PROBING DENSE NUCLEAR MATTER IN THE LABORATORY

Colliding heavy ions offer a glimpse of nuclear matter at densities and temperatures that previously were exclusive to the extraterrestrial domain.

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and Gary D. Westfall

A 150-meter-long outdoor beam line wanders through eucalyptus trees down the 45-meter slope connecting the SuperHILAC and the Bevatron at the Lawrence Berkeley Laboratory. The SuperHILAC is a linear accelerator designed to accelerate heavy nuclei at high intensities to energies of up to 8 MeV/nucleon for the purpose of studying heavy and superheavy elements. The Bevatron is a weak focusing synchrotron born in 1954 with the mission of discovering the antiproton and exploring the riddle of the hadrons using beams of 6.2-GeV protons. In 1974 these two machines were coupled to create the Bevalac, and with it a new field of research: relativistic heavy-ion reactions.

On Saturday, 21 February, of this year, the Bevatron beam was turned off for the last time by Edward Lofgren, who was in charge of the machine from 1954 till his retirement in 1979. The SuperHILAC had already been turned off, on 23 December 1992. So ended four decades of discoveries that earned four Nobel Prizes and gave us the antiproton and a profusion of hadronic resonances that have now become the basic vocabulary of particle physics. The Bevalac also gave us tantalizing glimpses of matter under conditions normally found only in stars.

Although the Bevalac was not the first machine to produce beams of relativistic heavy ions (the Princeton–Penn Accelerator was, in 1972), its beams of 2.1-GeV/nucleon oxygen and carbon, upgraded in 1981–82 to beams as heavy as uranium at 1 GeV/nucleon, supported three new endeavors. One was the study of the fragmentation of nuclei traveling at relativistic speeds in peripheral collisions, which is relevant to applications in astrophysics.[1] (See the article by Richard Boyd and Isao Tanihata in PHYSICS TODAY, June 1992, page 44.) Second, a successful program in treating tumors with heavy ion beams took advantage of the long range and sharp energy deposition characteristics of high-energy heavy nuclei. The third involved the production of very dense and highly excited nuclear matter during violent head-on nuclear collisions.[2,3] This last endeavor forms the subject of this article.

Central collisions hold the promise of studying new states of matter. The properties of many macroscopic substances can change radically with changing conditions of temperature and pressure. For example, as water vapor at room temperature is compressed, it reaches a pressure at which liquid begins to appear. Finally the gas is entirely liquefied, and the material becomes nearly incompressible. Such changes of properties under differ-

