dire economic straits. He was already very disappointed that the Japanese, starting from zero, had eradicated the US's early dominance of the semiconductor industry.

Interestingly, in all his arguments to stimulate a search for high-T_c superconductors, Bardeen never once mentioned a theoretical, abstract or philosophical motivation. He only stressed the myriad practical applications and the economic viability of the US. I found this very surprising for a physicist with such an abstract and theoretical reputation.

With Bardeen's aid, I finally succeeded in getting support for an experimental search for high-T_c superconductors at 3M Corp in 1985. Although we did discover new oxide superconductors, they were not high T_c. Later, when Georg Bednorz and Alex Müller discovered the cuprate superconductors, Bardeen extolled the commercial potential of high T_c and encouraged me to form my own company to exploit the opportunities. It is not clear that that was good advice.

A priori none of us foresaw the possible connection between oxide superconductors and the interfacial excitonic mechanism proposed by Ginzburg and his colleagues and by Allender, Bray and Bardeen. The layered characteristics of the high-T_c superconductors suggested such a mechanism, and we continued to discuss this and many other options until Bardeen's death. Both he and Ginzburg felt that an excitonic mechanism was as likely an explanation of high T_c as any other, although certainly not conclusive. Bardeen, always philosophical, said the truth would eventually be known and he was not going to argue with anybody, since he had already published his thoughts on high T_c. If this interfacial excitonic theory eventually proves successful (in the oxides or otherwise), it should earn Bardeen a third Nobel Prize in Physics. (At the "Woodstock of physics," it was widely promulgated that high T_c was totally unanticipated. Apparently most physicists do not read the literature or do not take it very seriously.)

Bardeen was totally devoted to physics, a subject his wife described as his mistress and obsession. I was always amazed at his enormous reading capacity and immediate comprehension. He was always up to date. He was also an extremely attentive listener, although he usually appeared to be asleep, a characteristic many people found very disturbing! Many misinterpreted his somnolent appearance as reflecting a lack of interest, but he was just deep in thought. In our last conversation he asked, as always, about business, and I said, "We are just trying to survive!" "Me too" was his characteristically laconic reply. I will remember him with great admiration as a physicist and human being of highest integrity and exemplary behavior.

Reference

1. D. Allender, J. Bray, J. Bardeen, Phys. Rev. B **7**, 1020 (1973).

CHARLES GALLO
Superconix
2/91 *Saint Paul, Minnesota*

Ferment over Beer Bubbles

Scientists have always been fascinated by macroscopic phenomena that provide insight into the underlying molecular mechanisms. The rising of a bubble in a glass of beer, as described by Neil E. Shafer and Richard N. Zare (October 1991, page 48) is a case in point. Shafer and Zare explain why a bubble grows as it rises, using phenomenological theory. At the same time, the rise of a bubble reveals underlying molecular motion.

Starting from an equation that describes the increase in the number N of CO_2 molecules in the bubble with time t,

$$\frac{dN}{dt} = \gamma 4\pi r^2 \qquad (1)$$

where γ is a phenomenological coefficient and r the radius of the bubble, Shafer and Zare find that

$$r = r_0 + v_r t \qquad (2)$$

where r_0 is the radius at $t=0$ and $v_r = \gamma k_B T/P$ is the rate of increase of the bubble's radius, experimentally found to be 4×10^{-5} m/sec. In the derivation it was assumed that γ and P, the pressure in the bubble, are constants. Strictly speaking, this is not true. P depends weakly on r, since in addition to the atmospheric pressure it contains a contribution due to the curvature of the bubble and a hydrostatic term. The coefficient γ can be identified with the flux of CO_2 molecules toward the rising bubble and can be calculated from standard transport theory:[1]

$$\gamma = D\frac{\Delta n}{\delta} = 0.623\, D^{2/3}\frac{u^{1/3}}{r^{2/3}}\Delta n \qquad (3)$$

Here D is the diffusion constant of a CO_2 molecule; Δn, the difference in CO_2 concentration far from the bubble, n_∞, and at the bubble surface, n_0;

continued on page 112

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δ, the (average) thickness of the diffusion boundary layer around the bubble; and u, the velocity of the bubble.

