CONSTRUCTING MODELS OF SPACE–TIME FOR SUPERSTRING THEORY

Calabi–Yau Manifolds: A Bestiary for Physicists

Tristan Hübsch
World Scientific, River Edge, N. J., 1992. 362 pp. $58.00 hc
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Reviewed by Edward Witten

Supersymmetry, or Bose–Fermi symmetry, has fascinated physicists since its discovery in the early 1970s. Part of this fascination comes from the possibility that supersymmetry might help solve the "large number problem" posed by Dirac—the problem of why ordinary particles are so light compared to the Planck mass. The idea is that masses of ordinary particles violate supersymmetry; if supersymmetry is spontaneously broken only at the rather low energies of our world, this would explain why ordinary particles are light.

In the context of superstring theory, which is now the primary approach to unification of all physical laws, the most straightforward way to obtain space–time supersymmetry in four dimensions involves compactification on a Calabi–Yau manifold. That is, one takes the underlying ten-dimensional space–time on which the strings propagate to be $M^4 \times K$, with M^4 being four-dimensional Minkowski space and K being a Calabi–Yau manifold, which is a compact six-dimensional manifold obeying certain mathematical conditions.

Models of particle physics based on compactification on Calabi–Yau manifolds were first constructed in 1984, and have some striking phenomenological successes. The simplest constructions give essentially the right gauge groups in four dimensions and the right fermion representations, so that the successes of conventional grand unified theories (for example, explaining the peculiar fractional charge of quarks) are built in, but with more predictive power. Moreover, the number of fermion representations is determined in an elegant way by the topology of K, and the Yukawa couplings (which together with other things determine quark and lepton masses) are also elegantly determined by the topology and geometry of K. And there are some unusual predictions, such as the existence in many of these models of unconfined fractional electric charges with Planck masses.

Following these developments, most activity in this area has been in one of two directions. One involves the study of algebraic constructions of models with low-energy supersymmetry (based on orbifolds, $N=2$ minimal models, free fermions, and so on). Some or perhaps even all of these models are equivalent to special cases of models based on Calabi–Yau manifolds, but the algebraic constructions have the great virtue of permitting easier and more complete calculations than does the general model based on a Calabi–Yau manifold. The other direction has involved studying mathematical properties of the Calabi–Yau models, largely motivated by the wish to make them more computable. Many beautiful discoveries have been made, which go in the direction of showing that in the context of a proper understanding of string theory, the concept of space–time as we know it will have to be replaced by a new concept that we do not yet understand.

The main obstacle to further fundamental progress is probably the absence of any real idea about why the cosmological constant vanishes after supersymmetry breaking. The vanishing of the cosmological constant should be a crucial clue to the correct mechanism of supersymmetry breaking, which in turn (if low-energy supersymmetry is right) is part of what controls the quark and lepton masses.

In the book under review here, Tristan Hübsch of Howard University has done a yeoman's job of surveying and expounding a wide range of background knowledge essential for understanding some of the developments I have just discussed. He focuses on describing the construction of Calabi-Yau manifolds and computing some of their properties, which—in this approach to elementary particle physics—determine such things as the number of fermion representations, the weak coupling limit of the Yukawa couplings and the possibilities for gauge symmetry breaking. Hübsch has patiently collected and described a sometimes bewilderingly large number of relevant techniques from differential geometry, and he thoroughly works out some quite illuminating examples, including many of physical interest. He explains how to extract from a given Calabi–Yau manifold not just the simplest properties—such as the number of quark and lepton generations—but also more subtle properties such as the number of approximately massless gauge singlets and the Yukawa couplings. His explanations are generally thorough and clear, though the wealth of concepts invoked will undoubtedly make the going bumpy in spots for most readers.

Most of this material has appeared in print before, but its presentation in book form will make the subject considerably more accessible. Hübsch's exposition, which is engaging and entertaining, is aimed primarily at the reader who is already familiar with how Calabi–Yau manifolds can be used in making models of elementary particle physics, and wants to understand the techniques used in constructing them and analyzing their properties. Hübsch presents a rather extensive sketch of the aspects of classical geometry of Calabi–Yau manifolds that are most relevant. In the final section of the book, Hübsch offers a tantalizing

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introduction to the many remarkable quantum properties of Calabi-Yau sigma models discovered in the last few years, such as their relation with supersymmetric minimal models, the Landau–Ginzburg correspondence and mirror symmetry. A second book surveying these developments would still be a valuable contribution, though the relatively rapid progress that is occurring would undoubtedly make such a book difficult to write.

Computational Methods in Physics and Engineering

Samuel S. M. Wong
Prentice Hall, Englewood Cliffs, N. J., 1992.
677 pp. $48.00 hc ISBN 0-13-155953-2 Disk included.

