ACOUSTICS OF DRUMS

Though such renowned physicists as Rayleigh and Raman have studied drums, our understanding of their acoustical behavior remains incomplete. Digital computers and holographic interferometry are now helping to fill in the missing pieces.

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Drums have played an important role in nearly all musical cultures. They have been used to transmit messages, convey the time of day, send soldiers into battle and warn of impending disaster. Although drums have been constructed of wood, stone and metal, the most familiar type of drum consists of a membrane of animal skin or synthetic material stretched over some type of air enclosure.

Modern drums can be divided into two groups: those that convey a strong sense of pitch and those that do not. In the former group are the kettledrums, tabla and boobams; in the latter group are the bass drum, snare drum, tenor drum, tom-toms, bongos, congas and countless other drums, mainly of African and Oriental origin. As vibrating systems, drums can be divided into three categories: those consisting of a single membrane coupled to an enclosed air cavity (such as kettledrums; see figure 1), those consisting of a single membrane open to the air on both sides (tom-toms, congas) and those consisting of two membranes coupled by an enclosed air cavity (bass drums, snare drums). One can also categorize drums by their ethnic origin—for instance, Oriental, African or Latin American—or according to the types of musical performance with which they are associated, such as symphonic, military, jazz, ethnic, dance and marching bands.[1]

The acoustical behavior of various drums has captured the interest of a number of physicists, including Lord Rayleigh, who analyzed the vibrational modes of kettledrum membranes. Early in his career, Nobel laureate C. V. Raman studied the acoustics of several musical instruments, including the traditional Indian tabla and mridanga. In the 1930s Jûicha Obata and his colleagues studied traditional Japanese drums such as the tudumi, o-daika and turi-daiko. In the United States, bass drums came under the scrutiny of Harvey Fletcher and his colleagues at Brigham Young University in the 1970s.

For an ideal circular membrane—that is, a completely flexible one vibrating in a vacuum—the normal mode frequencies are given by[2]

$$f_{mn} = \frac{ck_{mn}}{2\pi} = \frac{j_{mn}}{2\pi a}\sqrt{\frac{T}{\sigma}} \tag{1}$$

where j_{mn} is the nth root of the Bessel function $J_m(kr)$, k is the wavenumber, c is the wave speed, T is the membrane tension, a is the membrane radius, and σ is the mass per unit area.

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Pair of kettledrums being played by percussionist Garry Kvistad, who participated in some of the experiments described in the text. The pair shown here, which have copper bowls and calfskin heads, are reproductions of drums from the classical period, the late 18th century, and were designed and made by Ben Harms. **Figure 1**

The normal modes are designated by two numbers (m,n): The first indicates the number of nodal diameters, the second the number of nodal circles. The first 14 modes of an ideal membrane are shown in figure 2, along with their mode designations and relative modal frequencies. It is clear that none of the modal frequencies are integral multiples of a fundamental; the modal frequencies are therefore harmonically unrelated.

Snare drums

Snare drums are used in many types of music ensembles, including jazz bands, marching bands and symphony orchestras. The orchestral snare drum is a two-headed instrument about 35 cm in diameter and 13–20 cm deep. Strands of wire or gut stretch across the lower, or snare, head. When the upper, or batter, head is struck, the snare head vibrates against the snares. Alternatively, the snares can be moved away from the head to give a totally different sound.

In a two-headed drum there is appreciable coupling between the two heads, especially at low frequency. This coupling may take place acoustically through the enclosed air or mechanically through the shell and leads to the formation of mode pairs. In the first two modes of vibration of the drum the batter and snare heads move in the manner of the (0,1) membrane mode of an ideal membrane, as shown in figure 3. In the lower-frequency member of the pair, both heads move in the same direction, and in the higher-frequency mode they move in opposite directions.

A simple two-mass model describes these first two modes reasonably well. The batter head is represented by a mass m_b and a spring with stiffness K_b, the snare head by a mass m_s and a spring constant K_s; the enclosed air constitutes a third spring with constant K_c connecting the masses. This system is known to have two modes of vibration, whose frequencies are given by[3]

$$\omega^2 = \tfrac{1}{2}(\omega_b^2 + \omega_s^2 + \omega_{cb}^2 + \omega_{cs}^2) \pm \tfrac{1}{2}\sqrt{[(\omega_b^2 + \omega_{cb}^2) - (\omega_s^2 + \omega_{cs}^2)]^2 + 4\omega_{cb}^2\omega_{cs}^2} \qquad (2)$$

