

STRUCTURAL MECHANICS AND ITS DEBT TO ARCHITECTURE

An Introduction to the History of Structural Mechanics

Eduardo Benvenuto

Part I: Statics and Resistance of Solids

Springer-Verlag, New York,
1991. 306 pp. \$49.00 hc
ISBN 0-387-96227-1

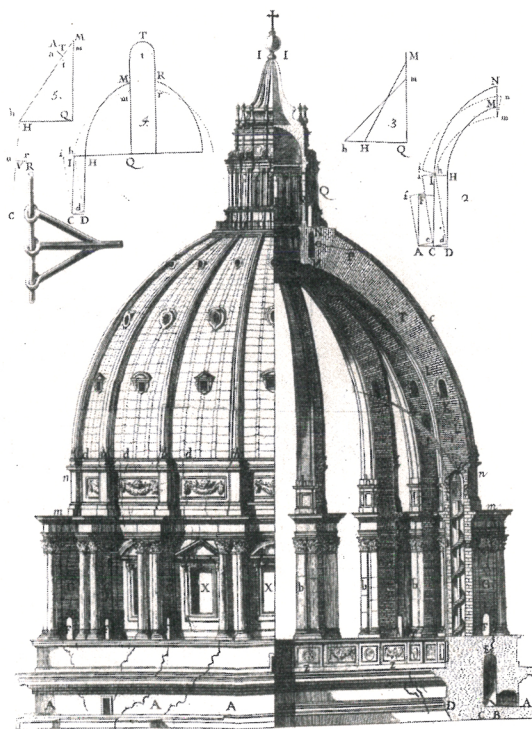
Part II: Vaulted Structures and Elastic Systems

Springer-Verlag, New York,
1991. 554 pp. \$49.00 hc
ISBN 0-387-97187-4

Reviewed by James Casey

It is easy to sketch an arch whose shape is pleasing to the eye. But will it stand? And where is it weakest? Such questions, which must have troubled the great Classical and Renaissance architects and engineers, can only be answered after a correct statical analysis is made. Yet no such analysis was available when Filippo Brunelleschi completed his magnificent dome for the Cathedral of Santa Maria del Fiore in Florence in 1434, nor when Michelangelo constructed the dome of St. Peter's Basilica in Rome more than 100 years later. The early builders had to rely on their intuition and perhaps a few rules of thumb garnered from experience. In his two volumes on the history of structural mechanics, Eduardo Benvenuto argues that it was from the rich and enduring experience of architectural construction that the theory of structural mechanics gradually emerged. It is as if the great buildings were the crucial experiments upon which the theory was originally founded. Benvenuto writes: "The

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The dome of St. Peter's Basilica in Rome had begun to show signs of damage as early as 1631. Yet it was not until 1743, when Pope Benedict XIV accepted the proposal for restoration submitted by Giovanni Poleni, that restoration was begun in earnest. This diagram of the dome appeared in a Venetian book published in 1744. (From *An Introduction to the History of Structural Mechanics*, Part II.)

Dome of St. Peter's Basilica sprang heavenward without the benefit of theory. It not only preceded mathematical analysis but begot it."

The pair of volumes is organized into four main sections, in which are treated the principles of statics, the theory of the strength of materials, the statics of arched structures and the theory of elastic systems. Benvenuto, a professor of structural engineering at the University of Genoa, draws from primary sources, using well-chosen drawings and equations as necessary, and presents a comprehensive portrayal of the growth of statics over the last two millennia. During most of this vast period statics was regarded as a serious intellectual pursuit, and new ideas, some right but many wrong, were suggested at each

stage of its development. It is sobering to think that it has taken us so long to wrest the governing principles of statics from Nature. Furthermore, because of the current concentration of interest in 17th-century dynamics, one is apt to underestimate the complexity of the history of statics, important parts of which developed even after the birth of Newtonian dynamics. From a historiographic viewpoint, the evolutionary growth of statics presents quite a different model from the revolutionary emergence of dynamics and certainly deserves scholarly attention of the type to be found in Benvenuto's work.