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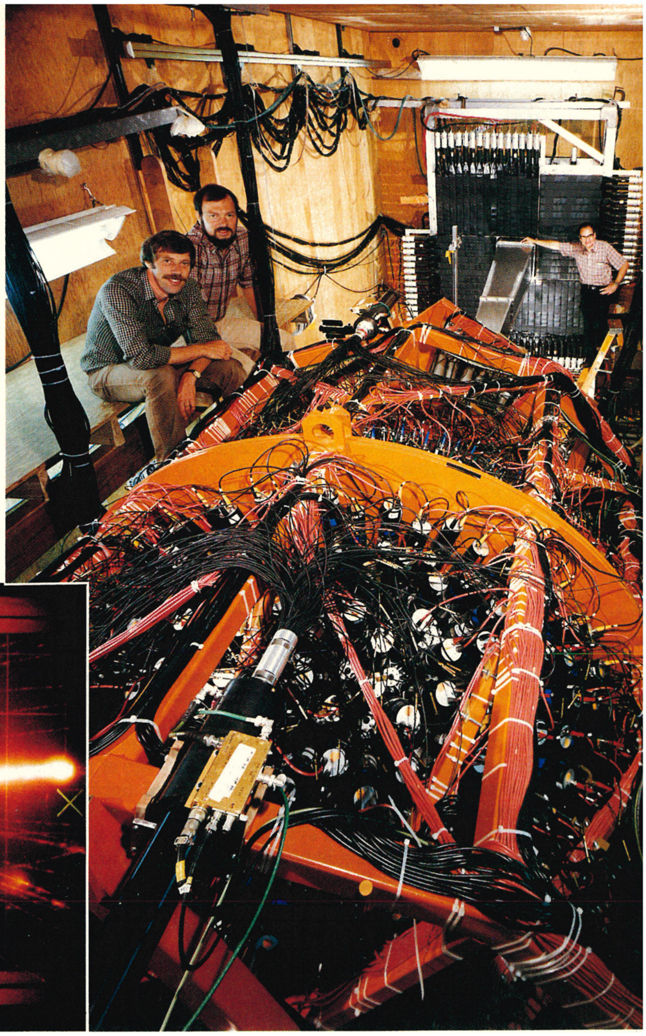
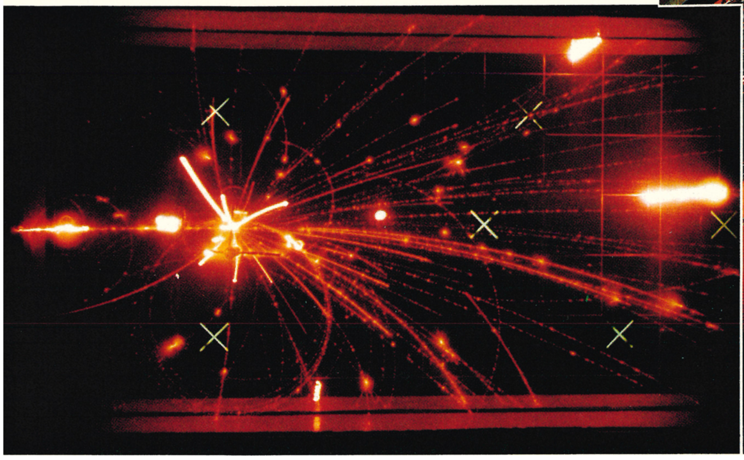
Detectors. Right: The Plastic Ball detector, a major workhorse in Bevalac experiments. From left to right are Hans Georg Ritter, Hans Gutbrod and Arthur Poskanzer. Below: Tracks from a violent collision as seen in the Bevalac Streamer Chamber, consisting of doped neon gas inside a solenoid magnet. Positive charges curve down, and negative charges curve up. **Figure 1**





ent temperatures and pressures are routinely studied for many materials. But the corresponding experiments are not easy to perform with a rather basic material: nuclear matter, the stuff that makes up nuclei and neutron stars.

In the early 1970s Tsung Dao Lee and Gian-Carlo Wick discussed the possibility that a new phase of nuclear matter might exist at high density, and that this new phase of matter might lie lower in energy than the more common type of matter in a nucleus. The Bevalac seemed to be the ideal instrument with which to make and discover this new matter: If it existed and was more stable than ordinary matter, it would accrete ordinary matter and grow. Eventually it would become so massive that it would fall to the floor of the experimental hall and be easily observed. But what would stop it from eating the Earth? Knowledge of dense nuclear matter was so poor at that time that the possibility of this disaster was taken seriously. Meetings were held behind closed doors to decide whether or not the proposed experiments should be aborted.

Experiments were eventually performed, and fortunately no such disaster has yet occurred. The behavior of nuclear matter in heavy-ion collisions turns out to be very different from this early picture. When two large nuclei hit each other, matter is compressed; because of two-body collisions the temperature rises. Theoretically expected maximum densities from such collisions are about 3 to 4 times that inside an atomic nucleus (0.15 nucleons/fm^3, or 2×10^{14} grams/cm^3), and maximum effective temperatures are in the range of 50–100 MeV (1 MeV$\approx10^{10}$ K). The matter, far from accreting to apocalyptic dimensions, simply cannot stay in this form for very long. It expands and disintegrates. A typical time scale for the whole process is 60 fm/$c = 2\times10^{-22}$ sec. The compressed and heated nuclear matter therefore will go through a decompression phase and turn into more normal forms by the time it hits the detectors. Thus the signals that contain information about the dense phase must be chosen with care, and the experiments appropriately designed to test theoretical predictions.