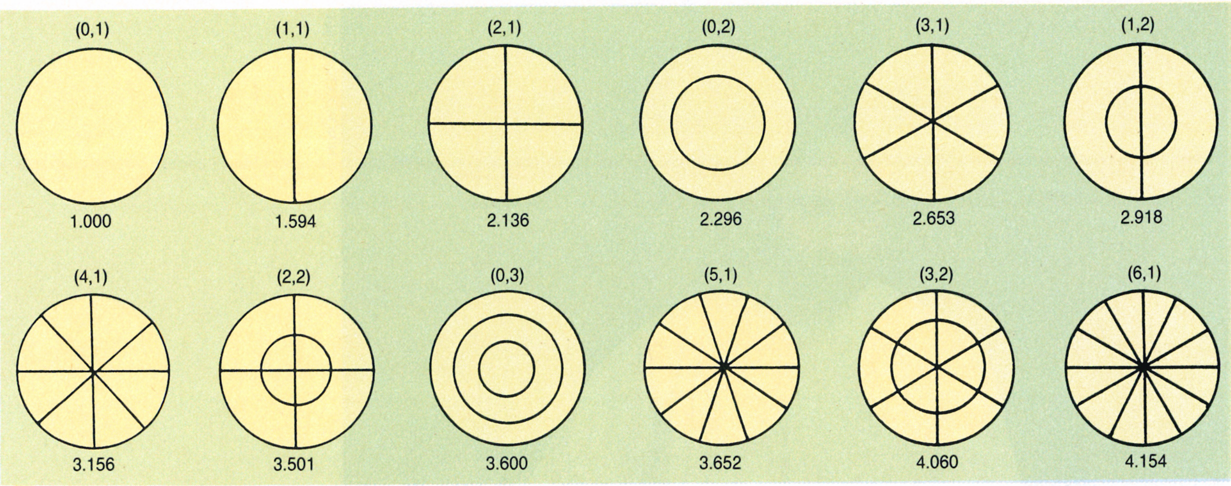
where

$$\omega_b^2 = \frac{K_b}{m_b},\ \omega_s^2 = \frac{K_s}{m_s},\ \omega_{cb}^2 = \frac{K_c}{m_b},\ \omega_{cs}^2 = \frac{K_c}{m_s}$$

If the snare head (or the batter head) is damped so that it cannot vibrate, one obtains a single frequency ω_b' for the batter head (or ω_s' for the snare head):

$$\omega_b' = \sqrt{\omega_b^2 + \omega_{cb}^2},\quad \omega_s' = \sqrt{\omega_s^2 + \omega_{cs}^2} \qquad (3)$$

The third and fourth modes in figure 3, in which the heads move in the manner of the (1,1) membrane mode, are more difficult to model. In the lower-frequency (1,1)-



Modes of an ideal membrane are not harmonically related in frequency. For the first 14 modes, shown here, the mode designation (m,n) is given above each figure and the relative frequency below. To convert these to actual frequencies multiply by $(2.405/2\pi a)\sqrt{T/\sigma}$, where a is the membrane radius, T is the tension and σ is the mass per unit area. (Adapted from ref. 1.) **Figure 2**

like mode, in which the heads move in opposite directions, air "sloshes" from side to side, and the mass of the air acts to lower the frequency. In the higher-frequency (1,1)-like mode the air moves a smaller distance. Because the mass loading is thus diminished, the frequency of this mode is raised.

Huan Zhao, Ingolf Bork, Dell Fystrom and I have studied the snare drum's vibrational modes separately in the batter head, the snare head, the cylindrical drum shell and the complete drum using four techniques: scanning the near-field sound, Chladni-pattern generation, holographic interferometry, and modal analysis with impact excitation. Each of these methods, described below, has certain advantages and disadvantages.

The first three methods involve excitation at a single frequency with a sinusoidal force. To analyze the near-field sound we generally attach a small (0.1–0.2 gram) permanent magnet made of NeFeB or SmCo to the membrane and supply a sinusoidal magnetic field with a small coil driven by an audio generator. A 5-mm-diameter electret microphone held about 3–5 mm from the membrane is convenient for scanning the near-field sound to locate the nodal lines for each mode of vibration. Chladni patterns, which are produced by a fine powder that accumulates at nodes on the vibrating membrane, are also useful for this purpose.