Before the bottle was opened, the CO_2 concentration in the beer was in equilibrium with the CO_2 gas under the cap, which usually has a pressure of several atmospheres. This concentration initially changes little, since the diffusion process is relatively slow. The equilibrium between the CO_2 concentrations near and in the bubble establishes itself rapidly. Thus n_0 is the equilibrium concentration corresponding to a pressure of 1 atmosphere. Hence $\Delta n \simeq n_0(S-1)$, where S is the supersaturation. The velocity u of the bubble, assuming Stokes flow, is

$$u = \frac{2}{9}\frac{\Delta\rho\, g r^2}{\eta} \qquad (4)$$

$\Delta\rho$ being the density difference between beer and CO_2 gas, g the gravitational constant and η the viscosity of beer. Combining equations 3 and 4 shows that γ is indeed independent of r. In reality deviations from Stokes flow (see figure 4 of Shafer and Zare's article) introduce a weak dependence of γ on r.

Neglecting the weak dependence of γ and P on r, it follows from equations 3 and 4 that one can determine D, the diffusion coefficient of CO_2, from the experimental value of γ, 1×10^{21} $m^{-2}\,sec^{-1}$. Assuming $S=2$, taking[2] the value of 0.9 m^3 CO_2/m^3 for the amount of dissolved CO_2, which corresponds to $n_0 \simeq 3\times10^{25}$ m^{-3}, and using the known values of $\Delta\rho$ and η leads to $D \simeq 5\times10^{-10}$ $m^{-2}\,sec^{-1}$. From the Stokes–Einstein equation, $D = k_B T/6\pi\eta a$, one calculates a radius a for the CO_2 molecule of around 3 Å. (Note that while one does not need to know the Boltzmann constant k_B to obtain the value of D, one does need it to calculate a.) This size is the correct order of magnitude, although slightly too large, presumably as a result of errors in the estimate of Δn, not using the correct expression for u and possible complications in the way CO_2 attaches itself to bubbles coated with surfactants. Nevertheless this argument shows that it is possible to deduce molecular sizes from contemplating the rise of a bubble in a glass of beer.

We would like to point out that our colleagues were lucky in their choice of beer: American beer is bottled under higher pressure than Canadian beer, resulting in more pronounced bubble growth south of the border.

References

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2. K. P. Miszczenki, A. A. Rawdiela, *Zbior Wielkosci Fizykochemicznych* [*Tables of Physicochemical Constants*], State Scientific P., Warsaw (1974).

T. G. M. van de Ven
S. S. Dukhin
Paprican and McGill University
11/91 *Montreal, Canada*

I read with interest the recently reported experimental results and simple analysis of Neil E. Shafer and Richard N. Zare on the behavior of bubbles in beer. In their observations of the brew, however, they neglected to discuss two important phenomena well known to both beer drinkers and droplet dynamics researchers: unsteady and added-mass effects.

For an observer fixed in the reference frame of the tavern, the motion of the droplets is unsteady because, as Shafer and Zare note, the ratio of the buoyant to the drag force increases as the bubble grows. The bubble also starts from rest. Since the bubble/beer density ratio is low, the added-mass effect (due to acceleration of the surrounding fluid along with the bubble) can be large. Likewise, the apparent force from changes in the far-field velocity with time, the Basset force,[1] can be large.

I hope this resolves any inconsistencies between theory and observation that other readers may have noticed while observing the early stages of bubble motion from the bottom of a glass of beer.

Reference

1. A. B. Basset, *A Treatise on Hydrodynamics*, vol. 2, Deighton Bell, Cambridge, England (1888), republished by Dover, New York (1961).

Gregory T. Linteris
12/91 *University of California, San Diego*

The article "Through a Beer Glass Darkly" by Neil E. Shafer and Richard N. Zare finds much interesting science in receptacles not widely studied in the laboratory.

