OPTICAL PROBES IN THE QUANTUM HALL REGIME

Luminescence and inelastic light scattering provide new ways to study the properties of a two-dimensional electron gas and have recently shed light on the collective states responsible for the fractional quantum Hall effect.

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Semiconductor nanostructures in which mobile electrons reside in a specific spatial region, such as a thin quantum well formed by heterojunction layers on the order of 100 Å in thickness, constitute a particularly striking physical system. The mobile electrons in such a system form a "two-dimensional electron gas." In a uniform semiconductor electrons scatter off the ionized donor dopants, but in a two-dimensional electron gas the electrons can be separate from the dopants and enjoy extraordinarily long mean free paths (many micrometers) at cryogenic temperatures. In the past dozen years or so, this unique property has provided an enormously rich field for the study of electron transport phenomena, ranging from ballistic motion in small device structures to many-electron correlations in strong magnetic fields.

In a transport experiment one obtains information about the electron system by coupling it to large reservoirs—that is, by connecting leads from an external circuit to the active layers in the semiconductor heterostructure—and measuring voltages and currents. Such an experiment communicates with those electrons that have energies within kT of the chemical potential, or Fermi energy E_F, and the signals are averaged over the entire specimen. In this article we outline how the modern arsenal of optical spectroscopy enables one to study two-dimensional electron systems without contacts to the material, in a way that complements and reaches beyond the world of transport phenomena. We focus on systems in strong magnetic fields, in which the quantization of electron motion produces the celebrated quantum Hall effects in transport experiments.[1] Recent optical experiments have for the first time observed modes believed to be magnetorotons—neutral excitations of fractionally charged quasiparticles—associated with the fractional quantum Hall effect and the behavior of the electrons as an incompressible quantum fluid.[2]

Spectroscopy and Landau levels

Luminescence and inelastic light scattering are the two principal spectroscopic methods in which photons emitted near the bandgap energy of a semiconductor can literally "light up" the material's two-dimensional electron gas. Large enhancements in the optical cross sections occur at the bandgap energy, which is the primary electronic tran-

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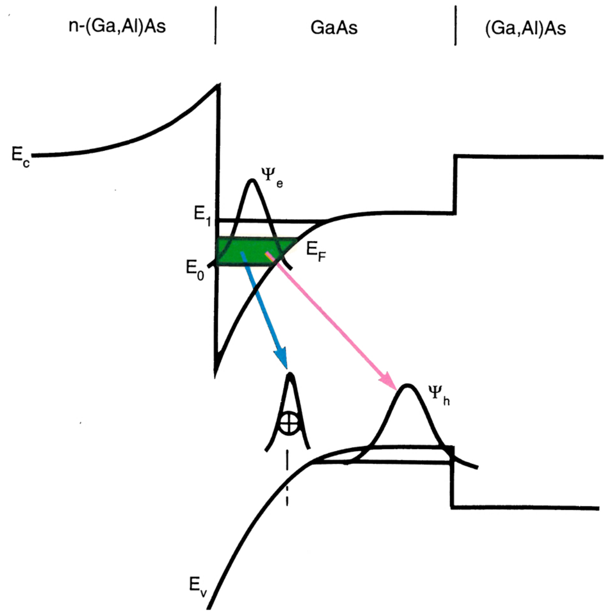


Radiative recombination of an electron (wavefunction Ψ_e) in the conduction band (potential E_c) of a semiconductor may follow two paths. The red arrow indicates recombination with a free hole (Ψ_h) in the valence band (E_v); the blue arrow shows recombination with an additional acceptor dopant (⊕) introduced at a specific location in the GaAs quantum well (δ doping). Both the conduction and the valence band of the two-dimensional electron gas are split into subbands (for example, E_0 and E_1) by the quantum effects of the small well size. **Figure 1**

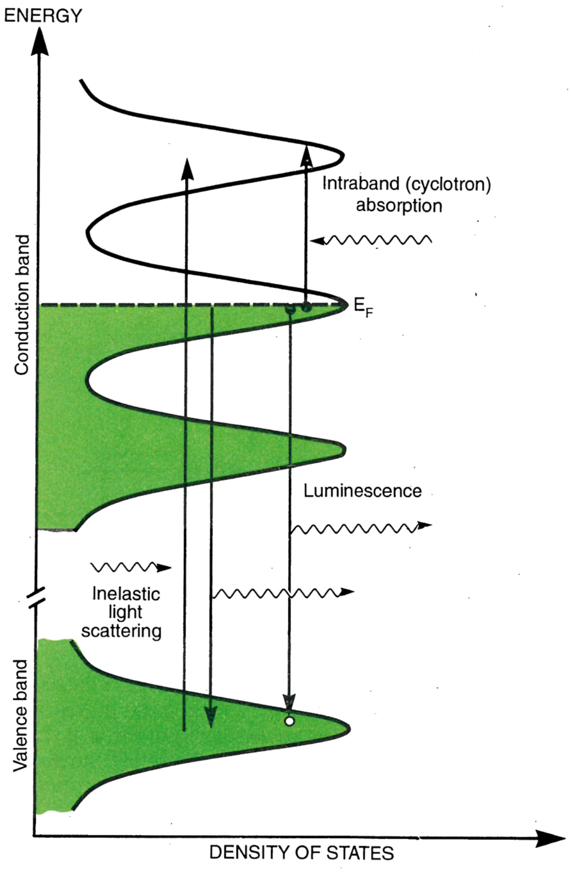
sition of the host semiconductor material. These interband transitions connect the conduction band states E_c (where the electrons of interest to us reside) to those of the valence band, E_v. Hence optical spectroscopy can access a much wider range of electronic states than are accessed in a transport experiment. However, the coupling of the incoming and outgoing photons via the strong bandgap resonance also perturbs the overall electronic system.

