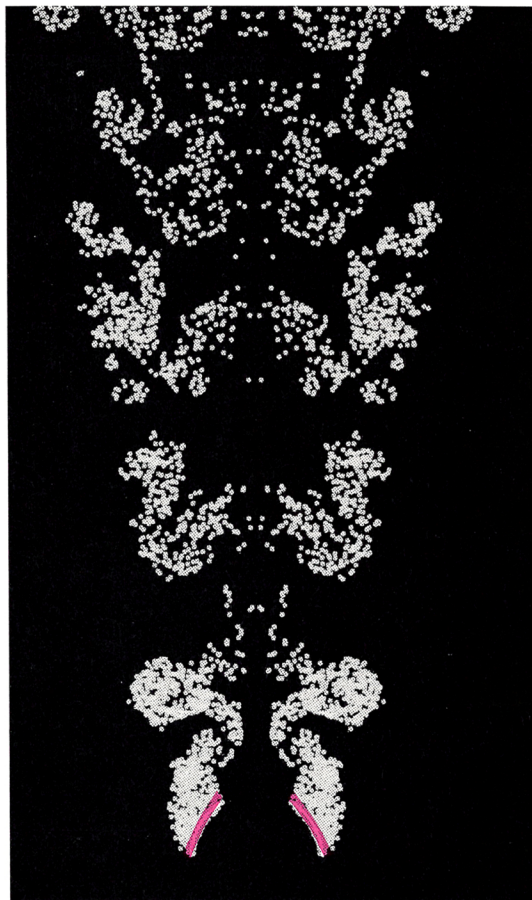
More approximate approaches. While research on modeling parachute inflation with the Navier-Stokes equations continues, aerodynamicists are pursuing a variety of more approximate numerical approaches. James Strickland of Sandia includes the time-dependent aspects of parachute inflation by taking the curl of the Navier-

Stokes equations and assuming that the air density and viscosity are both constant.³ The resulting vorticity transport equation is further simplified by neglecting the viscous terms and recasting the scalar vorticity in terms of the circulation Γ associated with a group of air "particles" called vortices. The equation describing the time-dependent change of circulation in a velocity field $\mathbf{u}$ is Kelvin's theorem:

$$\frac{\partial \Gamma}{\partial t} + (\mathbf{u} \cdot \nabla) \Gamma = 0$$

Strickland uses Kelvin's theorem to solve for the time-dependent motion of vortices generated by a parachute

Vorticity generated by an inflating, decelerating parachute, computed using the VPARA code. The dots mark the locations of vortices originally shed from a hemispherical "parachute" (color) with a large vent hole. As the computations continue, the movement of each vortex is calculated under the influence of all the other vortices, and new vortices are shed from the surface. **Figure 7**



decelerating a payload. At the start of his computations, there are no free vortices in the flow. To satisfy boundary conditions on the parachute canopy, vortices are placed along the inner and outer surfaces. The circulation strength Γ of these vortices is calculated so as to satisfy the boundary conditions on the solid or porous surfaces. Next, the flow solution is advanced by one time step. During this time step, the body surfaces move to a new position in some prescribed manner. Vortices on the surface are allowed to move off the surface into the flow field. The vortices are convected to new positions in the flow at the local fluid velocity, which is calculated from the previous positions and strengths of the vortices. Then a new set of vortices is introduced at the surface. The circulation strengths are again calculated so as to satisfy the surface boundary conditions in conjunction with the set of vortices that were shed previously. This process is repeated for the desired number of time steps. Pressure distributions along the surface are calculated at each time step based on the strengths of the new surface vortices.

Output from Strickland's VPARA code (vortex method for parachutes in axisymmetric flow), which performs these calculations, is shown in figure 7 for a hemispherical shell with a vent radius R_i equal to 40% of the skirt radius R_o , each measured along the surface from the axis of symmetry. The hemisphere is started impulsively from rest and in figure 7 has moved 20 radii R_o to the bottom along the axis of symmetry. Periodic structures are shed from the vented hemispherical shell. The instantaneous pressure coefficient C_p , a measure of the difference between the pressures inside and outside the canopy, changes constantly due to the unsteady wake. The drag coefficient C_D also shows pronounced periodicity. Calculations for vented hemispheres in general show good agreement with experimental results. Comparisons of VPARA drag history predictions with data on a 2-ft-diameter hemispherical shell towed in a water tank with a prescribed velocity history also show very good agreement.

The complexity of the parachute inflation problem, the magnitude of the research that must be done and the small size of the parachute community all suggest that the development of parachute inflation technology will not be completed soon. Obtaining high priority for this work is itself a challenge, because until recently the parachute community has treated parachute design as an art rather than a science. No single laboratory has either the resources or the mix of talent to solve all of the critical parachute technology problems by itself. The problems may not be solved at all unless the work is pursued as a national or international collaborative effort

by scientists and engineers.

If we can indeed understand and describe the fluid physics of parachute inflation on a computer, then future operational parachutes will likely feature "electronic canopies" with sensors and controls to measure flight conditions and on-board computers that use the inflation models to tailor the inflation process to meet performance requirements within system constraints. Electronic canopies will be essential for planetary exploration because of the *a priori* uncertainty in atmospheric conditions and landing terrain, and because the round-trip transit time of control signals to and from Earth is longer than the time required for inflation and landing. Parachutes will continue to find extensive use, if not top billing, on even more exotic flight vehicles. Leonardo would be pleased.

* * *

The work performed at Sandia National Laboratories is supported by the US Department of Energy.

References

1. J. W. Purvis, *J. Aircraft* **20**, 940 (November 1983); "Improved Prediction of Parachute Line Sail During Lines-First Deployment," pub. AIAA 84-0786, Am. Inst. of Aeronautics and Astronautics, Washington, D. C. (April 1984).
2. K. R. Stein, R. J. Benney, E. C. Steeves, "A Computational Model That Couples Aerodynamic and Structural Dynamic Behavior of Parachutes During the Opening Process," tech. rep. Natick/TR-93/029, US Army Natick Research, Development and Engineering Center, Natick, Mass. (April 1993).
3. J. H. Strickland, "Axisymmetric Bluff-Body Flow: A Vortex Solver for Thin Shells," rep. SAND91-2760, Sandia Natl. Labs., Albuquerque, N. M. (May 1992). ■