TRANSFORMING CLASSICAL LINEAR SYSTEMS TO QUANTUM ONES

Introduction to Algebraic and Constructive Quantum Field Theory

J. C. Baez, I. E. Segal and Z. Zhou
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Reviewed by Mark A. Kon

Mathematical quantum field theory, in its many current forms, has become an established and diverse area of both physics and mathematics. Work in the area began in earnest in the 1950s, after the realization that elaborate and apparently ad hoc mathematical constructs were required for devising a quantum field theory of phenomenological value. As with Richard Feynman's path integral approach to quantum mechanics, there are in quantum field theory strong and almost overwhelming suggestions of an underlying mathematically cohesive and relatively simple description of the phenomenology. Serious difficulties with divergences arise, however, when such apparently canonical theories are implemented in the infinite-dimensional home of quantum field theory. Dealing with such problems in a mathematically rigorous and understandable way was the motivation of the mathematical approach.

The primary workers in this field have included James Glimm, Arthur Jaffe, Oscar Lanford, Edward Nelson, Barry Simon and Arthur Wightman. Irving Segal, a major contributor since the area's inception, was one of the first to introduce a probabilistic approach, which has become widely used in the development of the field. In *Introduction to Algebraic and Constructive Field Theory*, Segal, John Baez and Zhengfang Zhou approach the construction of quantum fields through a very general algebraically oriented approach, which is then specialized to fields in two spacetime dimensions.

This book emphasizes a methodology, algebraic quantization, for quantizing arbitrary linear systems—that is, transforming classical linear systems to quantum ones. It gives a description of Fock space in its particle representation, and describes the extension of single-particle operators to many-particle operators on Fock space. Weyl systems, the exponentiated versions of the canonical commutation relations, are introduced rigorously; functional integral representations of Fock space are presented as well. In addition to the boson structures studied initially, the book also introduces the analogs of such systems for fermions. Quantizations of general symplectic and orthogonal systems are also introduced in this context. The issue of whether canonical transformations are unitarily implementable is discussed throughout the book, and a remedy to this problem for infinite-dimensional transformations is considered in a study of the C*-algebraic approach to quantum field theory.

Special cases of the algebraic approach are given for quantizations of linear differential equations such as the Schrödinger, Klein–Gordon, and Dirac equations, and then the process of constructing a nonlinear field theory begins. Renormalized products of quantum field operators $\phi(x,t)$ are defined and then used to rigorously construct a continuum quantum field in two space-time dimensions.

A benefit of the abstract algebraic approach is that it describes in a general context results that sometimes seem quite specialized. The book elegantly presents the quantization process for the two-dimensional spacetime theory, using a single-particle Hilbert space with a distinguished family of physical operators. This differs from other approaches, which specify physical wave equations and their related Hilbert spaces earlier on and use analytic techniques to quantize the equations. The book clarifies the process of quantizing hyperbolic equations. In the end, there is a renormalized quantum field theory in two spacetime dimensions, within a general mathematical context.

The book's intended audience consists of mathematicians and mathematically inclined physicists. The level of the book presumes a knowledge of functional analysis, though there is a very useful and novel glossary that explicates a large portion of the background required and explains terms in the book.

The book is quite well put together, with few typographical errors, and it includes an index. In any text with mathematical notation, a list of symbols is a great convenience, and this book has one. To me, the book provides an important overview and a useful synthesis of some significant contributions to the mathematical understanding of quantum field theory.

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Scanning Tunneling Microscopy I

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Scanning tunneling microscopy is a rapidly growing technique that is having an impact on many disciplines—physics, chemistry, materials science and biology. This book, the first of a two-volume series, provides a useful compilation of reviews of key STM research.

This book is not a "how-to" hand-