Consider first a luminescence experiment. An initial optical excitation of electrons across the bandgap produces a small, nonequilibrium excess electron–hole pair population. Although the additional *electrons* may be inconsequential, given the much larger pool of two-dimensional electrons in the quantum well, the excess *holes* recombine radiatively with all the available electrons, and the associated spontaneous emission provides the spectroscopic imprint of the electron gas. (See figure 1.) This type of experiment, however, has an idiosyncrasy: The positively charged holes cause a Coulomb perturbation of the electron system, which must be independently characterized before the experimental results can be understood in detail. With this important caveat, luminescence probes provide an exciting and timely opportunity for investigating low-dimensional electron systems in semiconductor nanostructures. A great deal of sophisticated instrumentation is available for such spectroscopy. In principle it should be possible to perform time-resolved measurements at the picosecond scale and spatially resolved measurements at the micrometer scale or better.

Inelastic light scattering is a powerful, complementary alternative to luminescence. Interest in using this technique to study two-dimensional electrons began in the late 1970s, when Elias Burstein and collaborators proposed that the cross section for the normally weak light scattering involving electronic excitations could be enhanced near the bandgap and at other "critical point" resonances in the interband density of states. Major advances in sensitive photodetection have made it possible to implement these ideas and study the optically thin two-dimensional electron systems by inelastic light scattering. Photon-counting detector arrays and stable-frequency tunable lasers permit the parallel detection of spectra with scattered photon fluxes as low as 0.01 photons/sec for each detector element! In this way inelastic light scattering gives access to elementary excitations that luminescence (or absorption) spectroscopy cannot reach. Also, while photoluminescence is an incoherent process, light scattering is fundamentally coherent.

In a strong perpendicular magnetic field, the transport phenomena of the integral quantum Hall effect can be regarded as arising from a sequence of metal–insulator transitions in the two-dimensional electron gas as the magnetic field is increased.[1] In a single-electron picture, the conduction (and valence) band states are concentrated into the familiar Landau levels. (See figure 2.) The Landau-level filling factor ν is equal to n/d, where n is the electron density and $d = Be/hc$ is the density of electron states of a given spin per Landau level. The Zeeman effect splits the spin-½ Landau levels into pairs of sublevels (not shown in figure 2) known as "spin-split Landau levels." At even integer values of ν the lowest Landau levels are fully occupied with both spin orientations, and all the higher levels are empty. At odd integer values the highest occupied Landau level is spin polarized, and only its lower spin-split level is fully occupied. In either case the two-dimensional electron system is in an insulating state similar to that of a semiconductor with a small forbidden energy gap. In the integral quantum Hall effect the gap is the energy required to promote one electron from the highest (fully) occupied spin-split Landau level to the next empty one.

The fractional quantum Hall effect occurs at "magic" fractional values of the filling factor. In the standard fractional quantum Hall effect the fractions take the form $\nu = \kappa/m$, where m and κ are integers and m is odd. The fractional quantum Hall effect arises because strong electron–electron (Coulomb) interactions favor condensation of the two-dimensional electron gas into highly correlated many-body states. These states include a collective ground state and quasiparticles, excited states that carry fractional charge and obey "fractional" statistics (neither fermionic nor bosonic statistics). The strongest effect occurs for $\nu = 1/3$. At very small fractional filling factors, the Coulomb interactions are expected to drive the two-dimensional electron system into a solid phase, the Wigner crystal, if the effects of nonzero disorder can be minimized. In the fractional quantum Hall regime, however, the two-dimensional electron gas is an incompressible quantum fluid. Each elementary excitation, a "gap excitation," is separated from the collective ground state by a finite energy gap. Optical spectroscopy is well suited to studying these excitations.



Landau levels form in the conduction and valence bands when a magnetic field is applied perpendicular to the quantum well plane. Possible optical processes include interband transitions such as luminescence and inelastic light scattering. Intraband, or cyclotron, absorption involves longer-wavelength infrared light. The spin splitting of the Landau levels is neglected in this figure. **Figure 2**

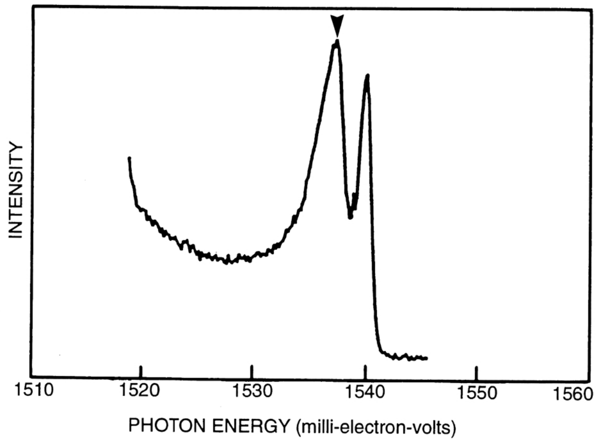
The Fermi-edge singularity

The prototype layered structure used in both luminescence and inelastic light scattering experiments is shown in figure 1. GaAs, AlGaAs and InGaAs are the most commonly chosen materials. The two-dimensional electron gas is located in an asymmetric potential, a quantum well formed by the electric potential differences at the heterojunction and by space-charge effects. The conduction and valence bands acquire additional structure (subbands) because of the quantum effects of the small size of the well. Figure 1 shows two of the lowest quantized conduction subbands (E_0 and E_1) and the highest valence band (lowest energy hole) state. States in the subband E_0 are filled up to the Fermi level E_F. (The components of the electrons' momenta in the two directions not indicated in the figure range freely.)

In a luminescence experiment one can control the spatial overlap between the photohole states in the valence band and the electron states in the conduction band by, for example, varying the width of the quantum well. This provides some control over the recombination of conduction electrons with valence holes (red arrow in figure 1). Alternatively, conduction electrons may recombine with acceptors in an atomic monolayer of acceptor impurities in the vicinity of the electron gas (blue arrow). The location of such a "δ-doping" layer controls the spatial overlap of the states involved.

