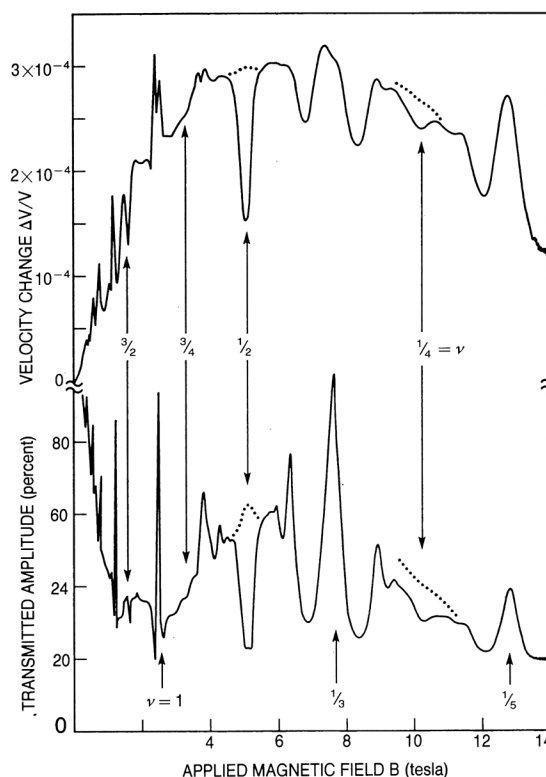


HALF-FILLED LANDAU LEVEL YIELDS INTRIGUING DATA AND THEORY

Surprises abound when strong magnetic fields are imposed on two-dimensional electron gases at low temperature. When the electron density approaches an integral multiple ν of the density of magnetic flux quanta threading the plane, the resistivity of the electron system begins to vanish and its Hall resistance takes on the quantized value $h/\nu e^2$ to a spectacular level of precision. That's the integral quantum Hall effect, for whose serendipitous discovery at Grenoble in 1980 Klaus von Klitzing won the 1985 Nobel Prize in Physics.

The integral quantum Hall effect is now well understood in terms of the filling of successive Landau levels of the quantized energies of the electrons circling in tight little cyclotron orbits. The first Landau level, for example, is full when there's precisely one electron for every flux quantum. If, at that point, one wants to add or scatter an electron, one must promote it to the next cyclotron energy level or else add more magnetic flux.

That was only the beginning. In the succeeding years the availability of ever more perfect semiconductor interfaces has unearthed a panoply of further surprises that continue to challenge the theorists. Looking to see what happens in very strong magnetic fields where the flux quanta outnumber the electrons, Daniel Tsui, Horst Stormer and Arthur Gossard at Bell Labs in 1982 found states of quantized Hall resistance when the "filling factor" ν had the fractional values $1/3$ and $2/3$. The theory of the integral quantum Hall effect had given no inkling of such fractionally filled states. Many more fractional quantum Hall states have been found since then, almost all of them with odd-denominator filling factors. At each of these fractional filling factors, it seems, the two-dimensional electron gas forms an incompressible quantum liquid with an energy gap in the density of states. But the wavefunction proposed by Robert Laughlin



Acoustic waves

propagating in a two-dimensional electron gas exhibit variable velocity (top curve) and attenuation (bottom curve) as a function of the magnetic field strength B . Arrows indicate corresponding fractional and integral filling factors ν . Both curves are close to what's predicted from the resistivity measured in dc transport experiments, except at $\nu = 1/2, 1/4$ and $1/3$. At those filling factors the velocity and transmission dip unexpectedly. Dotted curves indicate behavior predicted from dc resistivity. (Adapted from ref. 1.)

in 1983 to explain the fractional quantum Hall effect puts only the original $1/3$ and $2/3$ states on a really firm theoretical foundation. (See PHYSICS TODAY, July 1983, page 19.) Though interactions between electrons play almost no role in the integral quantum Hall effect, they are thought to be crucial to the existence of the quantum Hall states at fractional filling factors.

