WANNIER–STARK LADDERS AND BLOCH OSCILLATIONS IN SUPERLATTICES

Optical experiments in semiconductor superlattices in electric fields have shown the existence of a long debated quantum mechanical phenomenon, bringing us closer to demonstrating an extremely fast emitter of radiation.

Emilio E. Mendez and Gérald Bastard

Conduction electrons in real crystalline solids behave very much like electrons in free space, moving in straight lines between collisions when subject to an electric field. But in an ideal (although cold) world, free from scattering by impurities, imperfections and thermal vibrations of the lattice, how would conduction electrons behave? That question, answered in principle long ago in light of the then newly developed quantum mechanics, was purely academic until recently.

The proposal and demonstration of superlattice effects in semiconductors at the beginning of the 1970s and subsequent improvements in the perfection of materials during the 1980s (see figure 1) have revived interest in that old question and in its implications—the most important of which is the possibility of making an extremely fast emitter of electromagnetic radiation, the so-called Bloch oscillator. Although there is no experimental evidence of such an oscillator yet, progress in the optics of superlattices during the last five years has brought us closer to its realization. The biggest advance was observation of the Wannier–Stark ladder, a quantum mechanical concept proposed about 40 years ago and strongly debated since then. As the search for the Bloch oscillator continues, the consequences of the Wannier–Stark ladder are being explored for possible application in optical devices.

To understand what a Bloch oscillator would be, it is first necessary to realize that the motion of an electron in a periodic potential, such as that provided by atoms in a crystal, is governed by a dispersion law describing the relation between the wavevector and the energy of the electron within the range of possible energies—the energy band.

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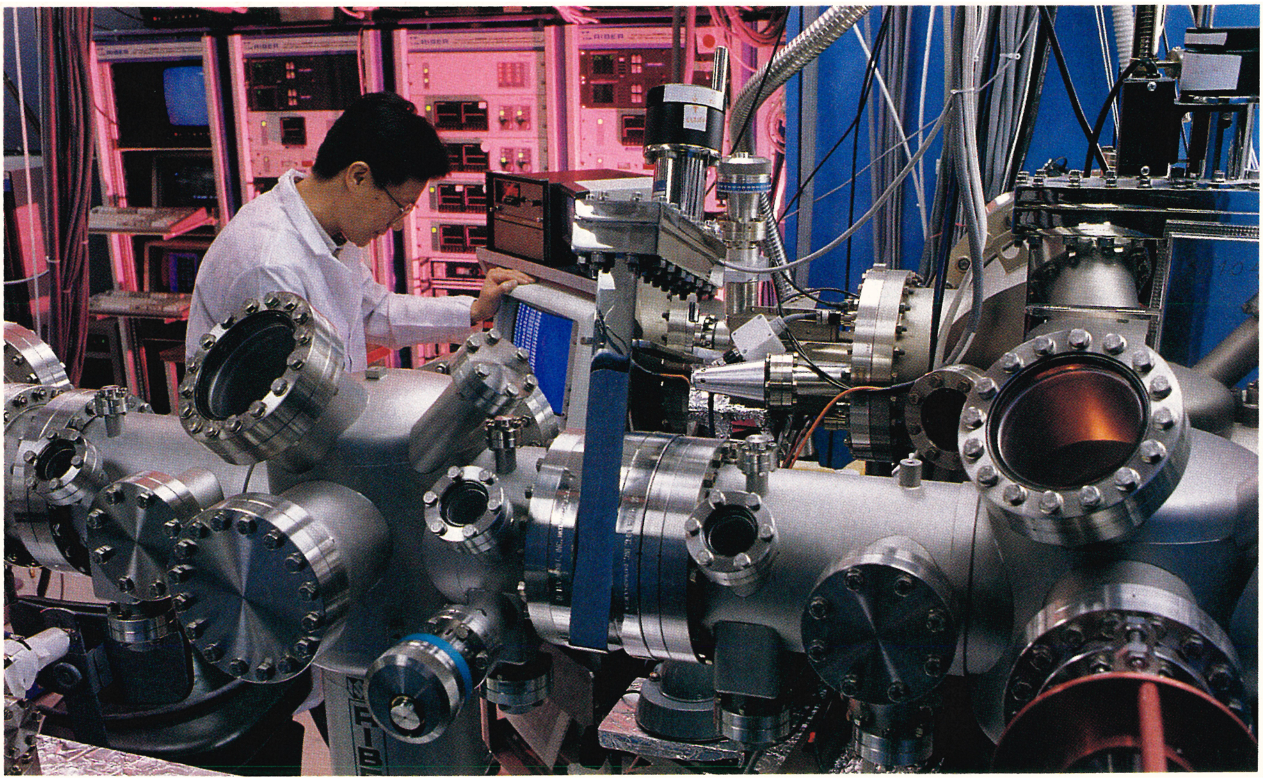
Figure 2 shows a simple example of the behavior of an electron of effective mass m^* moving along a one-dimensional potential of period D up to a maximum energy Δ.

Under the application of an electric field $\mathcal{E}$, in a semiclassical picture, the wavevector and correspondingly the kinetic energy of a collisionless electron would increase along the band. In contrast to what would happen in free space, where the kinetic energy would increase indefinitely with the square of an ever increasing velocity, the electron in the crystal would be first accelerated and then slowed down until it reached the top of the energy band. At this point the charge would reverse direction, retracing its steps and reaching the bottom of the energy band. In real space, the electron would move back and forth between its initial position and an end point, becoming localized in a finite region of the crystal whose dimension would be in direct proportion to the bandwidth Δ but inversely proportional to the field. The motion would be repeated periodically with a frequency ν_B (the Bloch frequency) proportional to the field and to the period of the potential ($\nu_B = e\mathcal{E}D/h$, where h is Planck's constant), and the charge oscillations would give rise to emission of electromagnetic dipole radiation. The Bloch oscillator is a hypothetical device based on this effect.