At low temperatures the low-density photoholes populate mostly the states with small wavenumbers $k \ll k_F$. Because the momentum of the luminescent photon is small, conservation of momentum implies that the luminescence amplitude diminishes rapidly as the conduction electron's wavenumber k increases from 0 toward k_F, unless there is additional scattering or localization by impurities or by potential fluctuations caused by compositional disorder. In contrast to this expectation, Maurice Skolnick and coworkers at the Royal Signals and Radar Establishment in England observed in 1987 that the luminescence spectrum from a modulation-doped InGaAs quantum well showed an unusual sharp increase in intensity at its high-energy edge (E_F), corresponding to recombination of electrons from near the Fermi level.[3] This effect is a striking demonstration of the spectral consequences of Coulomb interactions between a single hole and many electrons acting collectively. Such spectral enhancement near the Fermi level is now considered to be a form of Fermi-edge singularity in semiconductor heterostructures, roughly analogous to edge singularities observed in x-ray photoemission as final-state interaction effects due to the core hole. Gabriela Livescu and coworkers at AT&T Bell Laboratories also have seen this effect, in absorption spectra of an InGaAs quantum well.[4] The origin of such a "many-body exciton" concept in semiconductor theory can be traced to Gerald Mahan, now at the University of Tennessee, who argued 20 years ago that excitonic effects should be found in degenerate semiconductors.[5] The effects of these many-body interactions run counter to semiclassical expectations, which predict that electrostatic screening will dissociate normal (two-particle) excitons in a plasma if the Mott criteria are met.

The many-electron–one-hole recombination process also can be influenced by the detailed subband energy level structure and by dimensionality effects. Wei Chen and coworkers at Brown University demonstrated an example of the former.[6] They noted a very strong en-



Luminescence spectrum from a GaAs quantum well at a temperature of 0.4 K, in which the second conduction subband E_1 lies within a few meV of the Fermi level E_F. The peak marked with an arrow is caused by the "Fermi-edge singularity" and provides a distinctive spectroscopic signature of the electron gas at E_F. (Adapted from ref. 6.) **Figure 3**

hancement in the luminescence spectrum near the Fermi level when an unoccupied conduction subband was designed to lie within a few meV of E_F. Figure 3 shows such a spectrum for a GaAs quantum well at 0.4 K. The peak indicated by the arrow corresponds to the edge-singularity feature at E_F, while the higher peak is the luminescence originating from a population of nonequilibrium photoelectrons deliberately injected into the second subband. Such a distinct spectral signature of the electron gas at E_F is desirable for the subsequent spectroscopic study of the many-electron system, but its presence implies a finite Coulomb perturbation of the equilibrium two-dimensional electron system—the measurement invasively modifies the system. The late Stefan Schmitt-Rink and coworkers at AT&T Bell Laboratories and Pawel Hawrylak at the National Research Council in Ottawa, Canada, put forth a theoretical description of the subband resonance enhancement effect in terms of Coulomb scattering of virtual electron–hole pairs through the second subband channel. By using the subband effect as a tunable element, Chen and coworkers have measured the strength of the electron–hole interaction for a typical quantum well width of 200 Å to be as large as 0.5 meV. (This well width also corresponds to the mean electron–hole separation.)

Luminescence in a magnetic field

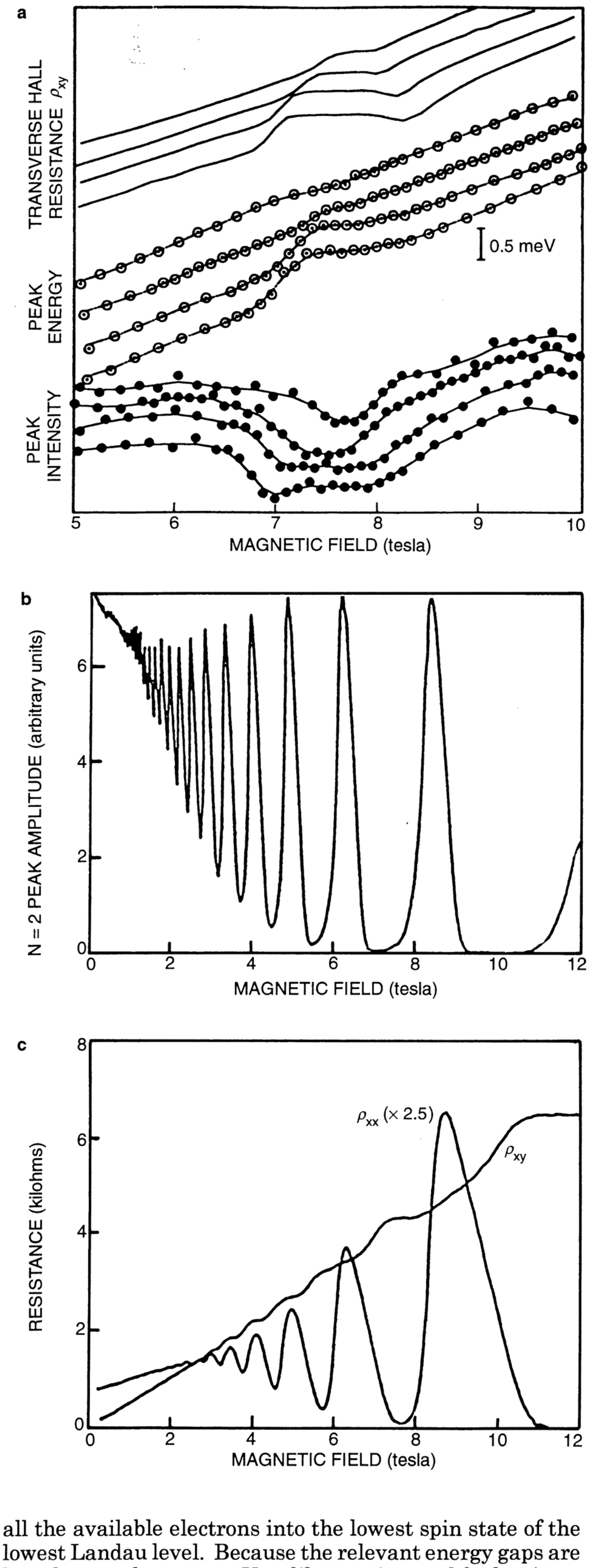
Figure 2 illustrates the main optical processes of a two-dimensional electron gas in a quantizing magnetic field according to the single-electron picture. The densities of states of the electrons and holes are compacted into Landau levels. An underlying question common to both the integer and fractional quantum Hall regimes is how the presence of a positively charged photohole "dresses" the state of the two-dimensional electron gas in a magnetic field. The issue has elements of a chicken-and-egg question: Does the electron gas screen the charge of the photohole and modify its state (and energy), or is it the perturbation of the electron gas by the photohole that provides a spectrocopic object that is sensitive to magnetic fields? At thermal equilibrium, increasing the strength of the magnetic field induces abrupt jumps of the Fermi level at particular field values. These jumps correspond to transitions from extended to localized states in the (single or collective) electron spectrum. Adding a photohole leads to a complicated hierarchy of screening and self-energy corrections, the details of which depend on the strength of the initial Coulomb perturbation by the hole.