In recent months the spotlight has been on the $1/2$ state, even though a true quantum Hall state is neither seen nor expected at that filling factor. The elementary flux quantum

embodies a magnetic flux of h/e . At $\nu = 1/2$ the first Landau level is half full; there are two flux quanta perpendicular to the plane for every electron in it. The Fermi-statistics requirement that Laughlin's multiparticle wavefunction be antisymmetric under the exchange of a pair of electrons restricts the incompressible quantum liquids it describes to odd-denominator filling factors. Nonetheless, the $1/2$ state turns out to be full of enlightening surprises. Three recent experiments^{1,2,3} have pointed out really striking behavior of two-dimensional electron systems at $\nu = 1/2$, and

a bold new theory of the half-filled Landau level is putting the entire fractional quantum Hall regime into a new perspective.⁴

Very clean interfaces

The fractional quantum Hall effect can be observed when a very clean interface in a high-mobility semiconductor heterostructure at temperatures below 1 K is subjected to a perpendicular magnetic field on the order of 10 tesla. A concentration of conduction electrons is confined near the interface by a potential well created by modulation doping and the discontinuity between the conduction bands of the two different semiconducting materials—typically GaAs and AlGaAs. The electrons are free to move in the plane, but at these temperatures they are confined to the ground state with respect to motion perpendicular to the interface. Because the two-dimensional electron density n is usually fixed in these heterostructures, one varies the filling factor $\nu = (n/B) \times (h/e)$ by varying the magnetic field strength B .

When a current I flows in the interface plane in the presence of a magnetic field normal to the plane, the Lorentz force generates a Hall-effect voltage V_H across the plane in the direction $I \times B$. The Hall resistance R_H is defined as V_H/I . The general trend is for R_H to increase linearly with increasing B (which means decreasing ν). But in the vicinity of integral and certain favored fractional values of ν , this linear climb levels off to form “quantum Hall plateaus” on which R_H temporarily remains constant at $h/\nu e^2$. These plateaus, each accompanied by a deep dip in the ordinary longitudinal resistivity of the system, are the characteristic signatures of the quantum Hall states.

No such plateau is seen at $\nu = 1/2$. In 1989 a Bell Labs-Princeton-MIT collaboration did find a dip in longitudinal resistivity at $\nu = 1/2$, but it wasn't really deep enough for a quantum Hall state, and its weak temperature dependence was more suggestive of the behavior of an ordinary metal in the absence of a magnetic field. Having seen quantum Hall states at so many rational filling factors, including the even-denominator $5/2$ state (PHYSICS TODAY, January 1988, page 17), one might well have expected the resistivity at $\nu = 1/2$ to exhibit an exponential temperature dependence indicative of an energy gap. There was at the time no theory of the half-filled Landau level.

The apparent absence of an energy gap at $\nu = 1/2$ despite the strong mag-

netic field led Yale theorist Nicholas Read to point out that the system was behaving as if two flux quanta were somehow fastened onto each electron. At $\nu = 1/2$ that would just precisely use up all the magnetic flux.

Composite fermions

The magnetic field of a flux quantum is a delta function in the plane, and one assumes that an electron would not feel the field of a flux line attached to itself. Therefore these putative composite particles would move as if they were in a field-free environment. Furthermore, an electron dressed with an even number of flux quanta remains a fermion. So one would have, in effect, a gas of fermions wandering around a field-free plane, with a Fermi surface and no energy gap. That would explain the metallic temperature dependence of the resistivity at filling factor $1/2$.

Jainendra Jain at Stony Brook had already applied just such composite fermions, each one sporting two flux quanta, to a broad problem of the fractional quantum Hall regime: Even though the Laughlin wavefunction yields fractional quantum Hall states only at filling factors $1/p$ and $1 - 1/p$, where p is any odd integer, Jain pointed out that the most prominent plateaus are seen at filling factors of $p/(2p \pm 1)$.

The best the theorists had been able come up with to generate fractional states other than the Laughlin states was the rather cumbersome “hierarchy” scheme. It offered no ready explanation for the special prominence of the $p/(2p \pm 1)$ “principal sequence” states. The hierarchy theory regards all the fractional states for which there are no Laughlin wavefunctions as daughters, granddaughters and more distant descendants of the Laughlin states.

In the Laughlin theory, the incompressibility of the quantum liquid at $\nu = 1/p$ is a manifestation of the gap energy required to create “quasiparticle” vortex excitations carrying fractional electric charge $\pm e/p$ when one tries to add or subtract electrons. More and more of these quasiparticles are generated as one pushes the filling factor further and further away from $1/p$. Eventually, according to the hierarchy scenario, the quasiparticles themselves form an incompressible quantum liquid. If, for example, one starts out at $\nu = 1/3$, one arrives at a Laughlin-analog state of $e/3$ -charged vortices when ν reaches $2/5$. Playing the same game again, one generates excitations of the $2/5$ daughter state until there are enough of them to create a granddaughter quantum Hall

state at $\nu = 3/7$. Eventually one can generate all the fractional quantum Hall states by this scheme.