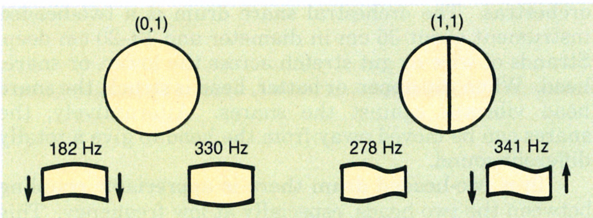
It is difficult to measure the modes of vibration of the heads independently of the rest of the drum. Therefore we studied each head individually by damping the other head and the drum shell with sandbags. The modes thus measured are those of the heads loaded by the mass of the air on each side and acted on by the restoring force of the air enclosed in the shell (corresponding to ω_b' and ω_s' in equation 3).

Time-averaged holographic interferometry[4] provides a convenient method for studying the vibrational modes of the drum shell. Because this technique uses exposure times of several seconds (during which time the shell has oscillated through many cycles), millions of holograms are, in essence, superimposed. Nodal lines appear as bright lines, and interference fringes create a sort of contour map of the vibrations. We have applied this method both to the snare drum shell alone and to the shell of a complete drum.

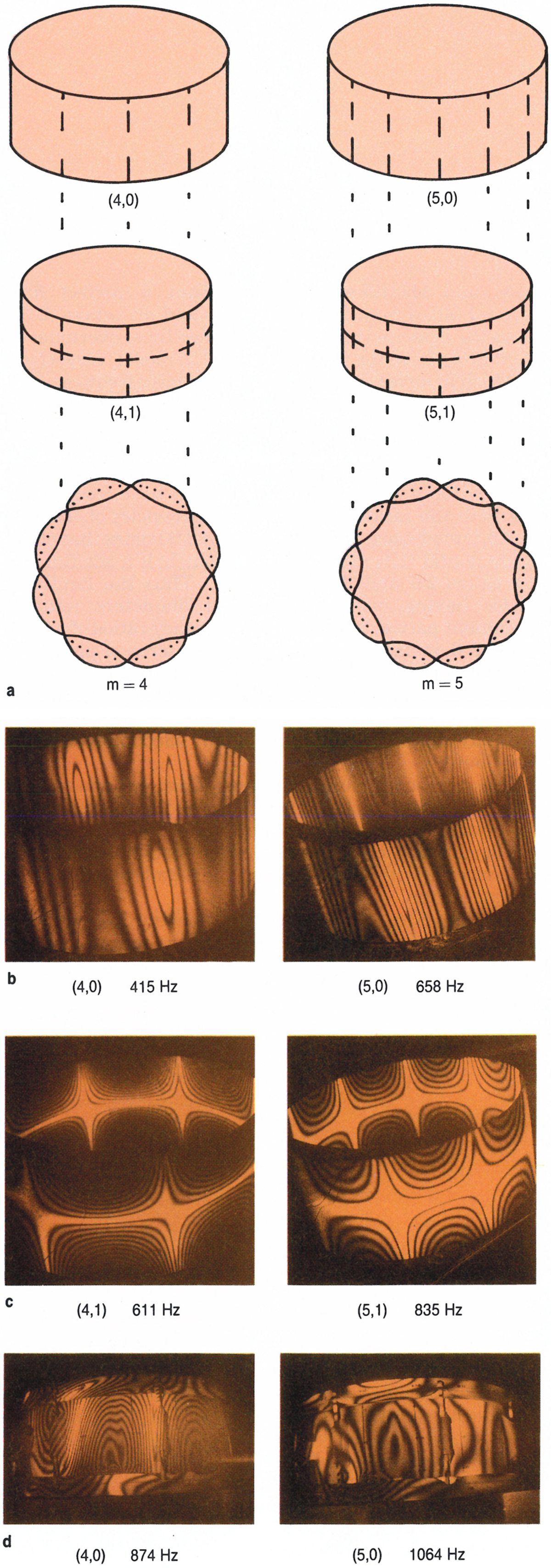
To illuminate both the inside and outside surfaces of the shell coherently, we used two object beams with slightly different path lengths; this required a second beam splitter. The entire apparatus was mounted on a vibration-isolating table made of alternating layers of concrete, foam rubber and steel.

The lowest modes of the free drum shell are the cylindrical shell modes having m nodes parallel to the axis and n circular nodes, as shown in figure 4a. Holographic interferograms of two $(m,0)$ modes and two $(m,1)$ modes are shown in figure 4b. We recorded the outside and inside surfaces simultaneously using the two object beams. Figure 4c shows two modes of a complete drum that are mainly shell modes.