Donald Heiman and coworkers at the Francis Bitter National Magnet Lab at MIT showed in 1988 that small but distinct anomalous variations in the amplitude and spectrum of the luminescence occurred near integer filling factors ν of a two-dimensional electron gas, that is, in the magnetic field range where the conductance shows quantized Hall behavior.[7] In subsequent work, Bennett Goldberg and collaborators at Boston University in 1990 studied these changes in detail near the $\nu = 1$ state (the so-called spin gap).[8] (See figure 4a.) A luminescence feature identified with the electrons in the lowest spin-split Landau level is blueshifted and has a minimum in intensity within the field range corresponding to the plateau in the transverse Hall resistance ρ_{xy}. Quenching of these effects with increasing temperature is also well correlated between transport and luminescence observations. Since an intensity minimum occurs when the Fermi level of the (unperturbed) electron gas is in one of the localized states (according to the standard description of the integer quantum Hall effect), it is tempting to attribute this effect to variations in the screening of the hole by the electrons. The many-body exciton effects discussed above make theoretical analysis of the system in magnetic fields considerably more complicated. Among recent models and calculations are the work by Takeshi Uenoyama and Lu Sham at the University of California at San Diego and by Tsuneya Ando and collaborators at the University of Tokyo.

For samples whose zero-field luminescence spectra show edge-singularity enhancement caused by the conduction subband structure, the effects of a quantizing magnetic field can be quite spectacular. Figure 4b shows the variations in the amplitude of the many-body exciton obtained by Mike Fritze and collaborators at Brown University[9] from an InGaAs quantum well sample similar to the design that produced the spectrum shown in figure 3. In this special case, the near coincidence of the second conduction subband with the Fermi level makes it possible to observe variations in luminescence intensity over two orders of magnitude almost in synchrony with variations in the longitudinal Hall resistance in the integer quantum Hall regime. Andrew Turberfield and coworkers at Oxford University also have seen large amplitude variations at low integer filling factors, in structures of considerably higher electron mobility.[10] (See figure 5a.) These results are apparently also caused by boosting of the amplitude of the luminescence probe due to resonant enhancement of the second conduction subband.

Luminescence in the fractional regime

The fundamental distinction between the integer and the fractional quantum Hall regimes is the dominance of many-electron (collective) interactions in the latter, which is reached when the magnetic field is high enough to place