In this idealized case even an infinitesimally small field would be sufficient to produce Bloch oscillations—although of very low frequency. For a typical semiconductor with a lattice unit cell of 5 Å and a conduction bandwidth of 1 eV, a very moderate field of 100 V/cm would localize electrons within a 100-micron region and make them oscillate at a 1-GHz frequency. In a more realistic situation in which scattering processes are taken into account, a minimum field is required for electrons to reach the top of the energy band. This field, which is inversely proportional to the scattering time and period of the lattice potential, is on the order of 10^6 V/cm.

Splitting of the continuum

At such high fields, the validity of the semiclassical approach outlined above to describe the electron's motion



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State-of-the-art molecular-beam epitaxy system used to prepare periodic multilayers with the quality necessary for observation of the Wannier–Stark ladder in semiconductors. Minghuang Hong at IBM uses the system to grow some of the superlattices discussed in this article. **Figure 1**

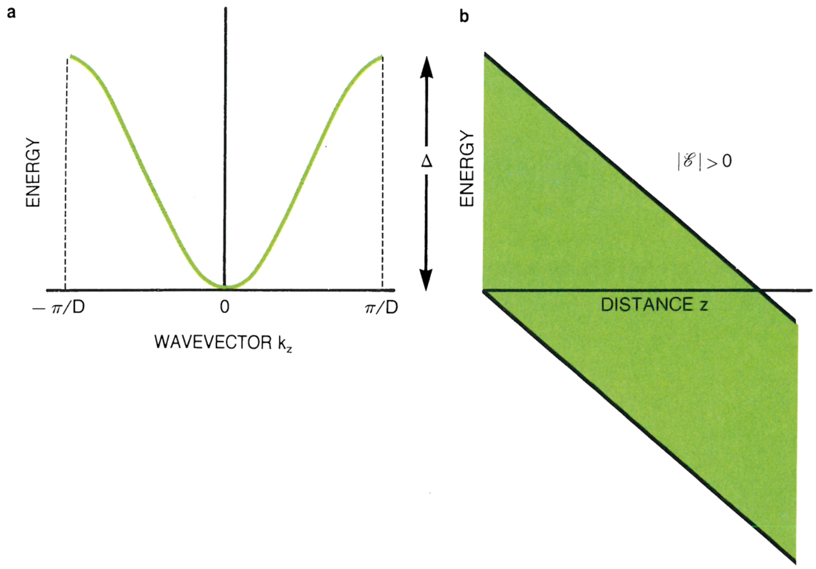
is doubtful. Already in 1949 Hubert James argued that in the presence of an electric field the continuum of band states would split into a series of levels with an equidistant energy separation proportional to the field. Ten years later Gregory H. Wannier systematically studied[1] the motion of electrons in a periodic potential and a constant electric field, finding that the Bloch frequency could be expressed in quantum theory as an energy splitting within each band, which he called "Stark splitting." He showed that if the electronic wavefunction $\psi(z)$ is a solution of the Schrödinger equation with energy E_0, then $\psi(z-nD)$ is also a solution of the equation, with energy $E_0+ne\mathscr{E}D$, where n is an integer. The set of such solutions constitutes the Wannier–Stark ladder of energy levels, whose separation $\Delta E = e\mathscr{E}D$ can be written in terms of the Bloch frequency as $\Delta E = \hbar\omega_B$, where $\omega_B = 2\pi\nu_B$. The wavefunction $\psi(z)$ extends throughout the periodic solid in the absence of the applied electric field, but in its presence is localized to a region whose dimension, as in the semiclassical approximation, decreases with increasing field. In the high-field limit the localization is extreme, with the electron confined to an atomic site. In this formalism the Bloch oscillations are the counterpart in the time domain of the stationary-state Wannier–Stark ladder in the energy domain.

The quantum mechanical description of eigenstates is valid when the energy separation between states is larger than the broadening Γ induced by scattering. In the time domain the condition for quantization can be written as $\omega_B\tau > 1$, where τ is a characteristic collision time equal to $\hbar/\Gamma$. This inequality simply states the requirement that an electron complete at least a full Bloch period before it is scattered.

The quantum mechanical effects of an electric field on a periodic solid have some similarities with those of a magnetic field. As in the case of electric fields, discrete energy levels (Landau levels) are formed in the case of magnetic fields, with a separation that is proportional to the strength of the field. The condition for quantization is identical in both cases once the Bloch frequency is replaced by the cyclotron frequency. However, one has to be careful not to stretch the analogy too far, because the Bloch oscillator is peculiar in that it does not have a zero-point energy, or, equivalently, the Wannier–Stark ladder is infinite in both directions.

For the Bloch oscillator, as in many other instances in physics, it is possible to establish a correspondence between the quantum mechanical and semiclassical descriptions. The equations that govern the average position and velocity of a wavepacket formed by the superposition of Wannier–Stark states are similar to those of the classical Bloch oscillator. From a comparison of the average extension of the wavepacket with a semiclassical result, one can conclude that the latter approximation is adequate if the drop in potential energy drop per period is smaller than about one-third of the bandwidth Δ.

The existence of the Wannier–Stark levels remained controversial for a long time, even as a concept, because the original treatment relied on the assumption of a single conduction band, thus precluding the possibility of interband tunneling. (More recent theoretical analyses have concluded that the Wannier–Stark levels can be regarded as approximate eigenstates, similar to scattering resonances.[2]) Several transport and optical experiments during the 1960s and 1970s, although initially explained in terms of the Wannier–Stark ladder, were inconclusive, mainly because of the smallness of the observed effects.



Dispersion relation and energy plot. **a:** Schematic representation of a generic dispersion relation for electrons in a periodic potential of period D in the z direction. **b:** Sketch of the spatial variation of the two energy extrema when an electric field $\mathscr{E}$ is applied in the $-z$ direction. Semiclassically an electric field parallel to z would ideally increase the wavevector k_z of an electron, raising its kinetic energy to the top of the energy band. A Bragg reflection would bring the electron to the other end of the Brillouin zone, driving it into Bloch oscillations. In real space, the kinetic energy of an electron starting at rest on the left side of **b** would first increase and then decrease, and eventually the electron would stop at the top of the band. The motion would then be reversed, and the electron would oscillate between the two end points. **Figure 2**

This is not surprising, because in a bulk semiconductor the interlevel spacing would be just a few milli-electron-volts for a 100-kV/cm field. In sharp contrast, in semiconductor superlattices, for which the superperiod is typically 50–100 Å, that spacing is an order of magnitude larger. Such an increase has been sufficient to permit the recent observation of the Wannier–Stark ladder, even at room temperature.