The first studies of hot, dense nuclear matter used spectra of light nuclear fragments from relatively simple devices that detected only one out of the hundred or more particles that each collision produces. The loss of information from integrating over the undetected particles made it difficult to select between competing physical models of the collisions. Thus a progression of two-particle measurements and multiplicity-associated single-particle measurements[4] were made under the guidance of Arthur Poskanzer, Hans Gutbrod, Reinhard Stock and Shoji Nagamiya. These measurements led to the two major detection systems used at the Bevalac, the Plastic Ball and the

Streamer Chamber. (See figure 1.)

Like the Bevatron itself, these detection systems borrowed heavily from high-energy physics. The Plastic Ball design was based on the Crystal Ball at SLAC. It consisted of about 1000 light-particle detectors arranged in a sphere, with a more highly segmented forward mall (for better angular resolution in the forward direction) augmented by a large, highly segmented time-of-flight wall, also in the forward direction. Its construction was spearheaded by Gutbrod, Poskanzer and Hans Georg Ritter. The Streamer Chamber was taken over lock, stock and barrel from the UCLA–LBL collaboration that constructed it. It consisted of a box filled with doped neon gas and lying inside a large solenoid magnet. When this detector was triggered on violent collisions in which very little of the original projectile nucleus survived intact, brilliant red tracks recorded on film by three high-speed flight cameras gave evidence of the multitude of particles emanating from the collisions. Stock and John Harris bore most of the responsibility for the Streamer Chamber project. These detection systems provided the necessary tools for discriminating between different physical models and led the way to understanding the nuclear equation of state.

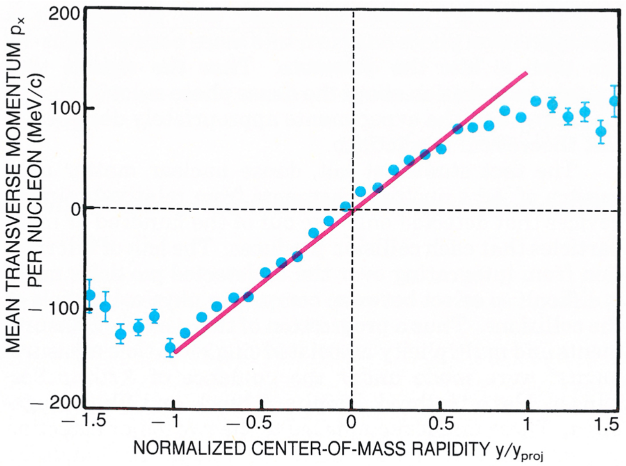
Collective flow

Heavy-ion collision physics may be classified according to the beam energy. This article deals mainly with the physics learned from central and semicentral collisions at the Bevalac at beam energies between 100 and 1000 MeV/nucleon. In this energy region the number of pions produced is small, the internal structure of the nucleon is unimportant, and one expects to reach a highest-compression phase of 2 to 3 times the normal nuclear density. The key issue in this energy regime is the behavior of nuclear matter at densities greater than normal and at excitation energies high compared with the binding energy.

In a peripheral collision, the projectile nucleus grazes the target nucleus and proceeds on with nearly its original velocity, transferring very little of its kinetic energy to the target nucleus. In a central collision, the projectile nucleus strikes the target nucleus nearly head-on. The resulting compression pushes nuclear matter away from the interaction region, producing a flow of energetic particles. The pattern of this energy and matter flow depends on the detailed properties of nuclear matter. One would expect that the higher the incompressibility of the matter, the greater is the push that the collision will produce. This effect is the primary motive for studying the phenomena of collective flow.

Quantifying collective flow in nucleus–nucleus collisions begins with determining the impact parameter b, the perpendicular distance from the target nucleus to the initial line of motion of the incident nucleus. That allows one to characterize collisions as central (head-on), near central, near peripheral or peripheral. To accomplish this characterization experimentally, one can use a variety of observables that relate directly to the centrality of the collision, such as charged-particle multiplicity, total transverse momentum and total transverse energy. For example, for an impact parameter b high enough that the nuclei miss each other completely, the transverse momentum $\mathbf{p}_\perp$ of the product particles is 0. For smaller b, the average $\mathbf{p}_\perp$ increases, goes through a maximum and decreases again. The quantitative relationship between the impact parameter and an observable can be established with the help of a model filtered through the acceptance window of the detection system. Typically b cannot be determined to better than 1 fm, so even the strictest central-collision trigger will allow a distribution of impact parameters to pass.