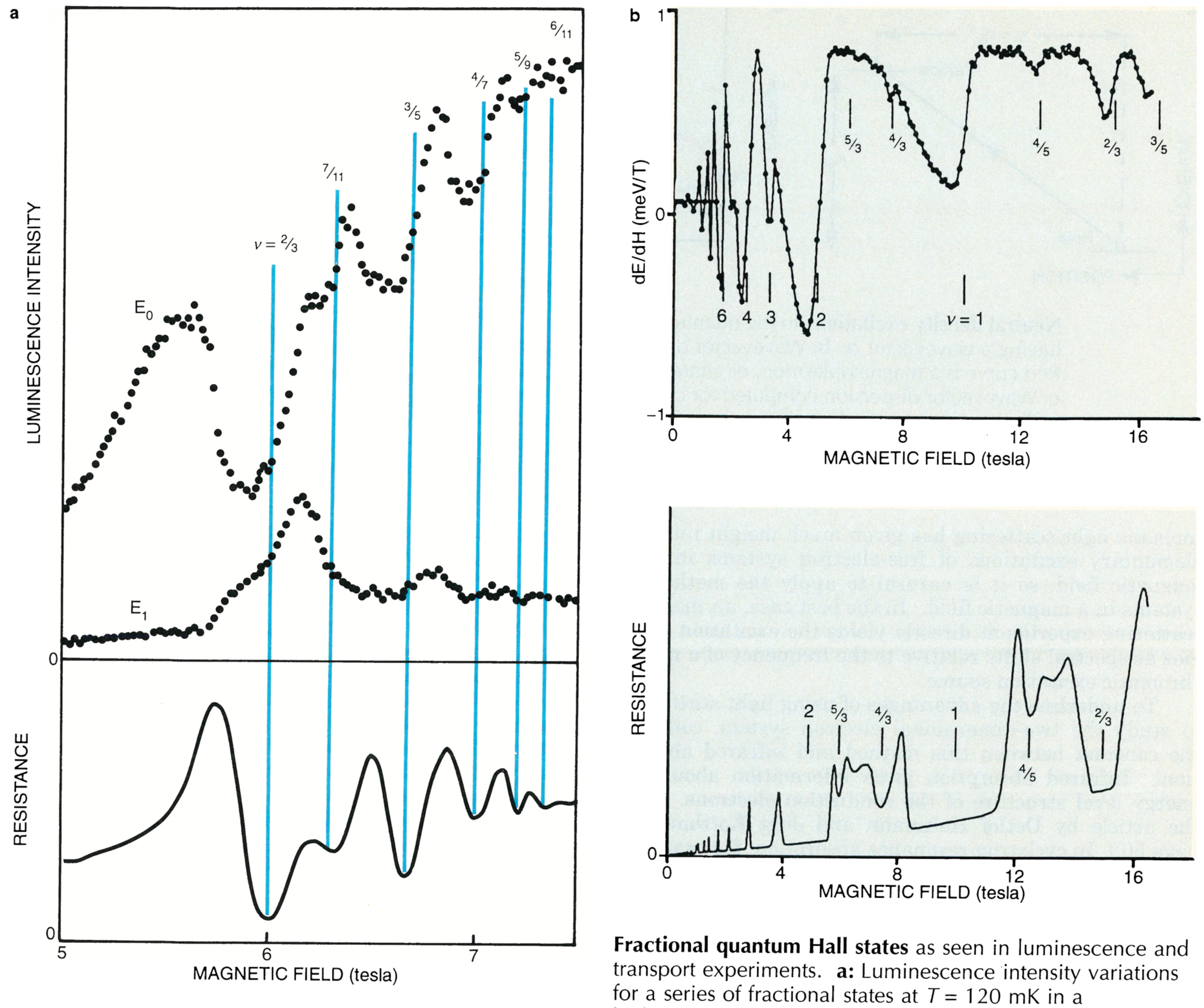


Luminescence signals of the integer quantum Hall effect in high-mobility GaAs quantum wells. **a:** In a magnetic field range where only the lowest Landau level is occupied ($\nu = 1$), the luminescence intensity (bottom set of curves) and the photon energy (middle curves) show synchrony with the transverse Hall resistance ρ_{xy} (top curves). From top to bottom, the temperatures for the four curves in each set are 2.5 K, 1.3 K, 0.6 K and 0.4 K. The curves within each set are offset vertically by a constant amount for clarity. The minima in peak intensity are about 30% less than the intensities away from the minima. (Adapted from ref. 8.) **b:** In special cases where the sample structure is designed to enhance the Fermi-edge-singularity effect, the luminescence amplitude can undergo very large amplitude variations. **c:** These variations occur in near synchrony with the quantum Hall effect (specifically the longitudinal Hall resistance ρ_{xx}) for a range of integer filling factors ν. (Adapted from ref. 9.) **Figure 4**

all the available electrons into the lowest spin state of the lowest Landau level. Because the relevant energy gaps are less than or about an meV, a dilemma is posed for luminescence spectroscopy: The photohole is necessary for the recombination process but causes a Coulomb perturbation with interaction energies typically comparable to the fractional gaps. The properties of the many-electron–one-hole complexes in this magnetic quantum limit are also of basic interest in their own right.

Several research groups have recently made remarkable observations using luminescence spectroscopy on different structures in the fractional quantum Hall regime. The group including Goldberg, Heiman and Pinczuk reported luminescence anomalies in multiple quantum wells[7] in 1988 and in single asymmetric wells[8] in 1990. That group also has tried to study the perturbation by the photohole systematically by varying the thickness of the confining quantum well. The Oxford University group of Turberfield, John Ryan and Robert Clark (now continuing the work independently at the University of New South Wales) used similar structures but with a very wide quantum well immersed in a weakly doped background.[10] The Russo-German group of Igor V. Kukushkin, Vladislav B. Timofeev, Hartmut Buhmann, Nicholas Pulsford, Klaus von Klitzing and coworkers has focused on designs where the δ-doped sheet of acceptor states provides more control over where the recombination occurs.[11,12] In all the experiments, optical excitation levels must typically remain below 10^{-4} W/cm^2 to prevent heating of either the electronic or the lattice baths. Each group has observed luminescence anomalies in the form of amplitude variations or spectral shifts, in fascinating synchrony with the fractional states as monitored by simultaneous Hall measurements. Figure 5 shows results obtained by the Oxford and Russo-German groups for selected fractional filling factors. Despite the differing designs of the experiments, it is clear that electron–hole recombination is modulated at or very near particular filling factors of the underlying highly degenerate electron gas.

Figure 5a shows an example of variations in the luminescence amplitudes that the Oxford group observed in its wide heterostructures. The potential energy profile for this low-density, two-dimensional electron gas is such that the two closely spaced, lowest conduction subbands (E_0 and E_1) contribute to the luminescence. Hence subband resonance effects are expected to accentuate the photohole–many-electron Coulomb interaction, to an extent that depends on the initial position of E_F with respect to the subbands, in the spirit of the edge-singularity phenomena discussed above. The strong relationship between the luminescence amplitude and the longitudinal Hall resistance ρ_{xx} points toward a way to identify the



Fractional quantum Hall states as seen in luminescence and transport experiments. **a:** Luminescence intensity variations for a series of fractional states at $T = 120$ mK in a high-mobility GaAs structure where the interplay between the lowest two conduction subbands, E_0 and E_1, is important. (Adapted from ref. 10.) **b:** First moment of spectral shift dE/dH for a number of fractional states in structures that employ the "acceptor δ doping" technique for localizing holes. (Adapted from ref. 12.) In both **a** and **b** the specific filling factors are identified from the resistance data at bottom. **Figure 5**

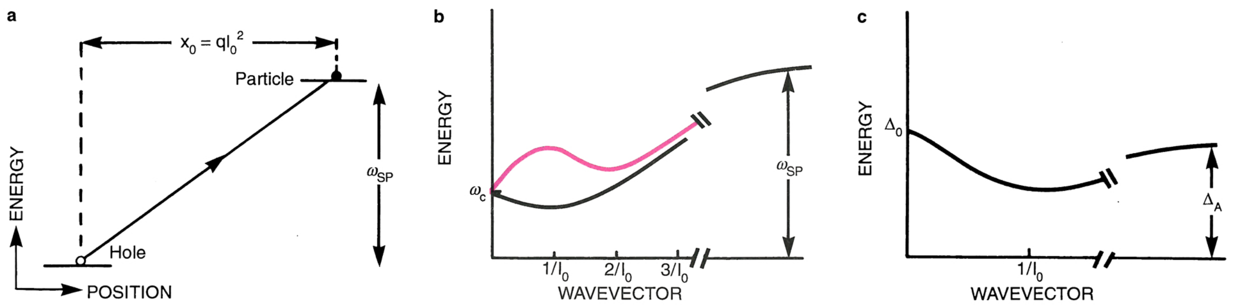
fractional quantum Hall states optically.