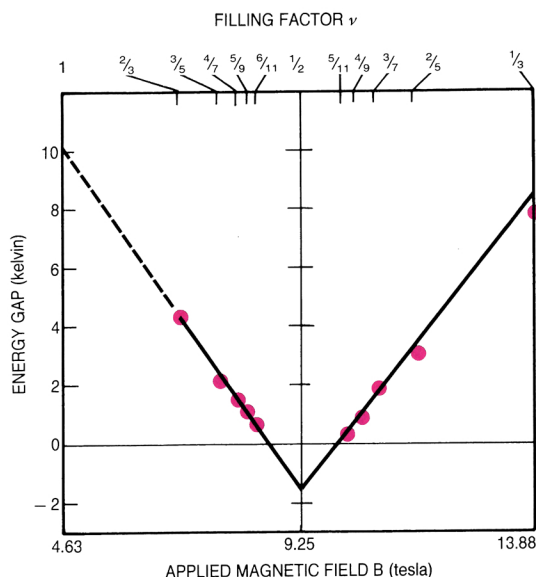
Seeking an alternative to the hierarchy theory in 1989, Jain pointed out that the principal-sequence fractional states could be thought of as integral quantum Hall states of the composite fermions. The $2/5$ state, for example, with 5 flux quanta for every 2 electrons, could be viewed as two filled Landau levels of composite fermions. Jain argued that the analytic form of the Laughlin multi-electron wavefunction was very suggestive of the idea that every electron in a Laughlin state comes with two flux quanta attached.

Acoustic wave anomalies

For a time Jain's approach met with considerable resistance among the theorists. But then experimenters unearthed a striking anomaly at $\nu = 1/2$ that cried out for explanation. In 1990 a Bell Labs group led by Robert Willett set out to investigate the fractional quantum Hall regime by means of surface acoustic waves at gigahertz frequencies. Most of the previous information had come from static measurements of dc transport parameters. But in fact much can be learned about the two-dimensional electron system by examining the frequency and wavelength dependence of its resistivity in a time-varying field. In Willett's experiments the acoustic waves were generated at the surface of a GaAs/AlGaAs heterostructure only about 1000 Å above the interface. The strong piezoelectric character of the materials translates an acoustic surface wave into an electromagnetic wave at the interface below.

In two years of experiments, Willett and Mikko Paalanen measured the attenuation and velocity of the acoustic waves as a function of filling factor, wavelength and temperature.¹ At all of the odd-denominator filling factors at which one sees prominent quantum Hall plateaus, Willett's experiments show sharp decreases in the attenuation of the acoustic wave accompanied by sharp increases in sound velocity. That's equivalent to saying that the high-frequency resistivity in the fractional quantum Hall states is very close to the dc value measured in ordinary transport experiments.

That's also what one sees in the integral Hall states, but *not* at $\nu = 1/2$. The modest drop in the dc resistivity at $1/2$ suggests that the acoustic attenuation should also drop, and the sound velocity increase. But Willett and Paalanen found just the opposite:



Energy gaps of a two-dimensional electron system measured at various fractional filling factors and plotted against the corresponding magnetic field strengths exhibit linear dependence reminiscent of the integral quantum Hall effect. Thus the slope is interpreted as a measurement of an effective mass. (Adapted from ref. 2.)

The attenuation increases rather sharply at $\nu = 1/2$, and the sound velocity shows a distinct dip. That is to say, the ac resistivity at short wavelengths shows a marked enhancement at $\nu = 1/2$, with similar enhancements at other even-denominator filling factors such as $1/4$ and $3/2$. (See the figure on page 17.) They found that this anomalous enhancement increases like the reciprocal of the wavelength and that it persists to surprisingly high temperature. The robustness of these even-denominator peaks suggested a series of states complementary to the fractional quantum Hall states. But these new states, unlike the real quantum Hall states, would have no energy gap between the ground state and its excitations.

A gauge transformation

Harvard theorist Bertrand Halperin was on sabbatical leave at Bell Labs in the spring of 1992 when the surface-acoustic-wave experiment was yielding up some of its best surprises at $\nu = 1/2$. Stimulated by Jain's elucidation of the principal-sequence odd-denominator states, Halperin, Read and MIT theorist Patrick Lee had undertaken to develop a detailed theory of the half-filled Landau level. After all, $1/2$ is the limit of the principal sequences $p/(2p \pm 1)$, from above and below. Why then is there no energy gap at $\nu = 1/2$?

Halperin and his collaborators treated Jain's composite fermion as a mathematical artifact generated by a gauge transformation. The physical predictions of a gauge-invariant field theory remain unaltered when an arbitrary gauge transformation of the

particle wavefunction is accompanied by a compensatory transformation of the field. That's essentially what one is doing when one takes flux lines from the external field and attaches them to the electrons.