The impact parameter vector (chosen to be in the x direction) and the beam velocity vector (in the z direction) define a "reaction plane" for the collision. We expect collective phenomena to occur in this plane. Pawel Danielewicz and Grazyna Odyniec[5] provided the standard



Mean transverse momentum per nucleon projected onto the reaction plane as a function of the normalized center-of-mass rapidity for 400-MeV/nucleon semicentral collisions of niobium on niobium. The slope of the magenta line measures the collective flow in the system. The S shape and the slope of such curves are most pronounced at intermediate impact parameters. (Adapted from ref. 2.) **Figure 2**

method for determining the reaction plane of an observed collision. The method derives from the observation that the net transverse momentum $\mathbf{p}_\perp$ normal to the reaction plane averages to 0 by symmetry: If collective flow exists, that is, if $\mathbf{p}_\perp \neq 0$, then the vector

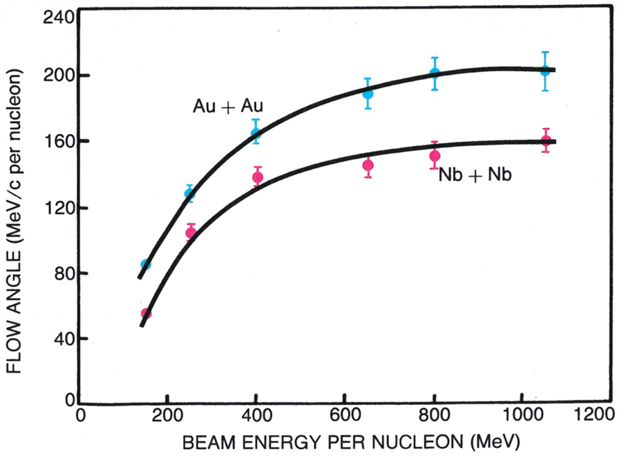
$$\mathbf{Q} = \sum_i \omega_i \mathbf{p}_\perp^i \tag{1}$$

will lie in the reaction plane. Here i designates a particle, and ω_i is a weighting factor usually defined as $+1$ for a particle going forward (along the z axis) in the center-of-mass frame and -1 for a particle going in the backward direction. Each transverse momentum vector is then projected onto this reaction plane. (The particle of interest must be excluded when determining $\mathbf{Q}$, to avoid undesirable autocorrelation effects.) The average transverse momentum per nucleon projected on the reaction plane, $\langle p^x \rangle$, is then determined as a function of a variable in the direction parallel to the beam, such as rapidity (y). Rapidity may be thought of as a relativistic generalization of velocity; it reduces to the true velocity divided by the speed of light in the nonrelativistic limit, but it is additive along the beam direction and has no upper bound. The distribution of $\langle p^x \rangle$ produces a characteristic S-shaped curve centered at mid-rapidity, the average of the projectile and target rapidities. (See figure 2.) The slope of this distribution at mid-rapidity is a measure of the "amount" of collective flow and is known as the flow angle or flow "value." Traditionally the center-of-mass rapidity has been scaled by the projectile rapidity to remove trivial scaling with the incident beam energy.[2]

In figure 2 the average transverse momentum in the reaction plane is plotted as a function of the center-of-mass rapidity for protons from 400-MeV/nucleon niobium-on-niobium collisions.[2] The slope of this distribution gives the flow value for this system. Figure 3 plots the flow value at different beam energies for several systems.[2] One sees that flow increases with beam energy and with the mass of the system.