A remarkable demonstration of the information that detailed spectral analysis of the luminescence can provide is shown in figure 5b, which illustrates the results of the Russo-German group. Based on theoretical suggestions by Emmanuel I. Rashba and coworkers in Moscow, the measured spectral shifts and amplitude changes are now expressed in terms of the magnetic field derivative d/dH of the first moment, $\int I(E) E dE / \int I(E) dE$. Comparing the field dependence of the derivative dE/dH with ρ_{xx} reveals fascinating structure for many fractional states. By investigating samples with different initial electron densities, Kukushkin, Pulsford and their colleagues have also made optical determinations of several quasiparticle gaps, including the principal fractional states, $\nu = 1/3$ and $2/3$.

While the optical detection of the integer quantum Hall states by luminescence is on relatively firm phenomenological ground, the case of the fractional states is much less well understood, despite the battery of fascinating experimental observations. The proper way to allow for the hole state (or hole states) remains elusive to both theory and experiment. Allan McDonald and coworkers have shown that theory does not anticipate any anomalies in the luminescence spectra of a system of coplanar electrons and holes. For an asymmetric case, treated also by Joseph Birman and collaborators, anomalies are expected in the fractional quantum Hall regime if there are spatially separated (isolated) electrons and holes. Rashba and Vadim Apalkov's calculations for the case of an acceptor sheet as a recombination site predict cusps in the luminescence spectra, with an explicit connection to the experiments by the Russo-German group. Hawrylak has enlarged on the subject for other low-dimensional geometries. An important and still largely unresolved question that needs experimental clarification is the internal structure of the photohole state in the fractional regime. Finite spectral splittings in luminescence observed by many groups in the high-field regime may be connected to such hole structure. Determining the nature of such structure will be crucial if we are to exploit the spectral information fully.

Inelastic light scattering

The determination of the gap excitations (transitions across the energy gap) in the two-dimensional electron gas is a major goal of spectroscopy in the quantum Hall regime.



Neutral density excitations in the quantum Hall regimes. **a:** Particle–hole pairs that take part in an excitation having a wavevector q. **b:** Wavevector dispersion computed for inter-Landau-level excitations at $\nu = 2$. Red curve is a magnetoplasmon, or charge-density excitation; black curve is a spin-density excitation. **c:** Wavevector dispersion computed for charge-density intra-Landau-level excitations in the fractional quantum Hall regime at $\nu = 1/3$. Δ_A is the energy of a widely separated quasiparticle–quasihole pair. This excitation determines the activated behavior observed in magnetotransport measurements of the fractional quantum Hall effect. The minima in **b** and **c** correspond to rotons or magnetorotons. **Figure 6**

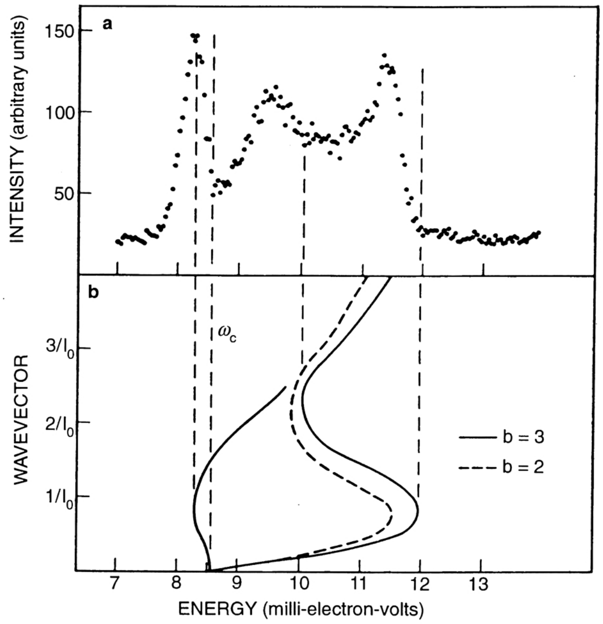
Inelastic light scattering has given much insight into the elementary excitations of free-electron systems in zero magnetic field, so it is natural to apply the method to systems in a magnetic field. In the best case, an inelastic scattering experiment directly yields the excitation energies as spectral shifts relative to the frequency of a monochromatic excitation source.

To underline the advantages of using light scattering to study the two-dimensional electron system, consider the contrast between this method and infrared absorption. Infrared absorption gives information about the energy level structure of the conduction electrons. (See the article by Detlef Heitmann and Jörg Kotthaus on page 56.) In cyclotron resonance absorption, for example, an electron is promoted to the next higher Landau level. (See figure 2.) The excitation energy is $h\omega = heB/m^*c$, where ω is the cyclotron frequency, B is the applied magnetic field, and m^* is the effective electron mass. In the language of gap excitations, the cyclotron energy can be thought of as the long-wavelength excitation energy gap associated with the quantum Hall effect at even-integer values of the Landau-level filling factor ν. (Recall that in the even-integer case the two possible orientations of electron spin in the highest populated Landau level are fully occupied, and consequently the lowest-energy excitation promotes an electron to the next Landau level.) Although infrared absorption gives precise values of the cyclotron energy (a one-electron property), it cannot measure the excitation energy gaps associated with the quantum Hall effect at the important odd integer values of the filling factor. In this case only one orientation of spin is fully occupied in the highest populated Landau level, and the gap excitations are associated with spin-flip processes. In the electric dipole approximation, which is valid for infrared absorption, such spin-flip processes are forbidden, a restriction not applicable to inelastic light scattering. Spin-flip transitions contain important physical information because the dominant contribution to the excitation gap is due to the exchange part of the electron–electron interactions, the part associated with antisymmetrization of the multielectron wavefunction. The methods based on infrared optical absorption are not expected to be useful for determining the gap excitations in the fractional quantum Hall regime either, because the optical oscillator strengths of its long-wavelength gap excitations vanish.