As they did with geometry, the ancient Greek mathematicians defined the way in which statics was to be pursued by subsequent genera-

tions. Credit is due to Archimedes (287–212 BC) for putting an essential part of the subject on an axiomatic foundation. Benvenuto carefully analyzes Archimedes's proof of the law of the lever and discusses variations and improvements upon Archimedes's work that appeared during the late Middle Ages, continued during the Renaissance and lasted into the 18th century. He also traces the development of the principle of virtual velocities and the parallelogram rule. For the history of statics up to the 18th century, we are fortunate in also now having available in English Pierre Duhem's masterpiece, *The Origins of Statics* (Kluwer, Boston, 1991).

Having assembled the basic ingredients of statics in the first section of his work, Benvenuto then takes up a question central to structural mechanics: Why is the resistance of a beam much greater for axial loading than for transverse loading? Galileo was the first author to raise this question, which is treated at length in the earlier part of his last and best work, *Two New Sciences* (1638). Galileo tries to calculate the strength of a beam, but falls into error. Benvenuto presents a thorough account of Galileo's ideas on beams. He then proceeds to discuss some physical theories of the resistance of solids, drawing on the writings of Marin Mersenne, René Descartes, atomist Donato Rossetti, Newton and Giuseppe Ruggiero Boscovich. He emphasizes the significance of molecular attraction as a model for elastic action and gives an account of Claude Louis Marie Henri Navier's important paper of 1821 in which the field equations for an elastic material were derived for the first time. (In later work, by Augustin Louis Cauchy, the molecular hypothesis was duly abandoned.)

In the last two chapters of Part 1, Benvenuto returns to the beam problem and describes ideas that were eventually to lead to its solution. These include the observation that fibers on the convex side of a bent beam are elongated, whereas fibers on the concave side are shortened. The experimental studies of Isaac Beeckman, Robert Hooke and Edmé Mariotte in the 17th century are described, and later experiments by Pieter van Musschenbroek (1729) and Pierre Simon Girard (1798) are also discussed. During the 18th century the mathematical study of the strength of materials blossomed into a golden age of mechanics. The contributions of Leonhard Euler and the Bernoulli family were phenomenal. Numerous difficult, but special prob-

lems were solved, and from this activity powerful methods and general theories emerged. Appreciation for 18th-century developments in mechanics has been growing in recent years, due largely to the incisive historical analyses of Clifford Truesdell. [See his memoir "The Rational Mechanics of Flexible or Elastic Bodies 1638–1788" in vol. 11:2 of Leonhard Euler's *Opera Omnia* (Orell Füssli, Turici, Italy, 1960) and his *Essays in the History of Mechanics* (Springer-Verlag, Berlin, 1968).] We are now further indebted to Truesdell for encouraging Benvenuto to bring out the present work in English and for writing its foreword.

At the beginning of Part 2, we find ourselves once again in Renaissance Italy, listening to Leonardo da Vinci ponder the mystery of the arch:

An arch is nothing but a strength caused by two weaknesses; that is why an arch in buildings is composed of two quarter-circles; these quarter-circles, each weak in itself, wish to fall, and opposing each other's ruin, convert weakness into a single strength.

Actually, Leonardo had a good grasp of how an arch transmits its load (see *The Notebooks of Leonardo da Vinci*, J. P. Richter, ed., vol. 2, Dover, New York, 1970). He even suggested a rule: "The arch will not break if the chord of the outer arch does not touch the inner arch." As Benvenuto explains, other rules of the same geometrical flavor also existed. Although of limited scientific value, they are still worthy of study for the light they shed on contemporary static thinking. It will surprise readers to learn that one must wait until the very end of the 17th century before one finds a decent analysis of the arch. And perhaps one would have had to wait even longer, had cracks not begun to appear in major buildings: Signs of structural distress were noticed in the dome of St. Peter's in 1631, and by 1693 the dome of Santa Maria del Fiore had accumulated considerable damage.