Theory

How can we extract properties of nuclear matter from data of this sort? The answer is that we require a dynamical model to simulate the collision without any assumptions concerning thermal equilibrium. The model should have some variable input parameters (for the two-body or many-body nuclear forces) that can be adjusted until a good representation of the data is achieved. The model in use today is based on microscopic transport equations; there were useful models with simplifying assumptions that preceded this approach. One of the earliest was based on hydrodynamics: It used the nuclear equation of state, but with a small mean free path so that a collision resembled droplets splashing on each other. At the opposite extreme were models representing the collisions of nearly transparent gas clouds. The microscopic model that we will outline can describe either



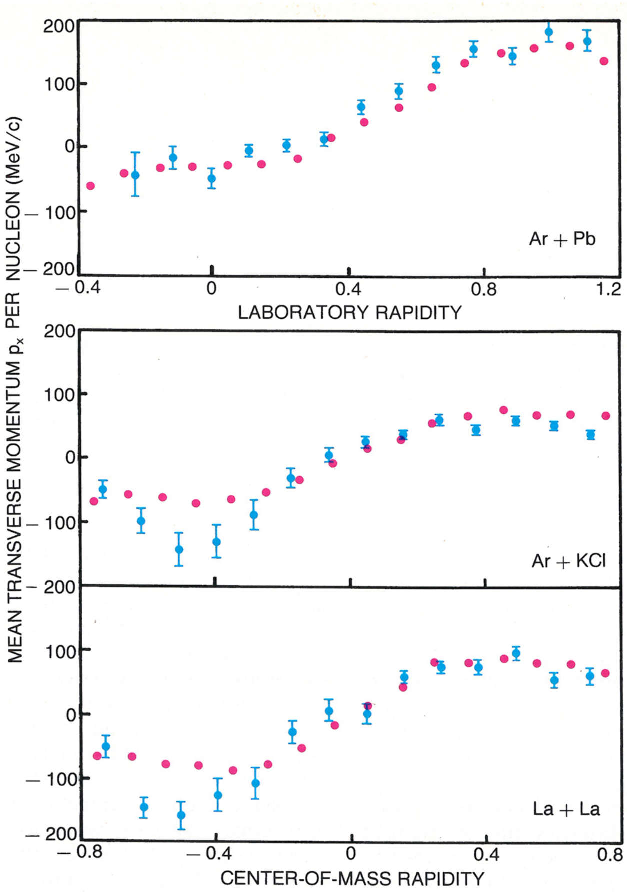
Flow angle—the slope at mid-rapidity of a curve such as the one in figure 2—as a function of beam energy for gold-on-gold and niobium-on-niobium semicentral collisions. (Adapted from ref. 2.) **Figure 3**

situation depending on the parameters used for the calculation. Quite often we know many features of these input parameters from elsewhere and insist on correct features' being reproduced at known densities and excitations while we extrapolate to unknown densities.

The microscopic model most often used is called the Boltzmann–Uehling–Uhlenbeck model (also known as the Vlasov–Uehling–Uhlenbeck or the Landau–Vlasov model).[6] The predecessor of the BUU model was the cascade model,[7] used by Joseph Cugnon and others to explain many features of the Bevalac data. The cascade model simulates each nucleus as a collection of point nucleons, with no Fermi motion or mean field. The two nuclei are shot toward each other. Numerically, in successive small time steps, we record the positions and momenta of the nucleons. As the nuclei pass through each other, two nucleons that approach within a distance of $\sqrt{\sigma_{\rm NN}^{\rm t}/\pi}$ are allowed to scatter. Here $\sigma_{\rm NN}^{\rm t}$ is an energy-dependent total cross section that is known from experiments on two-body scattering. The angle of scattering is similarly chosen from experimentally known differential cross sections, by Monte Carlo sampling. In general, $\sigma_{\rm NN}^{\rm t} = \sigma_{\rm NN}^{\rm e} + \sigma_{\rm NN}^{\rm in}$, where the superscripts "e" and "in" refer to elastic and inelastic channels. Pions are produced in the inelastic channel; at Bevalac energies the number of pions is observed to be small.

A big boost to microscopic models came when George Bertsch and coworkers devised a numerical recipe to extend the cascade model to include the nuclear mean field U as well. U is the average one-body potential in which the nucleons move. Many previous nuclear physics experiments have taught us some properties of U at saturation (or normal) density. Heavy-ion collisions sample in addition properties of the mean field away from the saturation density. The idea is then to use as input in the BUU equation different mean fields that give the same empirical saturation density and binding energy for nuclear matter but different behavior at higher-than-normal density; the mean field that has the best success in fitting the experimental data can be deemed the preferred one.