In the rest of this article we will discuss the formation of electronic states in superlattices, the experimental evidence of the last few years proving the existence of the Wannier–Stark ladder, and some potential applications in optical devices. We will conclude with a few remarks about the implications of Stark ladder formation for electronic transport and ultimately for the Bloch oscillator.

From coupled quantum wells to superlattices

As Leroy Chang and Leo Esaki have described in detail in these pages (see PHYSICS TODAY, October 1992, page 36), the potential profile created by the band structure of certain semiconductor heterostructures forms a one-dimensional quantum well for electrons (or holes) in one of the material constituents, while the other constituent provides the confining barrier. This profile is superimposed on the atomic potential and affects only the electronic envelope wavefunction perpendicular to the heterolayer interface. The system most studied is the GaAlAs–GaAs–GaAlAs trilayer, in which electron (and hole) states are mostly confined in GaAs—the component with the lowest energy gap between the conduction and valence bands, as sketched in figure 3a. (Although this configuration is quite common among other heterostructures and is the one we will focus on, it is not the only one.)

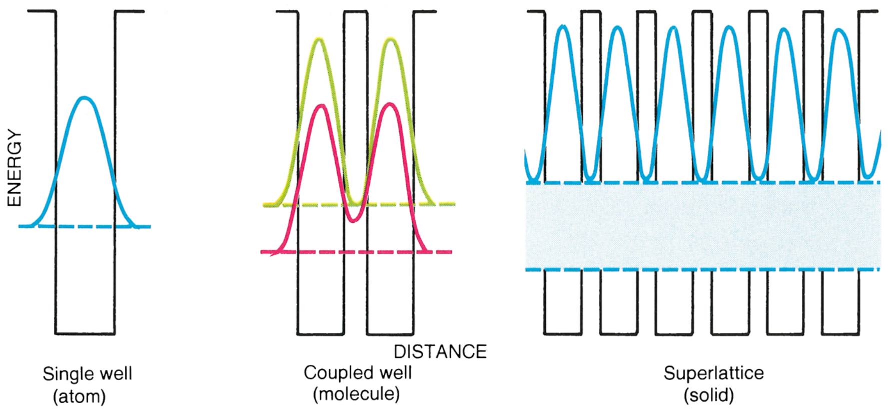
When a multiple heterostructure with two thin GaAs layers is prepared by epitaxial growth techniques such as molecular-beam epitaxy or metal–organic chemical vapor deposition, a double quantum well forms. If both wells are identical and the GaAlAs barrier between them is thin enough, the quantum states of individual wells will be strongly hybridized, and the corresponding wavefunctions will extend throughout both wells. In this case we speak of a coupled quantum well system (see figure 3b). The multiple repetition of the GaAs–GaAlAs building block leads to a structure in which there is a quasicontinuum of energy levels—a miniband—with the envelope wavefunctions completely delocalized. We can regard this arrangement as a one-dimensional periodic structure superimposed on the periodic atomic lattice—in other words, a superlattice (figure 3c).

To understand the effect of an electric field along the superlattice direction, it is easier to consider first how the field modifies energy levels and wavefunctions in a coupled double well. In this case, the relative energy shift of the wells imposed by the field reduces their tunnel coupling and changes the nature of the coupling from resonant at $\mathscr{E} = 0$ to nonresonant at $\mathscr{E} \neq 0$. The wavefunctions are increasingly localized in individual wells as the field increases, and at high fields the energy level separation increases linearly with the field.

The optical properties of such a heterostructure are drastically affected by the electric field. The threshold for optical absorption (determined by the quantum mechanical transition between the ground states of the conduction and valence band wells) is shifted to lower energies, but the onset of absorption simultaneously becomes less pronounced because of the decreased overlap between electron and hole wavefunctions. The electric field's effects on optical emission are somewhat different. The field also "redshifts" the energy of spontaneous (and stimulated, as we will see later) emission, but in contrast to the case for absorption, its steady-state intensity is unaffected by the field, at least ideally. However, the decrease of the interband transition probability translates into longer recombination times.

In addition to the structural periodicity that results from the multiple repetition of the double-well structure, the formation of a superlattice in the electronic sense requires that the electronic envelope wavefunctions be coherent throughout the entire heterostructure. That requirement imposes strict conditions on the degree of perfection of the various layers. Material imperfections such as deviations from periodicity, interface roughness, impurities and so on can reduce wavefunction coherence and, if too large, may produce a complete localization of the eigenstates to individual quantum wells.

Another, more controlled way of inducing localization in a superlattice is by means of an electric field, which, on the same principle as in a coupled quantum well, shifts each superlattice period by an energy $e\mathscr{E}D$ with respect to



Potential profiles (rectangular wells), energy levels (colored horizontal lines) and envelope-wavefunction probabilities (curves) for a single quantum well (**a**), a pair of coupled wells (**b**) and a superlattice (**c**). Such profiles, which crudely resemble atomic, molecular and crystalline potentials, respectively, are created at the conduction and valence bands of multilayers formed by alternating certain semiconductors—GaAs and GaAlAs, for instance. The former material acts as a quantum well for electrons and the latter as a potential barrier. **Figure 3**

its neighbor. (See figure 4.) The partial decoupling among the wells produced by the field reduces the spatial coherence of the wavefunctions, defined as their spatial extension throughout the superlattice. Moreover, the quasicontinuum of states is split into a set of discrete levels equally separated in energy by an amount $e\mathscr{E}D$. In the high-field limit—that is, when $e\mathscr{E}D$ becomes comparable to the energy width Δ of the miniband—the spatial coherence is limited to very few wells (and eventually to one), as if the consecutive wells were completely isolated from each other.