For low Reynolds numbers—too low to be of significance for the case of beer bubbles—the drag force F_d discussed in the article has a limiting form that does not accord with either the Stokes or Oseen law quoted (even after correcting for apparent misprints). Both those formulas are derived for a rigid sphere, assuming a "sticking" boundary condition. For a bubble, the result is different. The general case (a sphere of viscosity η' moving through a fluid of viscosity η) is solved in Landau and Lifshitz's *Fluid Mechanics* (section 20, problem 2). They quote W. Rybczynski (1911) as the source. For the gas bubble ($\eta' \ll \eta$), the result is $F_d = 4\pi\eta r v$.

This result belongs to the category of things "well, but not widely, known."

T. Michael Sanders
11/91 *University of Michigan, Ann Arbor*

I wish to thank you for the excellent article by Neil E. Shafer and Richard N. Zare. I was, however, distracted by what I consider to be sloppy physics in the "Buoyancy versus drag" section of the paper. I believe my physics colleagues would agree with me that equation 4 is not a correct expression of Archimedes' principle, that the weight of the bubble is missing from the right side of equation 7 (which is apparently a statement of Newton's second law) and that mass times acceleration is not a force (inertial or any other kind).

Jerry W. Moulder
Transylvania University
11/91 *Lexington, Kentucky*

Shafer and Zare reply: We thank the authors of the many letters and comments we have received on our article. We attempted to present the simplest explanation of our observations on beer bubble dynamics, but we delight in the corrections and embellishments of other fizzicists that reveal nuances of this system that we overlooked.

Jerry W. Moulder is correct in pointing out that equation 4, our expression for F_b, gives not the buoyancy force but the sum of the buoyancy force and the weight of the bubble. When equation 4 is substituted into equation 7 the weight of the bubble is indeed included.

T. Michael Sanders has found an error in the caption of our figure 4. The figure is correct, but the figure caption should read "... Stokes's law $F_d = 6\pi\eta v_z r$ (blue) or Oseen's law $F_d = 6\pi\eta v_z r\,(1 + {}^3\!/_{16}R)$ (red)...." In our article we emphasized that a rigid-sphere model for bubble ascent cannot be expected to explain the motion of very large bubbles. Sanders as well as Gregory T. Linteris makes the interesting comment that the rigid-sphere model must also fail to describe the motion of very small bubbles.

T. G. M. van de Ven and S. S. Dukhin show that the approximate size of a CO_2 molecule (treated as a sphere) can be estimated to be 3 Å from staring into a foaming glass of beer. This is consistent with the known C–O bond distance of 1.16 Å and is a remarkable demonstration of the applicability of hydrodynamics at the molecular level.

NEIL E. SHAFER
RICHARD N. ZARE
Stanford University
Stanford, California
1/92

More Educational Outreach Programs

I was delighted to learn, some time ago, that PHYSICS TODAY was planning a special issue on pre-college education to call the attention of the entire physics community to the many fine programs in this area as well as the need for participation by *all* concerned physicists. I hope that the September 1991 issue has achieved that goal.

The article "Pre-College Physics Education Programs from the Research Community" (page 48), by Brian Schwartz and James Wynne, understandably emphasized the fine programs sponsored by the authors' organizations (APS and IBM, respectively). Unavoidably, other fine programs were missed. In particular, the map in figure 3 recognized programs held by five of the AIP member societies but missed the workshops for high school teachers held by the Acoustical Society of America at its national meetings (Baltimore, Houston and Salt Lake City) and, perhaps even more importantly, the many workshops for high school teachers at every meeting of the American Association of Physics Teachers (of which I recently served as president).

For all its 60 years of existence, AAPT has promoted interaction between the research and teaching communities. Traditionally, this has affected mainly high school and college teachers, but there is now an active movement to promote interaction between research physicists and elementary school science teachers as well. At our sectional meetings (AAPT currently has 46 sections, which typically meet twice a year) as well as our national meetings, we bring together high school teachers and physicists from the university and research communities.