The BUU equation is a modifed Boltzmann equation for fermionic particles that includes a potential interaction and takes account of collisions. It equates the rate of change of phase space density f at the point $(\mathbf{r},\mathbf{p})$ in six-



Results of BUU simulations (magenta) incorporating momentum-dependent interactions as compared with experimental data (blue). Transverse momentum per nucleon is plotted as a function of rapidity in 800-MeV/nucleon central and semicentral collisions of different nuclei. The fits yield an incompressibility K of approximately 215 MeV. (Adapted from ref. 9.) **Figure 4**

dimensional phase space at time t to the loss or gain of f from Liouville flow and from collisions:

$$\frac{\partial f}{\partial t} + \left(\frac{\mathbf{p}}{m} + \nabla_{\mathbf{p}} U\right)\cdot\nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}} U\cdot\nabla_{\mathbf{p}} f = G - L \qquad (2)$$

where the gain G to f from collisions is

$$G = \int \frac{d\mathbf{p}_2\, d\mathbf{p}_3\, d\mathbf{p}_4}{(2\pi)^9}\, \omega(\mathbf{p}\mathbf{p}_2 \leftarrow \mathbf{p}_3\mathbf{p}_4)\, f_3 f_4 (1-f)(1-f_2)$$
$$\times \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)\, \delta(e(p) + e(p_2) - e(p_3) - e(p_4))$$

The delta functions conserve energy and momentum, and $f_i = f(\mathbf{r},\mathbf{p}_i,t)$, with $i = 2$, 3 or 4, are the phase space densities at different momenta. Particles of momenta $\mathbf{p}_3$ and $\mathbf{p}_4$ scatter to the momentum $\mathbf{p}$ of interest. The term L is similar to G except that the transition probability ω refers to the opposite scattering, that is, from the pair $(\mathbf{p},\mathbf{p}_2)$ to the pair $(\mathbf{p}_3,\mathbf{p}_4)$, leading to a loss in f:

$$L = \int \frac{d\mathbf{p}_2\, d\mathbf{p}_3\, d\mathbf{p}_4}{(2\pi)^9}\, \omega(\mathbf{p}\mathbf{p}_2 \rightarrow \mathbf{p}_3\mathbf{p}_4)\, f f_2 (1-f_3)(1-f_4)$$
$$\times \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)\, \delta(e(p) + e(p_2) - e(p_3) - e(p_4))$$

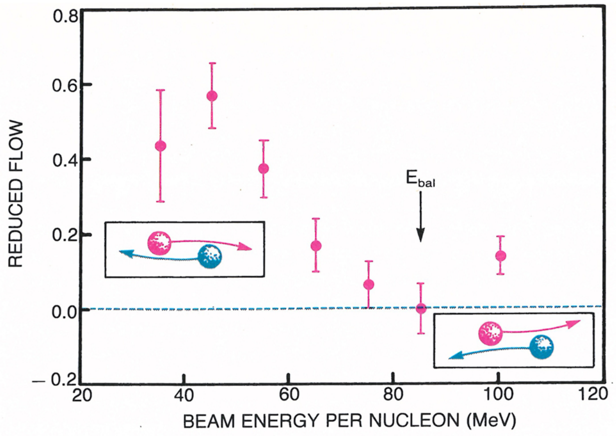
The $1-f$ factors prevent the fermionic particles from scattering into phase space elements that are already occupied. This is called Pauli blocking. The scattering contributions are integrated over all possible momenta for the net effect on f.

Setting the right-hand side, that is, collisions, equal to zero gives the Vlasov equation, a very useful approximation in plasma physics. In nuclear physics the semiclassical Vlasov description leads to a bulk dynamics very similar to that obtained from fully quantal time-dependent Hartree–Fock theory, extensively studied in the 1970s. Formally, the BUU equation has been known for a long time; the numerical Monte Carlo technique for solving it for large but finite numbers (approximately 300) of particles is new. The BUU equation makes no assumption of equilibrium, and thus there is no temperature in equation 2. *A posteriori* one can check if and when the concept of temperature became valid by examining the velocity distribution that the solution generates. It has been shown[8] that a large part of the transverse momentum is generated quite early in the history of the collision, far from equilibrium; thus theories that assume short mean free paths and instant local thermal equilibrium will be inadequate.