Evidence for the Wannier–Stark ladder

It is possible to deduce these results mathematically even with a tight-binding analysis in which the superlattice wavefunctions are linear combinations of the envelope wavefunctions of individual wells.[2] Under an electric field, the various terms of such a combination are Bessel functions of integer order, which give rise in the mathematical representation to the localization properties just outlined.

Optical experiments, especially absorption measurements, probe the electronic density of states directly, regardless of relaxation effects. So it is not surprising that some of the early attempts to observe the Wannier–Stark ladder in solids relied on optical techniques and that absorption and related measurements in superlattices finally demonstrated its existence.[3] Mathematically, the oscillator strength of the optical transitions between valence and conduction band states is proportional to the square of the overlap integral between their corresponding wavefunctions. In a tight-binding formalism the absorption coefficient α at a photon energy $\hbar\omega$ can be written as[4]

$$\alpha(\omega) = (2N+1)\alpha^{\mathrm{QW}} \times \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{\Delta_c + \Delta_v}{2e\mathscr{E}D}\right) Y(\hbar\omega - E_{\mathrm{gap}}^{\mathrm{QW}} - ne\mathscr{E}D) \quad (1)$$

Here α^{QW} represents the absorption of an isolated quantum well, N is the number of wells in the superlattice, J_n is the Bessel function of order n, and Δ_c and Δ_v are the bandwidths of the superlattice conduction and valence bands, respectively. $Y(z)$ is the step function, and $E_{\mathrm{gap}}^{\mathrm{QW}}$ is the energy bandgap (that is, the onset of the optical absorption) of an individual quantum well in the absence of a field.

The loss of translational invariance of the superlattice potentials due to the field causes the well-defined absorption band edge at $\mathscr{E} = 0$ to formally disappear. A staircase-shaped absorption spectrum emerges instead (see figure 5a), composed of a series of steps corresponding to interband optical transitions between electrons and holes separated by a distance nD, whose equidistant separation increases linearly with the field. (Here $n = 0, \pm 1, \pm 2, \ldots$ are the Wannier–Stark level indices.) The strength of the nonzero-order steps initially oscillates with increasing field and then gradually decreases until the high-field limit is reached, when electron and hole wavefunctions are localized to a single period and only the zero-order ($n = 0$) transition remains. Once the field has completely destroyed the coupling, the superlattice behaves as a set of independent quantum wells. The absorption coefficient regains a well-defined edge, and even more significantly, this edge is shifted by $(\Delta_c + \Delta_v)/2$ to higher energies relative to the zero-field absorption edge.[4]

All these predictions have been demonstrated during the last few years in GaAs–GaAlAs and related superlattices using a variety of optical techniques,[5] ranging from those that measure the density of states (such as photocurrent, absorption or electroreflectance spectroscopy) to those that probe it more indirectly (for instance, luminescence or Raman spectroscopy). Figure 5b reproduces selected low-temperature photocurrent spectra of a GaAs–$Ga_{0.65}Al_{0.35}As$ superlattice clad between n^+ and p^+ GaAs regions that serve as electrodes to apply an electric field along the superlattice direction. Very low fields simply drift the photogenerated carriers to the appropriate electrode, where they are collected as an electrical current. The corresponding optical spectrum (top of figure 5b) shows the absorption characteristics of a superlattice in a nearly flat-band condition ($\mathscr{E} = 0$), with peaks corresponding to the excitation of heavy and light valence band electrons into the conduction band. In this regime the broadening of Wannier–Stark states still is larger than their separation, so the superlattice quasicontinuum is retained.

At intermediate fields—that is, when the separation of the levels is larger than their broadening—the discrete nature of the Wannier–Stark ladder is fully revealed in the photocurrent spectra (middle of figure 5b). Equally spaced peaks of varying intensity develop, with separations and relative strengths determined by the field. As the field increases, so does the peak separation, while high-order transitions gradually disappear at both the low and high ends of the energy spectrum. In the high-field limit (bottom of figure 5b) only the zero-order transition remains, involving heavy valence band and conduction band electrons (as well as light valence band electrons). The absorption edge is definitely shifted to higher energies with respect to that in the low-field regime.

Although photocurrent and absorption measurements have presented by far the most unambiguous proof of the existence of the Wannier–Stark ladder, other optical experiments offer complementary advantages. Electroreflectance, for instance, shows especially high sensitivity, because the optical signal is proportional to the derivative of the dielectric function with respect to the field, which in turn is proportional to nD. Raman scattering does not have to rely on interband transitions, as the energy of optical phonons can be made resonant with the separation between various states of the conduction band Wannier–Stark ladder. Luminescence can give insight into recombination mechanisms between conduction and valence band states that have been spatially separated by the field. The characteristics of a photocurrent-versus-field plot, at a given photoenergy excitation, exhibit oscillations periodic with the inverse of the field, analogous to the Shubnikov–de Haas oscillations induced by a magnetic field. These oscillations, which reflect the presence of energy-equidistant Wannier–Stark states, are still quite visible at room temperature, at which many of the other techniques fail to show those states.

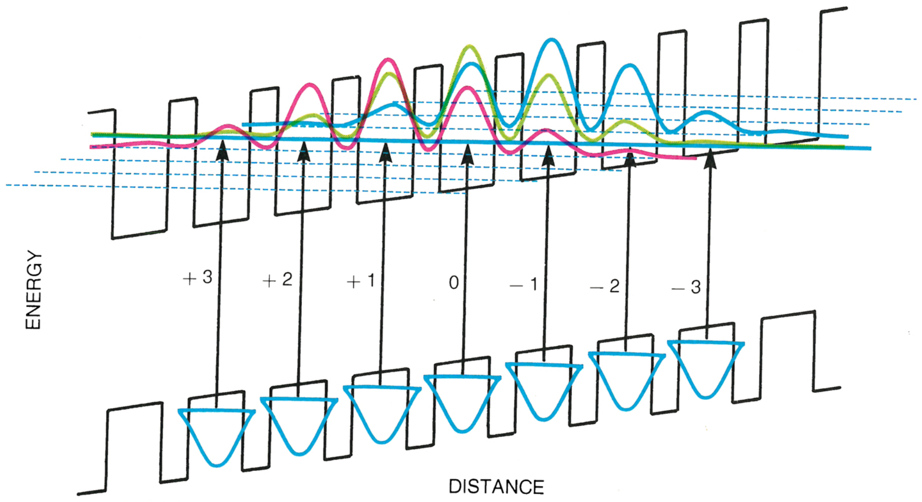
Mapping the ladder

In all cases, a chart of the photon energies at which optical features are observed as a function of electric field maps directly the Wannier–Stark ladder. As an example, we have plotted in figure 6 the peaks of the complete photocurrent spectra for the superlattice of figure 5b. The fan-like picture shows an array of states emerging at low fields from the quasicontinuum of the conduction miniband, extending down to the state with ladder index $n = -6$. The states gradually vanish with increasing field, starting with the ones of highest order, and ultimately only the $n = 0$ state, whose energy is to first approximation unaffected by the field, survives.