Modal analysis or modal testing with impact excitation is a convenient way to observe several vibrational modes of a structure at one time. In experimental modal testing, one excites a structure at one or more points and observes the response at one or more points. From these sets of data one usually determines the natural frequencies (eigenfrequencies), mode shapes (eigenfunctions) and damping parameters using multidimensional curve-fitting routines on a digital computer.[5]



Some vibrational modes of a snare drum are mainly modes of the drum shell. Modes with four or five axial nodes are shown here. **a:** Vibrational modes of a cylindrical drum shell. **b, c:** Holographic interferograms of a snare drum shell without the heads. **d:** Interferograms of the complete drum. (Adapted from ref. 3.) **Figure 4**



For modal testing of a snare drum we attached a small accelerometer to the rim of the drum and struck the drum with a miniature force hammer—a small hammer with a piezoelectric force transducer—at 226 selected points strategically distributed over the two heads, the rims and the shell.[3] We processed the electrical signals $a(t)$ and $F(t)$ from the accelerometer and force hammer, respectively, with a fast-Fourier-transform analyzer to obtain a transfer function proportional to $a(f)/F(f)$ (where f is the frequency) for each pair of points. We determined the normal modes of vibration using the curve-fitting routines in a commercial modal-analysis software package.

Figure 5 shows modal shapes of several modes, obtained from modal analysis with impact excitation. In the lowest (0,1) mode, at 183 Hz, the two heads move in phase and radiate strongly. In the lower (1,1) mode at 275 Hz, the heads move in opposite directions, so air sloshes from one side to the other inside the drum shell. In the higher (1,1) mode at 344 Hz, the two heads move in the same phase, so air is displaced only slightly in the axial direction; the lower effective air mass results in a higher modal frequency.

Kettledrums

Kettledrums, or timpani, are usually considered to be the most important drums in modern orchestras. Most modern timpani have a pedal-operated tensioning mechanism in addition to six or eight tensioning screws around the rim of the kettle. The pedal typically allows the player to vary the tension over a range of at least 3:1, which corresponds to a tuning range in excess of a musical sixth. At one time most timpani heads were made of calfskin, but this material has gradually been replaced by Mylar (polyethylene terephthalate). Because of its homogeneity, Mylar is insensitive to humidity and is easier to tune than are heads made of natural materials. A thickness of 0.19 mm is considered standard for Mylar timpani heads. Timpani kettles are roughly hemispherical; copper is the preferred material, but fiberglass and other materials are also used.

Although the modes of vibration of an ideal membrane are not harmonic, a carefully tuned kettledrum is known to sound a strong principal note as well as two or more harmonic overtones. Lord Rayleigh recognized the principal note (at frequency f_1) as coming from the (1,1) mode and identified overtones about a perfect fifth ($f/f_1 = 1.50$), a major seventh (1.88) and an octave (2.00) above the principal tone.[6] He identified these overtones as originating from the (2,1), (3,1) and (1,2) modes, respectively, which in an ideal membrane should have frequencies of 1.34, 1.66 and 1.83 times the frequency of the (1,1) mode. Rayleigh's results are quite remarkable, considering the equipment available to him.

More recent measurements[7] have indicated that the (1,1), (2,1) and (3,1) modes in a timpani have frequencies nearly in the ratios 1:1.5:2. The frequencies of the (4,1) and (5,1) modes are typically 2.44 and 2.90 times that of the fundamental (1,1) mode, within about half a semitone of the ratios 2.5 and 3, respectively. Thus the family of modes having one to five nodal diameters radiates prominent partial tones having frequency ratios of nearly 2:3:4:5:6, giving the timpani a strong sense of pitch.

How are the inharmonic modes of an ideal circular membrane shifted in frequency so that a series of prominent harmonic partials appears in the sound of a carefully tuned kettledrum? Four effects seem to to contribute:

▷ The membrane vibrates in a sea of air, and the mass of this air sloshing back and forth lowers the frequencies of the principal modes of vibration.

▷ The air enclosed by the kettle has resonances of its own that interact with those modes of the membrane that have similar shapes.