Choosing the one-body potential U for equation 2 that best fits the experimental data also fixes the potential energy density V; together with the kinetic energy, V determines the behavior of nuclear matter at any density. This is the kind of information we are looking for.

Stiffness

From known properties of nuclei one deduces that the saturation density of nuclear matter, ρ_0, is about 0.15/fm^3 and the binding energy W is about 16 MeV/nucleon. If we



Energy balance between attractive and repulsive parts of the nuclear interaction is evinced by the disappearance of flow. The reduced flow, defined as the average transverse momentum in the reaction plane divided by the total transverse momentum, is shown as a function of beam energy for central collisions of argon with vanadium. The measurements are of $Z = 2$ particles from the collision. At lower energy the constituent particles of the target and the projectile are attracted toward each other (inset to the left); at higher energy they are repelled (inset to the right). (Adapted from ref. 21.) **Figure 5**

Taylor-expand W about ρ_0 there is no linear term; the first term is quadratic. The coefficient of the quadratic term is therefore critical to our knowledge of nuclear matter: It describes the curvature at the energy minimum, or the "stiffness" of nuclear matter to compression. We parametrize the stiffness of cold nuclear matter by the incompressibility

$$K = 9\left.\frac{\mathrm{d}P}{\mathrm{d}\rho}\right|_{\rho_0} = 9\rho_0^2\left.\frac{\partial^2 W}{\partial^2\rho}\right|_{\rho_0} \tag{3}$$

evaluated at saturation density, where P is the pressure. K is a property of the mean field U and can be related to the "flow" in a collision: We might expect that the higher the value of K, the bigger the transverse momenta that will be detected.

The simplest choice for U is one where U is a function of ρ, the density, alone: $U(\mathbf{r}) = A\rho(\mathbf{r}) + B\rho(\mathbf{r})^\sigma$. In this simplified Skyrme interaction, the constants A, B and σ are adjusted to give the correct saturation density and binding energy and a preset value of K. Horst Stöcker and collaborators were the first to do extensive BUU calculations with such forces for comparison with experimental data on collective flow. They found that a high value of K, approximately 380 MeV, is required for a good fit.[9] Others reported similar results.

In addition to the density dependence, it is known empirically that the one-body potential felt by a nucleon inside a nucleus depends also upon the nucleon's momentum with respect to the surrounding medium. This momentum dependence can arise from many sources: the exchange term of a two-body force, higher-order many-body corrections, explicit momentum dependence in the two-body force, and so on. In experiments and in theoretical calculations the momentum dependence has been seen to be a strong effect in nuclear structure physics. Gerald Brown, Vijay Pandharipande and Philip Siemens have pointed out at conferences and in private discussions that this aspect of the optical potential must play an important role in collisions involving energetic ions. Momentum dependence was first put into BUU calculations by Charles Gale and coworkers; subsequent work at Stony Brook[8] has used an improved parametrization,

$$\mathrm{U}(\rho,\mathbf{p}) = A\rho(\mathbf{r}) + B\rho(\mathbf{r})^\sigma + C\int \mathrm{d}\mathbf{p}'\,\frac{f(\mathbf{r},\mathbf{p}')}{1+\left(\dfrac{\mathbf{p}-\mathbf{p}'}{\Lambda}\right)^2} \tag{4}$$

where the constants are chosen to reproduce the binding energy, the saturation density, the real part of the empirical optical potential and a preset value of K. It is then found that a lower value of K, approximately 215 MeV, is consistent with the flow properties measured in heavy-ion collisions. Such a fit is shown in figure 4. The parametrization of equation 4 is simple, yet the result is remarkably close to the $U(\rho,\mathbf{p})$ generated by a state-of-the-art many-body calculation that uses a multiparameter interaction tuned to fit scattering and few-body data and saturation properties of nuclear matter.[8]

We might ask how the incompressibility value of $K \approx 215$ MeV obtained from heavy-ion collisions compares with estimates obtained from other areas of nuclear physics. Some excitations in finite nuclei arise predominantly from small oscillations in density about the equilibrium value. Near the end of the 1970s and in the early 1980s there was intense activity by many groups aimed at deducing the value of K from positions of such "giant monopole" resonances. Using detailed Hartree–Fock plus random phase approximation (RPA) analysis they all came to the conclusion that the data are best fitted by $K \approx 210$ MeV. Recent work using a different model yielded preference for a higher value of K, but it has been shown[10] that a unique value for K cannot be extracted from that procedure: Any value from 120 to 350 MeV could have been deduced. Models of supernova explosion prefer a value of $K \approx 200$ MeV or lower.[11] Neutron star masses do not provide rigorous limits on K.