Implicit in this fan chart is the dependence on field of the electronic wavefunctions' spatial coherence. Thus the presence of the $n = -6$ transition at low fields attests to a wavefunction in the conduction band that extends six quantum wells "up to the right" of the well at which valence band states were excited, for a total coherence of 13 wells. (Subtle effects make the transitions "to the left" weaker and therefore less observable.) The disappearance of the high-order transitions indicates a progressive field-induced reduction of the spatial coherence and its final collapse to a single superlattice period.

The number of optical transitions with nonzero indices provides a simple and direct measure of the degree of localization of any superlattice's envelope wavefunctions. If the localization is due only to the electric field, then the extension of the wavefunctions is given approximately, in number of periods, by $\Delta/e\mathscr{E}D$. When other perturbations, such as random fluctuations, are at play, this maximum coherence may be reduced.

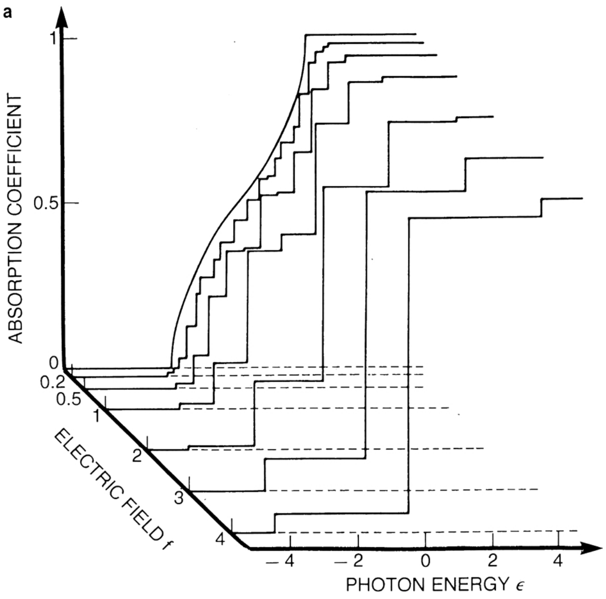
Coherence lengths as large as 17 periods have been



Partial localization of the envelope wavefunctions (colored periodic curves) for the conduction band of a GaAs–GaAlAs superlattice resulting from application of an electric field. The localization is stronger in the valence band because of the greater effective mass of its states. Interband transitions between conduction and valence band states lead to a series of lines in absorption or emission spectra that are equally separated in energy and of different intensity. From the number of spectral lines at a given electric field it is possible to determine the localization length. **Figure 4**

Absorption and photocurrent spectra. a: Calculated interband absorption spectra of a superlattice under a longitudinal electric field $\mathcal{E}$. The dimensionless quantities ϵ and f are directly proportional to the photon energy and the field, respectively. At very low fields (say, $f < 0.2$) Wannier–Stark localization has a small effect on the absorption spectra, but at intermediate fields (say, $f = 1$) several strong steps are clearly resolved. These steps correspond to resonant absorption of photons between valence and conduction band states separated by an average distance nD (where $n = 0, \pm 1, \pm 2, \ldots$). At very high fields ($f > 4$) Wannier-Stark localization is complete. In this limit, the absorption edge is "blueshifted" relative to the zero-field spectrum. (Adapted from ref. 4.)
b: Photocurrent spectra in three different regimes for a 40 Å–20 Å GaAs–$Ga_{0.65}Al_{0.35}As$ superlattice, taken at a temperature of 5 K. At very low fields the spectra are similar to those of a superlattice at zero field. The peak at low energy involves heavy-hole states (indicated by $\mathcal{h}$); that at high energy, light holes (indicated by ℓ). At intermediate fields the various peaks are for interband transitions between fully localized states of the valence band and states in the conduction band that are only partially localized. At very high fields, the quantum wells are effectively isolated and the peaks in the photocurrent spectra are identical to those of a single quantum well. **Figure 5**

a



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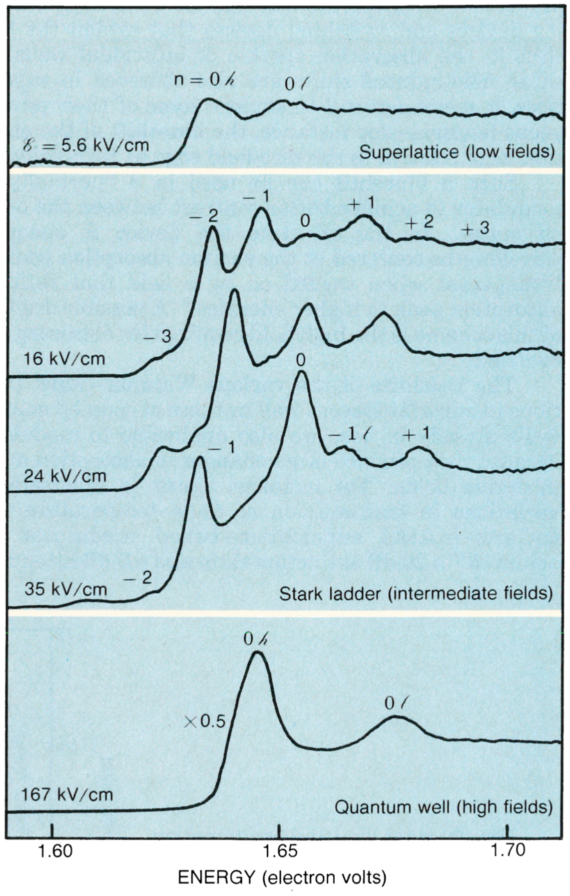