Toward the end of the 17th century, theoretical studies of arches began to emerge. In 1697 David Gregory discovered the affinity between an over-turned catenary and an arch made of infinitely small, smooth spheres. In 1712 Philippe de la Hire presented an analysis of the arch and proposed a failure mechanism. This analysis was improved upon by Claude Antoine Couplet in the late 1720s. The first static analysis of domes appeared in 1734 under the authorship of

Pierre Bouguer. By the mid-1740s, concern for the worsening state of St. Peter's dome reached critical proportions, and the dome became the focus of intense theoretical attention. Benvenuto gives a splendid account of the various arguments and recommendations that were put forward. With evident pleasure and admiration, he describes Giovanni Poleni's monumental treatise of 1748, which contains impressive theoretical and experimental studies on the problem. With Poleni's work, structural engineering had finally come of age.

Thus galvanized by a major practical problem, the theory of arches and domes underwent extraordinary development in the late 18th century. Benvenuto describes an important analysis by Charles Bossut in which a differential equation of equilibrium was deduced and used to find "the best figure of vaults." However, the crowning work on the subject came from Charles Antoine Coulomb. In his celebrated essay of 1773 (see J. Heyman, *Coulomb's Memoir on Statics*, Cambridge U.P., Cambridge, UK, 1972), Coulomb employed a combination of sound intuition, correct mechanics and good mathematics to set the subject on a secure footing and provided solutions to a number of important problems. Approaching the end of the third section of Benvenuto's work, we find accounts of later 18th-century studies of vaults and a summary of 19th-century developments.

The final section begins with the late-18th-century debate on static indeterminacy: If we set three rigid balls in a line on a horizontal plane, the reactions of the plane are completely determined, but if we join the balls together by rigid rods, why should the reactions not be uniquely determined? The philosophical and technical reactions provoked by this basic indeterminacy shed much light on prevailing views on the nature of physical theory. A satisfactory resolution of the problem was not found until Navier proved in 1825 that if the elasticity of a body is taken into account, the support reactions can be found.

There were many reasons, both practical and theoretical, to explore the phenomenon of elasticity, and as a result of the marvelous work by Cauchy in the 1820s, followed immediately by that of George Green, a deep understanding of the mechanics of deformable media emerged: The partial-differential equations of motion were established for an arbitrary continuum; a generalized notion of Hooke's law of elasticity was intro-

duced and the concept of elastic strain energy was created. Benvenuto does not dwell on the birth of continuum mechanics, but moves quickly to a discussion of the development of energy methods for elastic structures. During the latter half of the 19th century much exciting work in this field was done by Italian engineers and mathematicians, most notably Luigi Federico Menabrea and Alberto Castigliano. Castigliano's lasting contribution was his theorem that for a linearly elastic structure, by taking the partial derivative of the strain energy with respect to a load, one can obtain the component of displacement along the load at its point of application. Benvenuto also discusses other contributions to structural mechanics, including those of Alfred Clebsch, James Clerk Maxwell and Otto Mohr.

In the closing paragraphs of this fine history, Benvenuto expresses a note of disappointment. Modern engineers, he writes, know only the formulas of their profession: The circumstances of their derivations have been forgotten. In reply, I would suggest that the principles of mechanics are themselves monuments that may outlast the domes of the Renaissance. Like other monuments, these too have a fascinating history, and as long as there are dedicated scholars like Benvenuto, that history will be accurately recounted.

The Maxwellians

Bruce J. Hunt

*Cornell U. P., Ithaca, N. Y.,
1991. 266 pp. \$34.95 hc
ISBN 0-9814-2614-3*

The theory of electromagnetic phenomena presented in James Clerk Maxwell's culminating work on the subject, *A Treatise on Electricity and Magnetism* (1873), differs significantly from the theory that appears in modern textbooks on classical electromagnetic theory. In *The Maxwellians* Bruce Hunt presents a fascinating account of a central episode in the recasting and further development of Maxwell's theory, focusing on the work of his British followers—especially George Francis FitzGerald, Oliver Lodge and Oliver Heaviside—in the last quarter of the 19th century. FitzGerald, a graduate and later a professor of natural and experimental philosophy at Trinity College, Dublin, was the major architect of the broad intellectual vision of this group of three. Lodge, a graduate of University College, London, who became a professor of physics at University College, Liverpool, was the chief ex-

perimenter, interlocutor and propagandist. Heaviside, a self-educated telegrapher who was for the most part isolated from the academic community, was the mathematical brains of the outfit and the one who made the important technological connections.