At one time it was thought that the total number of pions produced in heavy-ion collisions would depend strongly upon the incompressibility. Calculations do not bear this out: The number of pions depends critically on the inelastic scattering cross section $\sigma_{\rm NN}^{\rm in}$ used, but very little on K. Calculations by Gale[12] illustrate this quite clearly. The renormalization of this free-space cross section by the nuclear medium has recently been discussed.[13]

Currently a great deal of activity is being devoted to producing and studying microscopic models for heavy-ion collisions. The models differ from one another in detail and often in the physics they address. The efforts include work by Pandharipande and coworkers,[14] J. Aichelin and a group at the University of Frankfurt,[15] Wolfgang Bauer and colleagues,[16] David Boal[17] and a group at McGill University.[18] It is comforting that the main results of these models are consistent.

The BUU equation can also be used to study the creation of entropy (disorder) in heavy-ion collisions. The usefulness of the concept of entropy in this system was pointed out by Siemens and Joseph Kapusta.[19] Subsequent analysis of the emission of hydrogen and helium isotopes in these collisions showed that the amount of entropy produced because of heating is in the range of 3 to 4 (dimensionless) units per nucleon.[20] This range is consistent with that predicted by the BUU calculations. In comparison, supernova explosions produce only about 1 unit.

Recently attempts have been made to add to the body of information about the nuclear medium by using new observables. One such observable is the balance energy,[21] $E_{\rm bal}$. The central collisions described above are at high energies, where there is a strongly repulsive interaction and the scattering angle is positive. However, at incident energies around 10 MeV/nucleon, the nucleus–nucleus potential is primarily attractive, resulting in negative-angle scattering. One can define and measure flow at these relatively low energies too. As the incident energy is raised, the interaction must switch from being dominantly attractive to being dominantly repulsive. $E_{\rm bal}$ is the energy at which these two interactions will balance each other. Figure 5 shows the reduced flow, defined as the average transverse momentum in the reaction plane divided by the total transverse momentum, as a function of the incident energy for collisions of argon and vanadium nuclei. Flow disappears at 85 ± 10 MeV/nucleon, indicating that balance has been achieved. BUU predictions for $E_{\rm bal}$ for this system turn out to be very sensitive to the assumed nucleon–nucleon interactions in the nuclear medium but only mildly sensitive to the parameters of the equation of state. Thus nucleus–nucleus interactions at these lower energies hold promise for yielding additional information on the microscopic properties of nucleons in nuclear matter.

Prospects

Working at the Bevalac, nuclear scientists have been able to subject significant amounts of nuclear matter to high density and excitation energy for the first time. The experience has brought us new technology, led to progress in our ability to solve for the complicated time evolution of a large but finite number of strongly interacting particles, and broadened our understanding of the underlying nuclear interaction. Investigations at Bevalac energies have stimulated further studies at both lower and at higher energies. At lower energies, signatures of a possible nuclear liquid–gas phase transition are being investigated at the National Superconducting Cyclotron Laboratory in East Lansing, Michigan, and at the Grand Accélérateur National d'Ions Lourds (GANIL) in Caen, France. At higher energies, signatures of the transition from hadronic matter to quark–gluon plasma are being investigated at the Alternating Gradient Synchrotron (AGS) at Brookhaven and the Super Proton Synchrotron (SPS) at CERN; future investigations will take place at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven. With the closure of the Bevalac in February, work on dense nuclear matter at energies of 100–1000 MeV/nucleon will be carried forward at the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt, Germany.

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We wish to thank Joseph Kapusta for providing the impetus for writing this article.

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