observed at low temperatures in superlattices composed of 25 periods each 55 Å thick. That envelope wavefunctions remain coherent almost throughout the entire superlattice is a feat undoubtedly made possible by the extraordinary progress during the last ten years in materials preparation. Phonon scattering, which increases with temperature, may reduce spatial coherence; however, it does not seem to be the limiting mechanism even at room temperature. Lengths of up to 11 periods have been measured in 37-Å-period superlattices at 300 K, and the detection limit in those measurements was imposed more by an increase in photocurrent background than by a thermal smearing of the resonances.

Tight-binding calculations explain the main effects of an electric field on superlattices, but some details in the experimental results can only be interpreted with a theory that includes the Coulombic interaction between photogenerated electrons and holes. The expression for the absorption coefficient in equation 1 predicts *steps* at photon energies resonant with optical transitions involving Wannier–Stark states; however, *peaks* are observed. Although this phenomenon also is present in bulk semiconductors, it is more patent in systems of reduced dimensionality, as the field-induced effects highlight.

At zero field, the itinerant nature of the envelope wavefunctions effectively makes a superlattice a three-dimensional (although anisotropic) system. Indeed, the Coulomb interaction in a GaAs–GaAlAs superlattice with period $D \leqslant 60$ Å is comparable to that of bulk GaAs, with a binding energy for the electron–hole pair of about 5 meV. However, the field-induced localization alters that similarity drastically. The confinement of electrons and holes to smaller spatial regions increases the attractive force between them and, correspondingly, their binding energy. In the high-field limit this energy is the same as that for an isolated quantum well, or about 12 meV for a well 40 Å thick. (The binding energy of an exciton in a quantum well increases with decreasing well width down to about 30 Å, at which point it starts decreasing as a result of the penetration of the wavefunctions into the barrier regions.) This enhancement of the Coulomb energy is reflected in the increase with field of tne strength of the absorption peaks (figure 5b) and in the deviations from linearity of the Wannier–Stark transitions, especially at low fields (figure 6).

Various calculations have been able to explain these results, among others those in which the exciton eigenstates of the superlattice are expanded in terms of localized exciton wavefunctions, which also account for the asymmetry of the upper and lower branches of the Wannier–Stark transitions.[6]

Probably the main conceptual objection in the past to the existence of the Wannier–Stark ladder was the neglect of high-energy bands in Wannier's theoretical predictions. In a single-band analysis, each miniband would give rise to its own ladder, with crossing between two levels of different ladders possible. However, near resonance—that is, when the energies of both independent levels coincide—the interaction between bands produces a repulsion and anticrossing between levels as well as a delocalization of the electron in the two bands.[7] Ultimately this interaction term is responsible for Zener breakdown—that is, field-induced interband tunneling. Anticrossing, mixing and field-induced delocalization—phenomena essential to transport along the growth axis—have already been observed in GaAs–AlAs superlattices.[8]

Applications of superlattices under fields

The sustained interest in quantum wells and superlattices lies not only in the advantages they offer for the testing of fundamental concepts but also in their potential use in applications derived from their electric-field-controlled properties. In optics, applications have included modulating devices and switching devices that exploit the Stark shift in the absorption spectra of individual wells. Because field-induced shifts are also observed in superlattices, it seems natural to explore some of their advantageous features—for instance, the blueshift of the absorption edge relative to the zero-field edge at high fields.

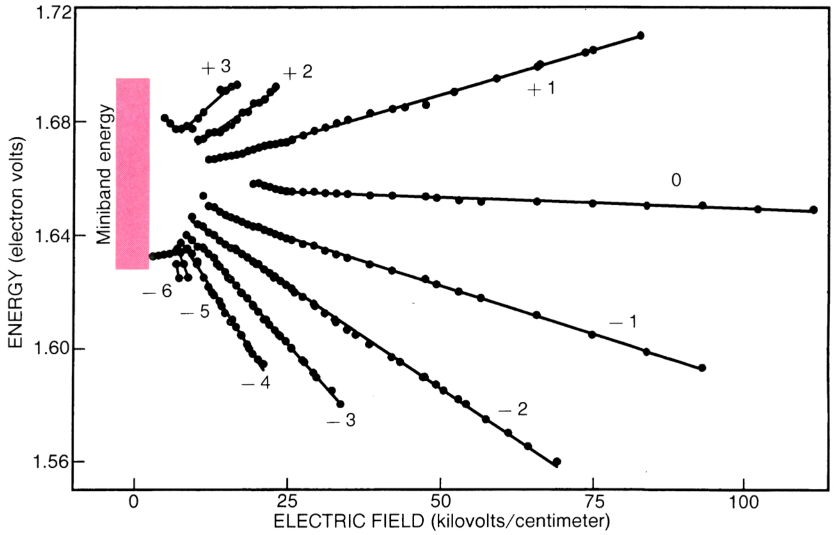
Such a blueshift can be used in a "normally off" modulator to achieve better contrast between the on and off states. In the off state the device is opaque to wavelengths centered at the exciton absorption peak and transparent when turned on by a field that shifts the absorption peak to higher energies.[9] A possible drawback of this scheme is the high field required for obtaining large contrast.

The redshifts of the various Wannier–Stark transitions in superlattices, as well as those of coupled quantum wells already in use, are also applicable to modulators, because they produce large changes in absorption at very moderate fields. For instance, figure 7a illustrates the variations in transmission at room temperature of an InGaAs–InAlAs superlattice-based modulator that achieved[10] a 20-dB extinction ratio and a 3-dB attenuation with a drive voltage as low as 0.8 V.