In their paper published back-to-back with Willett's experimental paper, Halperin, Lee and Read examined in detail the consequences of performing such a gauge transformation (known as a Chern-Simons transformation) on a half-filled Landau level.⁴ At $\nu = 1/2$, the transformed electrons, now sporting two flux quanta apiece, see a transformed magnetic field that is, on average, zero. Already at that "mean field" level one has an explanation of why the $1/2$ state looks so much like a field-free Fermi sea topped by a Fermi surface rather than an energy gap. But Halperin and his collaborators went on to higher-order, time-dependent approximations that take into account fluctuations of the gauge field about its mean value. One reason for this exercise was to assure themselves that the mean-field solution doesn't blow up in the face of small perturbations. The time-dependent treatment was also essential for explaining the anomalous response of the $1/2$ system to high-frequency acoustic waves.

"It's quite remarkable how much the dc transport properties of the half-filled Landau level look like what we see when there's no magnetic field," Halperin told us. "But when you get to the [time dependent] random-phase approximation, you find that high-frequency behavior is quite different from the field-free case." That's mostly due to the surprising slowness of

the relaxation response of the electron gas at $\nu = 1/2$. The random phase approximation predicts that the anomalous resistivity of the $1/2$ state in the presence of an acoustic wave increases linearly with wavenumber (reciprocal wavelength). That's just what Willett and Paalanen were finding in the laboratory. In fact the overall agreement between the surface-acoustic-wave experiment and the Halperin-Lee-Read theory turned out to be surprisingly good.

Weighing composite fermions

The theory also predicts what happens when the filling factor departs from $1/2$. Higher-order perturbation calculations beyond the random-phase approximation predict that interaction between electrons generates a surprisingly high effective mass for the composite fermions. This effective mass parametrizes the ν dependence of the energy gaps of the fractional quantum Hall states. "If you stayed at the mean-field approximation," says Halperin, "you'd just use the ordinary effective mass of conduction electrons in GaAs and you'd end up with gaps that are much too large."

With the advent of the Halperin-Lee-Read theory, it became particularly interesting to have good measurements of the energy gaps of all the accessible principal-sequence quantum Hall states. That task was undertaken last fall by Stormer, Tsui and Tsui's Princeton colleague Rui Du, using very high-mobility GaAs-AlGaAs heterostructures fabricated at Bell Labs by Loren Pfeiffer and Kenneth West.² The figure on this page summarizes their results. It plots the energy gaps for the prominent fractional quantum Hall states measured with one particular heterostructure against B .

The figure serves to underline a striking general property of the fractional quantum Hall states that is embodied in the composite-fermion theories. If one simply shifts the magnetic field axis of the figure to pretend that the field at $\nu = 1/2$ represents $B = 0$, then the fractional states look almost exactly like the integral quantum Hall states around $B = 0$. Because the ΔB between the $1/2$ and $1/3$ fractional states is just the B corresponding to the integral state at $\nu = 1$, the axis shift makes the $1/3$ state look like the first integral quantum Hall state. By the same argument the $2/5$ state looks like the $\nu = 2$ state, the $3/7$ state becomes the $\nu = 3$, and so on *ad infinitum* at the center. The left-hand side of the plot now represents the integral quantum Hall states with the magnetic field direction reversed.

The gap energies of the integral quantum Hall states are just the cyclotron energies. They, of course, scale linearly with magnetic field, and the resulting slope gives the ordinary effective mass of electrons in GaAs. As one expects from the Halperin-Lee-Read theory, the figure shows that the gap energies of the *fractional* states have just the same kind of linear dependence on magnetic field around $\nu = 1/2$. "That suggests a description of the fractional quantum Hall effect in terms of the mass of some new particle," Stormer argues. From the slope of these straight lines the Bell Labs-Princeton collaboration gets an effective mass about of 60% of the free-electron mass. As the theory predicts, that's an order of magnitude more than the usual effective mass that comes from the band structure of the semiconductor.

There is, of course, a bit of a fiddle here. The theory and the experiments tell us that the $1/2$ state has no energy gap. Therefore a straightforward interpretation of these slopes as measures of the composite-fermion effective mass would require all the data to extrapolate to zero gap at $\nu = 1/2$. But in fact we see that the data in the figure extrapolate to slightly negative energies at $1/2$. Stormer and company, however, regard this negative intercept as yet another analogy with the case of real electrons: The integral quantum-Hall levels are broadened by the scattering of electrons at impurities—otherwise one wouldn't see plateaus. That broadening reduces the Landau-level energy gaps and thus yields a negative intercept in the integral quantum-Hall case. Therefore, Stormer argues, the negative intercept in the plot of energy gaps at *fractional* filling factors is actually a

measure of the scattering rate of the composite fermions. That scattering rate turns out to be in good agreement with the Halperin-Lee-Read theory.