The central theme in Hunt's story is the shift from Maxwell's own emphasis on the vector and scalar potentials A and ψ as the central field variables of the theory—with the basic equations phrased in terms of them—to the familiar modern form of the theory, in which the electric and magnetic field vectors are the basic variables, the fundamental equations are the four symmetrical "Maxwell's equations" and the potentials are demoted to an auxiliary role. Heaviside is eponymously honored in this connection in that the four equations are sometimes referred to as the Heaviside-Hertz form of Maxwell's equations. (Heinrich Hertz's work on the reformulation of the equations was in part independent and in part influenced by Heaviside.)

FitzGerald, however, also played a central role in recasting the equations: Among the British interpreters of Maxwell he gave the most thought to the element of arbitrariness in the potentials and the related problem of potentials that are propagated instantaneously—as is ψ in the Coulomb gauge. These problems motivated what FitzGerald referred to as "the murder of ψ " and the attendant rephrasing of the equations. Also associated with this rephrasing was the work of Heaviside and John Henry Poynting on energy localization and transfer in the electromagnetic field, as expressed in terms of the electric and magnetic field vectors.

Branching off from the main theme of the book is a variety of interesting episodes and developments. A detailed account of the origins of the FitzGerald contraction hypothesis serves to show that this was something more—something deeper—than a mere *ad hoc* response to the Michelson-Morley experiment. In connection with the issue of the propagation of potentials and fields, as investigated by FitzGerald and others, Heaviside developed in 1888 a formula for the field around a rapidly moving electric charge, exhibiting contraction along the direction of motion by $\sqrt{1-v^2/c^2}$. Knowing this and assuming that intermolecular forces behaved in the same way, FitzGerald early in 1889, during a conversation with Lodge concerning the 1887 Michelson-Morley experiment, first formulated the contraction hypothesis. Turning to the more

immediately practical connections of electromagnetic theory, Hunt shows how concerns with telegraphy and telephony motivated many of Heaviside's theoretical advances and how in turn Heaviside made important contributions to the technology of transmission lines, such as the practice of inductive loading to reduce distortion of the signal, "now recognized," according to Hunt, "as the most important technical innovation in telephone transmission between [Alexander Graham] Bell's original invention in 1876 and the development of the first electronic amplifiers in 1912."

Throughout, the book is a good read—clear, cogent and interesting, with a good balance between the coverage of personalities and their interactions and that of technical issues. Extensive use of archival materials—correspondence, notebooks and working papers—enriches the narrative so that it is concrete, lively and convincing. One might have wished for a bit more engagement with the existing historical literature on the subject for the purpose of making stronger connections with the broader history of electromagnetic theory. This single caveat notwithstanding, *The Maxwellians* makes an important contribution to our understanding of the history of electromagnetic theory, and I highly recommend it to both physicists and historians.

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Quantum Signatures of Chaos

Fritz Haake

*Springer-Verlag, New York,
1991. 242 pp. \$59.00 hc
ISBN 0-387-53144-0*

The Transition to Chaos in Conservative Classical Systems: Quantum Manifestations

Linda Reichl

*Springer-Verlag, New York,
1992. 551 pp. \$45.00 hc
ISBN 0-387-97753-8*

Most physicists have grown up with the belief that elementary mechanics is represented by the well-worn standard examples of regular, or integrable, systems, such as a few linearly coupled pendula or a lone planet circling the Sun. This naive faith in the ultimate simplicity of nature extends even to the atomic and subatomic realm, where the Schrödinger