As mentioned above, the photocurrent through a superlattice illuminated with monochromatic light oscillates with increasing voltage. The region of negative differential conductance in each of these oscillations has been exploited to induce positive feedback in an optoelectronics device consisting simply of a large resistor and a superlattice under an external electric field (using a p–i–n configuration, for example). In this way, a device with multiple stable states, even at room temperature, has been built.

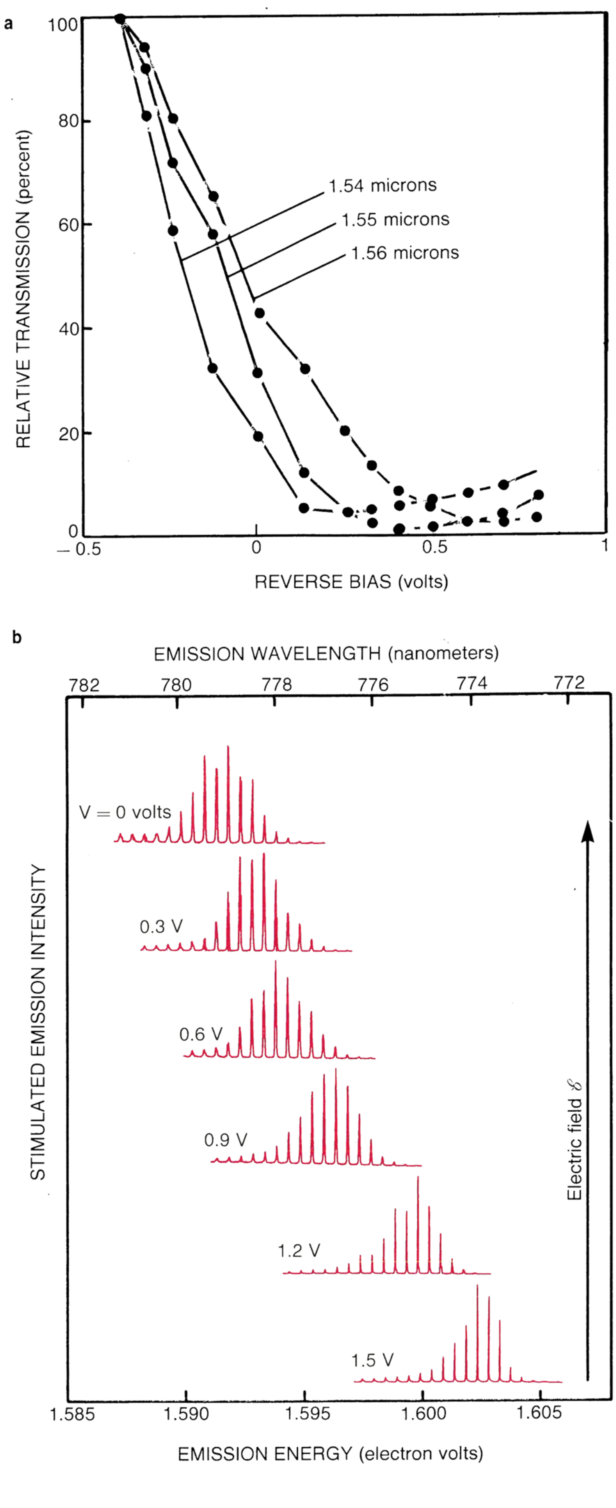
Until now, most of the schemes explored for applications have been based in one way or another on the effects of the field on optical absorption. Recently, however, researchers have employed the localizing action of a field to tune the wavelength of stimulated emission in a coupled-quantum-well laser structure. The scheme is in principle applicable to emitters with a superlattice in the active region of the device. As discussed above, the spontaneous emission of a pair of coupled quantum wells can be redshifted linearly by an electric field without loss of output intensity. (Any decrease of the total intensity is due to field-induced nonradiative processes such as interface recombination.) The stimulated emission from a coupled double well shows the same favorable characteristics, as demonstrated recently by the optical pumping of a p–i–n laser heterostructure whose active region consisted of two 50-Å GaAs wells separated by a 40-Å $Ga_{0.78}Al_{0.22}As$ barrier.[11] (See figure 7b.)

Although modest (about 7 nm), this shift is remarkable. Spontaneous emission is normally observed under conditions of very-low-power excitation, so that the few photogenerated carriers do not perturb appreciably the band bending created by the external field. Field-induced shifts are in this case expected. Stimulated emission, by comparison, requires a population inversion on the order of 10^{18} electrons/cm^3. The electric polarization induced by such a large, spatially separated density of electrons and holes tends to screen the external field, effectively decreasing its effects. Although this mechanism is always present and ultimately saturates the redshift, it still allows for a reasonable range of tunability. Nevertheless, for this effect to be practical, it must overcome two handicaps: It must be robust enough to survive at room temperature, and one has to devise a scheme to introduce carriers into the well electrically (for example, by lateral injection) while an external field is applied vertically.



Fan chart for the interband transition energies of a 40 Å–20 Å GaAs–$Ga_{0.65}Al_{0.35}As$ superlattice, obtained from photocurrent spectra at 5 K. The numbers correspond to the indices of the Wannier–Stark states of the conduction band involved in the transitions. The observation of the $n = -6$ line implies a minimum spatial coherence of 13 periods for the envelope wavefunction of the superlattice. (Adapted from ref. 5.) **Figure 6**

Transmission and emission data. **a:** Room temperature optical transmission, at three different wavelengths, of an InGaAs–InAlAs superlattice under an electric field. The field is produced by application of a reverse bias to the p electrode of a p–i–n diode, in which the heterostructure is imbedded. The large decrease in transmission is due to resonant absorption corresponding to the $n = -1$ transition of the Wannier–Stark ladder. (Adapted from ref. 10.) **b:** Optically stimulated emission spectra, at 5 K, for a laser structure with a coupled-quantum-well active region consisting of two 50-Å GaAs layers separated by a 20-Å $Al_{0.23}Ga_{0.77}As$ barrier. The stimulated emission shifts to lower energies when an electric field is applied along the growth direction of the heterostructure, as a result of the decrease of the heavy-hole and electron ground state energies induced by the field. (Adapted from ref. 11.) **Figure 7**