Numerical calculation has played a continuous and vital role in physics since at least the time of Kepler, yet the undergraduate curriculum for physics majors often does not address the issue at all. A generation ago, extensive calculation was so tedious or expensive that it made sense to postpone education in numerical methods until a research project required them. But the drastic change in computer power, price, and accessibility now calls out for incorporating computation into the undergraduate curriculum. The question is where.

Computational Methods in Physics and Engineering, by Samuel S. M. Wong, claims to be "mainly concerned with the ways that computers may be used to advance a student's understanding of physics." It is designed for use either as a text in an undergraduate course or for independent study by seniors and graduate students.

The book presents a large number of mathematical and numerical methods that are met in the physics curriculum and required later in research; some techniques are explained in the context of a specific physics application. The book covers the traditional topics of numerical analysis—integration, interpolation and extrapolation, ordinary and partial differential equations, numerical linear algebra and least squares fitting (though there is almost nothing on solving nonlinear equations). In addition, there are chapters on special functions, an introduction to computing (which moves rapidly from "computer literacy" questions to factoring prime numbers) and very cursory chapters on graphics and computer algebra. There is an extensive chapter on Monte Carlo calculations with numerous applications to rather specialized topics, including percolation, matrix ensembles and path integrals.

For most of the topics and methods discussed, Wong begins with a mathematical explanation that leads to an algorithm, expressed in a "box" in steps that are not quite self-contained. A floppy disk that comes with the text contains Fortran programs implementing all the algorithms presented. This approach is very appropriate, as it permits the text to discuss algorithms in conceptual terms while the working program lets the student explore the method and see all the details explicitly. References to further theoretical treatment are given for many methods, but references to suitable programs and libraries would have been helpful. Two numerical analysis packages, EISPACK and LINPACK, are mentioned in the introduction, but not LAPACK, which is an updated replacement for both of them. The introduction also mentions NETLIB, a repository for many useful numerical analysis programs. But no internet address is given, which will certainly make it harder for the unsophisticated network user to find this important source of material for further calculational techniques.

In writing (or reviewing) an undergraduate textbook, many issues need to be resolved. One must aim at a certain level of prerequisite knowledge, and decide between covering many topics briefly or fewer topics in a more leisurely fashion, and whether to lay a broad basis preparatory to more advanced methods or to present the methods explicitly. Wong has chosen to present a large number of diverse methods explicitly. While the foundation of each is discussed, I think more emphasis should have been placed on explaining the methods. In the treatment of many topics, I found it slow going to understand what the fundamental approach was. An extreme case was the treatment, under the section called "extrapolation," of "Richardson's deferred approach to the limit." I was confused about where extrapolation fit into this approach—apparently so was the author, because his program for implementing this method contains an extrapolation subroutine but never calls it!

I also think there is a problem in the level of presentation. For while the preface implies that the book is aiming at a junior level course, topics such as Clebsch–Gordon coefficients, critical phenomena and hydrogenic wave functions seem to aim at a higher level. I would prefer to see a book with fewer methods and topics and clearer explanations, aimed at a lower level of physics applications but at the same mathematical level.

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The Monte Carlo Method in Condensed Matter Physics

Edited by K. Binder
Springer-Verlag, New York, 1992. 392 pp. $59.50 hc ISBN 0-387-54369-4

Monte Carlo methods have grown tremendously in importance since their first serious use 50 years ago. Recent progress in statistical mechanics has come in the form of greater and more widespread understanding of the crucial importance of finite-size scaling of Monte Carlo data. In addition, specialists have developed new algorithms that in some cases can ameliorate the universal problem of computations slowing down near critical points. *The Monte Carlo Method in Condensed Matter Physics*, edited by Kurt Binder, surveys recent progress and the state of the art in a number of areas of condensed matter physics. In most of the 12 chapters there is a very brief overview of the subject in question, followed by a catalog of recent work with many hundreds of references to the original literature.

Chapter 1, written by Binder, has a good discussion of the sources of error in the context of a finite computational budget. Chapter 4 covers histogram techniques and algorithms to beat critical slowdown. These algorithms are very promising, although the fact that they are not (yet) adaptable for vector processors is a significant limitation. Also there are recent suggestions that these methods are sensitive to extremely small imperfections in random-number generators. Other chapters are more oriented toward computational techniques used for specific physical systems or situations, such as quantum spins, random growth, classical fluids, quantum fluids, macromolecules, percolation, interfaces and wetting, and glasses.

Like the software used in Monte Carlo methods, the current hardware situation is rather remarkable. Giant vector-processing mainframes are teetering on the brink of extinction, barely able to compete with small scalar workstations, which have grown faster and cheaper at a tremen-