AAPT has a strong tradition of holding joint meetings with physics research societies, partly to allow high school as well as college teachers to learn of the latest frontiers in physics research. From 1931 to 1991, AAPT and APS met jointly during January, a meeting that holds many fond memories. It was at these January meetings that we heard memorable lectures by Fermi, Feynman, Compton, Dirac, Bethe and many others. When it became apparent that APS no longer wished to meet in January, AAPT completely revamped its meeting schedule to include a joint April meeting with APS, beginning in 1992.

Many AAPT members are active in APS and other physics research societies. By the same token, many research physicists who are not classroom teachers are active members of AAPT. *We certainly would like to encourage more physicists to join.* (Please write in "Join AAPT" in the box of "activities open to the individual researcher" at the bottom of page 50 of the September issue!) We are anxious to assist other professional societies, national laboratories and industrial companies in planning programs to benefit physics education at all levels, pre-college and college alike. We consider AAPT to be the educational arm of the entire physics community, serving grades K–20!

THOMAS D. ROSSING
Northern Illinois University
DeKalb, Illinois
10/91

We noted with dismay the absence of Argonne National Laboratory in your recent edition devoted to pre-college education. A large number of Argonne's scientists are involved in a wide variety of educational programs. The lab has the largest and longest-funded set of DOE educational programs.

Here are a few of the many educational efforts here at Argonne:

▷ The local Sigma Xi chapter has been offering awards to outstanding science and mathematics teachers in the Chicago area since 1984. It has recognized 29 teachers so far, and junior high teachers will be included in the competition for next year.

▷ A program called the Chicago Science Explorers Program has been created, with Argonne as the lead institution. The program has given over 35 000 students firsthand exposure to science and science careers in the Chicago area. It combines "New Explorers" videos, created by CBS anchorman Bill Kurtis, with detailed teacher guides, created by teachers and scientists, and field trips to local science sites. The combination offers children a chance to see that scientists are human and interesting and that science could be a viable career. The "New Explorers" series will be broadcast on PBS in January, and the teacher guides and videos will be available for purchase from an 800 number displayed during the broadcast.

▷ Argonne and Fermilab were instrumental in the creation of the Chicago Teachers Academy for Mathematics and Science on the Illinois Institute of Technology campus. This academy attempts to provide support and retraining for the approximately 17 000 science and math teachers in the Chicago schools. Physicists in both laboratories were involved in the design and operation of the academy. One of us (Berry) served for several months as the first director of the academy.

A number of other programs at Argonne offer students and teachers a chance to experience science and to improve their science skills. This is true also of the large number of DOE-supported laboratories and facilities around the country.

We congratulate you on a fine edition on education.

GORDON BERRY
JOHN GREENE
SAM BOWEN
Argonne National Laboratory
Argonne, Illinois
12/91

The summary of educational programs sponsored by professional scientific societies did not mention that of the American Astronomical Society's Division for Planetary Sciences.

The DPS holds a $1\frac{1}{2}$-day workshop, Exploring Our Solar System, for 7th- to 12th-grade science teachers in conjunction with our annual meeting. The workshops feature talks about the latest discoveries in planetary science by DPS members, discussion sessions on "ideas that work," and hands-on activities for the classroom. A unit of graduate credit for participants is arranged via the continuing education office of the local state university.

The next workshop will be held in Boulder, Colorado, in 1993.

MARTHA S. HANNER
DPS Education Officer
Pasadena, California
1/92

SCHWARTZ AND WYNNE REPLY: In our article we emphasized the increasing role of the research community in pre-college science education. Unlike AAPT, which has remained focused on physics education for over 60 years, the other AIP member societies have only recently been developing formal outreach educational programs.

Currently the membership of the APS is over 43 000, and the AAPT membership is approximately 10 000. Joint APS–AAPT membership numbers less than 2500. Thus nearly 41 000 APS members are not formally