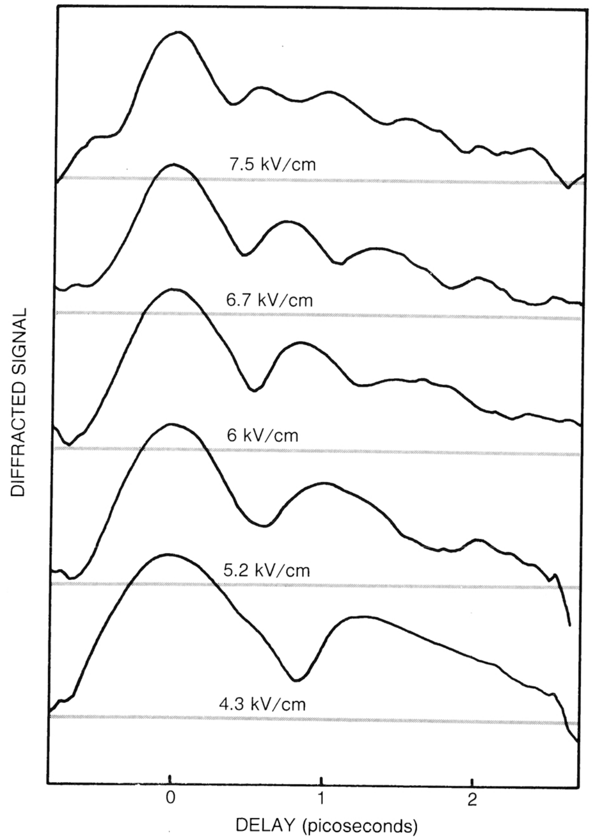
Wannier–Stark ladders and Bloch oscillations

Optical experiments, which as we have just seen can resolve the energy levels of the Wannier–Stark ladder, have also been used recently to study the dynamic behavior of electrons in a superlattice under an electric field.[12] This work follows the conclusion that the diffracted signal of a time-resolved four-wave-mixing experiment in bulk semiconductors reflects the temporal evolution of the electron wavevector,[13] and the suggestion that the method could be applied to superlattices to observe the Wannier–Stark ladder in the time domain—in other words, to detect Bloch oscillations.[14] As previously done in two coupled quantum wells and even before that in atoms and molecules, two short light pulses with wavevectors k_1 and k_2 separated by time t were employed to create a wavepacket formed by the superposition of several Wannier–Stark states. The diffracted signal in the direction $2k_2 - k_1$, which is a measure of the average of the square of the polarization, showed oscillations as a function of the delay t whose frequency increased almost linearly with the field (figure 8). These have been attributed to Bloch oscillations.

It remains to be answered whether such oscillations correspond to a real displacement of electric charge or are simply oscillations of the spatial width of the wavepacket. A semiclassical analysis for an infinite superlattice has concluded that because the velocity of the Bloch oscillator is limited to $\Delta D/2\hbar$, it cannot absorb an arbitrarily large energy from an electromagnetic wave of frequency ω, and consequently the absorbed power should not show any resonance at $\omega = \omega_B$. The same conclusion was reached using a quantum mechanical treatment, in which induced electromagnetic absorption and emission cancel each other as a result of there being identical probabilities for the $n \to n+1$ and $n \to n-1$ transitions.[15] However, for a finite superlattice, net absorption would be possible due to edge effects originating from the period at each of the superlattice's two ends.

Ultimately the answer to the question of the physical mechanism of the oscillations that have been seen lies in the observation of electromagnetic radiation. Such an emission has been detected in an experiment in which an electron wavepacket was optically excited in one of the wells of a double quantum well. The time-varying polarization that accompanied the tunneling of the wavepacket back and forth between the two wells was detected in the form of 1.5-terahertz electromagnetic radiation.[16] In a superlattice, the frequency of the emitted radiation should be proportional to the electric field.

One of the main motivations for the original proposal of an artificial superlattice was the possibility that electronic transport along the new periodicity direction might enter the Bloch oscillation regime.[17] In the presence of scattering, the semiclassical drift velocity of an electron would increase linearly with the field, reach a



Diffracted signal of a four-wave-mixing experiment as a function of the delay between two light beams incident on a 100 Å–17 Å GaAs–$Ga_{0.7}Al_{0.3}As$ superlattice subjected to an electric field at 8.5 K. The optical signal shows oscillations—interpreted as evidence of Bloch oscillations—whose frequency increases approximately linearly with the field. (Adapted from ref. 12.) **Figure 8**

maximum when $\omega_B \tau = 1$ and finally decrease as $(\omega_B \tau)^{-1}$ at large fields. Recently the current–voltage characteristics of GaAs–GaAlAs superlattices spanning a wider range of layer thicknesses than previously studied have been measured at various temperatures.[18] Low-field results show some of the attributes expected semiclassically—even the presence of negative differential conductance beyond a certain field.

Models for transport in the presence of localization (whether induced electrically or by disorder) can also predict negative differential conductance by relying on phonon-assisted hopping between quantum wells, but they seem inapplicable to the low-field measurements. Quite surprising is the experimental demonstration of the coexistence of negative differential conductance and optical transitions associated with the $n = \pm 1$ Wannier–Stark states.[18] In our view this result points out a conceptual problem with the idea of transport at fields for which the Wannier–Stark quantization condition $\omega_B \tau > 1$ is satisfied, because, as we have seen, such a field would localize the electronic wavefunctions.

Another open question is the feasibility of emission of electromagnetic radiation in finite superlattices. If this were demonstrated, the challenge from a practical point of view would be to make the process efficient and to find ways of injecting an electron wavepacket into the superlattice region without having to rely on optical pumping.

In spite of all these hurdles, we can say that progress in the last five years has been outstanding. We have passed from a situation in which the existence of the Wannier–Stark ladder was controversial to one in which coherent transport at the Bloch frequency—if not the Bloch oscillator itself—seems achievable. The development of new materials systems could now make basic ideas going back to the early days of quantum mechanics a reality. At the very least, to paraphrase Eugene Wigner, it has helped this generation of physicists to rediscover quantum mechanics by themselves.

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