As in all inelastic scattering processes, conservation of energy and momentum dictate the kinematics of light scattering. The high-mobility two-dimensional electron systems of interest here have nearly perfect translational invariance in the plane, so conservation of momentum can be reduced to conservation of the in-plane components of the wavevector. For typical experiments, the in-plane component of the light scattering wavevector (the difference between the wavevector of the incoming and outgoing photon wavevectors) is $k \leq 10^5$ cm$^{-1} \leq k_L$, where k_L is the wavevector of the incident photons. In a high perpendicular magnetic field the reciprocal of the magnetic length of the electron states, $1/l_0 = (eB/\hbar c)^{1/2}$, sets a scale for wavevectors of the elementary excitations. The length l_0 is less than 100 Å at typical fields $B \geq 5$ tesla. Light scattering with wavevector conservation measures excitations with wavevector $q = k$; these are the long-wavelength excitations, because $1/k \gg l_0$. A major feature of resonantly enhanced inelastic light scattering is the occurrence of intense processes when wavevector conservation breaks down. In high-quality two-dimensional electron systems such processes can be induced by the breaking of strict translational symmetry by residual disorder. With breakdown of wavevector conservation, light scattering gives access to excitations with wavevector $q \gg k$, that is, with a relatively broad range of wavevectors $q \geq 1/l_0$. In such measurements the spectra show structure associated with the critical points in the densities of states of the elementary excitations.

The energy gaps in the quantum Hall regimes are associated with neutral excitations of the two-dimensional electron gas in a large perpendicular magnetic field.[1,13] The excitations are constructed from particle–hole pairs like that shown in figure 6a. The particles may be either electrons or quasiparticles. At integer values of ν, the particle is excited to an empty Landau level, leaving behind a hole. In the Landau gauge, which is appropriate when the excitations are described by a wavevector q, the separation of the pair is $x_0 = ql_0^2$. In the integral quantum Hall regime the particle is an electron of charge $-e$, and neutrality requires that the hole have a charge $+e$. In the fractional quantum Hall regime the pairs consist of a quasiparticle and a quasihole with equal and opposite fractional charges. At integer filling factors the excitation energy $\omega(q,B)$ can be described within the time-dependent Hartree–Fock approximation as $\omega(q,B) = \omega_{sp}(B) + E_d(q,B) + E_x(q,B)$, where $\omega_{sp}(B)$ is the single-particle transition energy. The other two terms represent the couplings between the electron and the hole arising from strong electron–electron interactions. $E_d(q,B)$ is the contribution of the direct terms, and $E_x(q,B)$ is due to exchange effects. The single-par-

Inelastic light scattering spectra of inter-Landau-level excitations at filling factor $\nu = 2$ (**a**) and the calculated mode dispersions (**b**) in a 250-Å GaAs–AlGaAs quantum well at B = 5.16 T and $T = 1.6$ K. Here b parametrizes the finite width of the electron wavefunction in the quantum well. (Adapted from ref. 14.) **Figure 7**



ticle energies are given by $\omega_{sp}(B) = \omega_0(B) + \Sigma(B)$, where $\omega_0(B)$ is the transition energy in the absence of interactions, and $\Sigma(B)$ is a self-energy due to electron–electron interactions. While $\omega_0(B)$ and $\Sigma(B)$ are q independent, $E_d(q,B)$ and $E_x(q,B)$ have marked q dependences and vanish as $q \to \infty$. This happens because the couplings between the electron and the hole become negligible at large separations x_0. Physically, $E_d(q,B)$ is related to the longitudinal electric field associated with the fluctuation of charge density in the collective modes. $E_x(q,B)$ is negative and can be regarded as an attractive force analogous to that between an electron–hole pair (an exciton) in an ordinary semiconductor.

Figure 6b presents the theoretical prediction for inter-Landau-level excitations at $\nu = 2$. These modes represent gap excitations of the integral quantum Hall effect at this filling factor. The red curve corresponds to charge-density excitations, or magnetoplasmons. The black curve correponds to spin-density excitations, caused by the fluctuations in the spin degrees of freedom. Such separation of the excitations into modes of fluctuating charge density or spin density has been carried out at integer values of the filling factor, where the Hartree–Fock approximation is applicable. In the limit of $q \to 0$, the energies of the magnetoplasmons and the spin-density excitations become equal: $\omega_{MP}(0,B) = \omega_{SD}(0,B) = \omega_c$. That is, one recovers the bare cyclotron energy. This result, known as Kohn's theorem after Walter Kohn, is a consequence of translational symmetry in the plane of the two-dimensional system. In the framework of the Hartree–Fock approximation, Kohn's theorem manifests as the cancellation at $q = 0$ of the positive self-energy $\Sigma(B)$ by the excitonic term $E_x(0,B)$.

Rotons

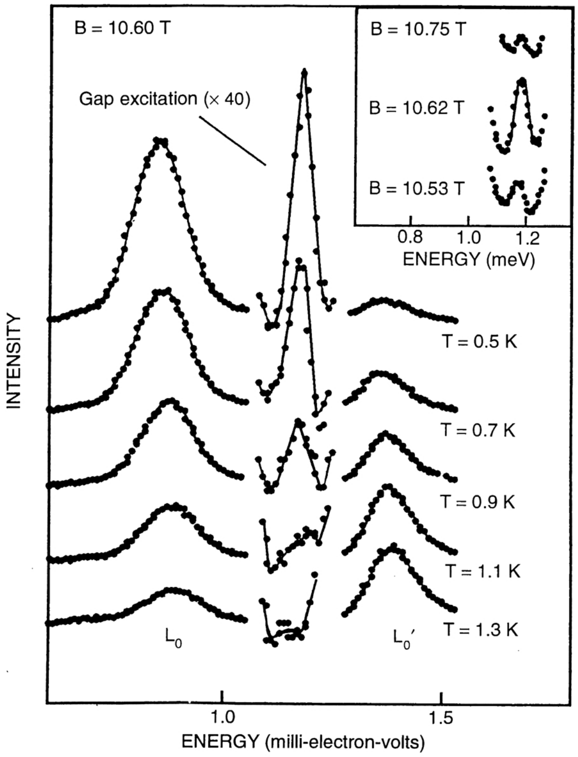
The other significant feature of the calculations displayed in figure 6b is the presence of minima associated with a roton or a magnetoroton at wavevectors $q > 1/l_0$. Rotons are a type of quasiparticle behavior that also occur in superfluid liquid helium. The existence of roton minima in the two-dimensional electron gas is one of the major predictions of current theories of collective excitations in the quantum Hall regime. The roton of inter-Landau-level excitations is due to the reduction of the excitonic binding energy $E_x(q,B)$ at large wavevectors $q > 1/l_0$. As we will discuss below, light scattering experiments gave the first direct demonstration of the roton density of states. Excitonic binding also plays a major role in the theories of gap excitations of the fractional quantum Hall effect. The magnetoroton minimum in the dispersion of intra-Landau-level excitations is due to the attraction between the fractionally charged quasiparticles. Figure 6c shows an example of a magnetoroton at $\nu = 1/3$. The energy Δ_A is that of an infinitely separated quasiparticle–quasihole pair; it is believed that Δ_A would be measured as the activation energy in a magnetotransport experiment.