Plots of Hall resistivity against B also exhibit this extraordinary similarity between the fractional and integral quantum Hall states, even though the physical bases of the two effects look so different at first glance. But it's the second glance, embodied in the Jain and Halperin-Lee-Read theories, that clarifies the resemblance. "It's given us a whole new way of looking at the hierarchy of fractional quantum Hall effect," explains Stormer. "The fractional states are the integral quantum Hall states of these composite fermions—the electrons with two flux quanta attached. This new framework makes the whole subject so much easier to think about." The paper in which the Bell Labs-Princeton collaboration purports to measure the effective mass of the composite fermions by way of the energy gaps² is entitled "Experimental Evidence for New Particles in the Fractional Quantum Hall Effect." How real are these "new particles"? "They're as real as Cooper pairs," asserts Stormer.

In the midst of this euphoria, Princeton theorist Duncan Haldane springs to the defense of the much maligned hierarchy theory. "The hierarchy scheme and composite fermions are not competing theories," he told us. "They're just different ways of organizing the same construction. Each has its own particular usefulness" A recent paper by Jian Yang and Wu-pei Su at the University of Houston argues that the two theories are mathematically equivalent.⁵

Tunneling

Pfeiffer, West and their Bell Labs

colleague James Eisenstein have recently developed yet another experimental means of studying the quantum Hall regime. They measure the tunneling current between two heterostructure interfaces separated by a few hundred angstroms. If the $1/2$ state indeed has a Fermi surface one would expect to see an anomaly in the tunneling current at $\nu = 1/2$ because of the absence of an energy gap at that filling factor. But Eisenstein and company reported last December³ their observation of a broad energy gap covering an extensive range of fractional filling factors *including* $1/2$.

In a recent preprint, Halperin and Bell Labs theorists Song He and Philip Platzman invoke the Halperin-Lee-Read theory to save the special character of the half-filled Landau level.⁶ They conclude that the observed tunneling gap is due to the energy dependence of the overlap between the quantum state of the intruding electron and low-energy states of the half-filled Landau level into which it is tunneling. Platzman, He and Halperin calculate that this overlap falls off very fast as the low-energy states approach the Fermi level, thus simulating an energy gap.

—BERTRAM SCHWARZSCHILD

References

1. R. Willett, R. Ruel, M. Paalanen, K. West, L. Pfeiffer, Phys. Rev. B. **47**, 7344 (1993).
2. R. Du, H. Stormer, D. Tsui, L. Pfeiffer, K. West, Phys. Rev. Lett. **70**, 2944 (1993).
3. J. Eisenstein, L. Pfeiffer, K. West, Phys. Rev. Lett. **69**, 3804 (1992).
4. B. Halperin, P. Lee, N. Read, Phys. Rev. B. **47**, 7312 (1993).
5. J. Yang, W. P. Wu, Phys. Rev. Lett. **70**, 1163 (1993).
6. S. He, P. Platzman, B. Halperin, Submitted to Phys. Rev. Lett. (1993).

CRITICAL TEMPERATURE NEARS 135 K IN A MERCURY-BASED SUPERCONDUCTOR

No superconductor has broken the record for the highest critical temperature since 1988, when a thallium-bearing compound, exhibiting resistanceless conduction at temperatures as high as 125 K, laid claim to the title.¹ (See PHYSICS TODAY, April, 1988, page 21.) But in early May a group from ETH in Zurich reported measuring a superconducting transition several degrees above 130 K in a compound containing mercury together with barium, calcium, copper and oxygen.² The researchers were not able to specify the exact stoichi-

ometry of the new champion, because their sample consisted of several phases of the compound and they have been unable so far to separate out the phase that is responsible for the high T_c .

High T_c is not the whole story. The increment is not that great, especially given that thallium-based material can be made, albeit with some difficulty, to superconduct at temperatures as high as 130 K. But in addition to high T_c , at least one member of the mercury-based copper oxide family appears to have favorable behavior

in magnetic fields. Furthermore the material is both simple and novel, offering perhaps some new clues to the mechanism of superconductivity in the class of copper oxides.

There is a dark side, however. Thallium has frightened away some researchers because of its extreme toxicity, and mercury is not much of an improvement. Even if the so-far-unknown phase proves to have favorable electrical, magnetic and materials properties, it will have to be bound into a stable compound if it is to see widespread application. Robert Cava