Figure 7a shows how inter-Landau-level excitations reveal themselves in light scattering experiments,[14] here at filling factor $\nu = 2$. The spectrum consists of three relatively broad but clearly defined intensity maxima. Figure 7b displays the mode dispersions as calculated for the parameters corresponding to the data in figure 7a. The parameter b of the calculation is used to take into account the finite width of the electron wavefunction along the direction perpendicular to the layer. The three structures resolved in the spectra have energies close to the positions of the calculated critical points of the mode dispersions. At those points, there are peaks in the density of states, because $d\omega/dq = 0$. Since the critical points occur at wavevectors $q \geq 1/l_0 = 10^6\,\text{cm}^{-1} \gg k$, the spectra imply a massive breakdown of wavevector conservation. This breakdown suggests that translational symmetry is lost at $\nu = 2$. This is not surprising, because the insulating quantum Hall state at $\nu = 2$ is not effective in screening the residual disorder in the underlying semiconductor.

The results in figure 7 exemplify the usefulness of the light scattering method for determining the gap excitations in the quantum Hall regime. These observations give the first direct evidence of the existence of magnetorotons, which is a major prediction of theories of electron–electron interactions in two-dimensional systems. The two peaks seen at $\omega > \omega_c$ correspond to the multiple critical points associated with the roton minimum in the mode dispersion of the magnetoplasmons. Recent light scattering results have also shed light on the electron–electron interactions at $\nu = 1$.

Light scattering in the fractional regime

Figure 6c shows the predicted gap excitation of the fractional quantum Hall effect.[1] This mode is constructed from particle–hole states in the lowest Landau level, which is partially populated in the fractional regime. The direct measurement of these excitations is one of the Holy Grails of the field. To assess the impact of optical experiments, it would be helpful to have a conceptual understanding of gap excitations at $q \leq 1/l_0$. The character of $q = 0$ gap excitations is still a subject of conjecture. Steven Girvin and McDonald at Indiana University, in collaboration with Philip Platzman at AT&T Bell Laboratories, argued that gap excitations with wavevectors near the magnetoroton minimum near $1/l_0$ might pair to give a two-roton bound state with $q = 0$. Dung-Hai Lee and Shoucheng Zhang of



Collective excitations in the incompressible quantum fluid revealed by inelastic light scattering in a GaAs–AlGaAs quantum well. The sharp peak is caused by scattering off a $q = 0$ intra-Landau-level charge-density excitation (a long-wavelength gap excitation) of the fractional quantum Hall effect at $\nu = 1/3$. The broad peaks labeled L_0 and L_0' are due to luminescence of the two-dimensional electron gas. Inset shows dependence of the peak on tuning to the magnetic field that corresponds to $\nu = 1/3$. (Adapted from ref. 2.) **Figure 8**

IBM have speculated that the $q = 0$ gap excitation consists of two dipoles of size l_0 arranged in a configuration that has a quadrupole moment and no net electric dipole moment. The $q = 0$ gap excitations, being excitations within a Landau level, have vanishing oscillator strength, so optical absorption experiments are not expected to be effective for observing them.

Because it is a two-photon process, inelastic light scattering can be used to measure excitations that lack an electric dipole and to observe directly the collective excitations of the fractional quantum Hall effect. Figure 8 shows the results of a light scattering measurement[2] of a low-energy excitation of the fractional quantum Hall state at $\nu = 1/3$. One sees a very sharp peak (its full width at half-maximum is only 0.04 meV) between two luminescence bands L_0 and L_0'. This has been interpreted as a $q = 0$ collective gap excitation in the incompressible fluid. The mode is associated with the fractional quantum Hall effect at $\nu = 1/3$ because of its temperature and magnetic field dependences, which are characteristic of the incompressible states:[15] The peak is observed only at temperatures $T \leq 1$ K and in the narrow magnetic field range $\Delta B \approx 0.5$ T centered at the field corresponding to $\nu = 1/3$. (See the inset of figure 8.) The measured energy is close to the calculated values of Δ_0. The light scattering from such gap excitations is relatively weak, and one needs very high-quality samples to observe it. For example, at $T = 0.5$ K the peak's integrated intensity (the area under the peak) is about 200 times smaller than that of the luminescence band L_0. The results in the figure lead the way to further study of the fractional quantum Hall regime by inelastic light scattering methods.

Outlook

Interband spectroscopic probes are providing revelations about the behavior of highly mobile two-dimensional electron systems in strongly quantizing magnetic fields. Luminescence probes are very versatile experimental techniques, but the correct interpretation of their results requires a careful accounting of the invasive nature of the injected holes. Nonetheless Pulsford, Clark and their coworkers have recently argued that the elusive Wigner-crystal regime may be observable with luminescence probes. Inelastic light scattering presents challenges in the laboratory but in principle allows a direct measurement of the gap excitations that are fundamental in the fractional quantum Hall regime. Ongoing work using both approaches is also focusing on electron systems in one and zero dimensions—quantum wires and dots. In the future one can imagine ultrafast real-time and space-resolved experiments in which the kinetics of the many-electron excitations may be directly optically tracked within the confines of a semiconductor nanostructure.

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