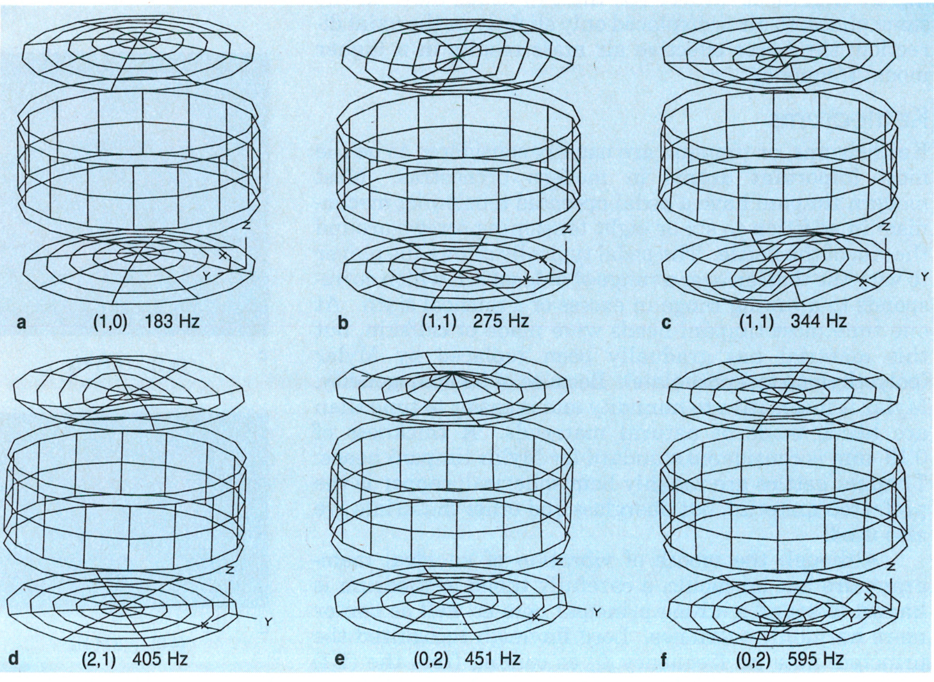
▷ The bending stiffness of the membrane, like the stiffness of piano strings, raises the frequencies of the higher overtones.

▷ Drumheads have a rather large stiffness to shear, so they resist the type of distortion needed to deflect a membrane (as if it were being wrapped around a bowling ball) without wrinkling it.[8]

Our studies have shown that air loading, which lowers the low-frequency modes, is mainly responsible for establishing the harmonic relationship of kettledrum modes. Other effects act merely to fine-tune the frequencies but may have considerable effect on the rate of decay of the sound.[9] The stiffness (or pressure) of the air enclosed in the kettle raises the frequencies of the axially symmetric modes, especially the (0,1) mode.[10]

Although modeling the vibrating membrane as a piston of appropriate area in an infinite baffle gives a reasonably good estimate of the air-mass loading,[1] a much more accurate determination of the effects of air loading can be made by applying the Green function technique.[11] The modal frequencies calculated by this method agree very well with those measured for a timpani membrane both attached to and separate from the kettle.

Our studies indicate that changing the volume of air in the kettle changes the normal-mode frequencies of the membrane, but changing the kettle shape has little or no effect[1] (contrary to a rather widely held opinion). Reducing the kettle volume raises the frequency of the axially symmetric (0,1), (0,2) and (0,3) modes (by increasing the effective stiffness of the enclosed air), but lowers the frequency of the musically important (1,1), (2,1), (3,1) and (4,1) modes by increasing the effective air-mass loading.



Modal shapes of a snare drum determined by modal analysis with impact excitation include the lowest (1,0) mode (**a**), a pair of (1,1) modes (**b** and **c**), the (2,1) mode in the batter head (**d**), the (0,2) mode in the batter head (**e**) and the (0,2) mode in the snare head (**f**). (Adapted from ref. 3.) **Figure 5**

Tabla and mridanga (right), traditional drums of India, are tuned by applying a starchy paste to the drum head. **Figure 6**

This increase in air-mass loading comes about because the average air flow velocity (and hence the effective momentum) is slightly greater when the available volume is decreased. The volume of modern orchestral kettledrums is generally near the optimum value for harmonic tuning of the $(m,1)$ modes over the recommended range of pitch for each size drum.

Bass drums

The bass drum can radiate more power than any other instrument in the orchestra. (A peak acoustical power of 20 watts was observed in one experiment.[12]) A concert bass drum usually has a diameter of 80–100 cm, although smaller drums (50–75 cm in diameter) are popular in marching bands. Most bass drums have two heads, set at different tensions, but single-headed "gong" drums are used when a more defined pitch is appropriate. Mylar heads with a thickness of 0.25 mm are common, although calfskin heads are preferred by some percussionists for large concert bass drums.

Most drummers tune the batter, or beating, head to a greater tension than the carry, or resonating, head; some percussionists suggest that the difference be as much as 75%, giving an interval in pitch of about a fourth between the two heads. A distinctive timbre results from setting both heads at the same tension, but the prominent partial tones between 70 and 300 Hz appear to be stronger initially and to decay faster when the carry head is tuned to a lower pitch than the batter head.

Modal frequencies of the (0,1), (1,1), (2,1), (3,1) and (4,1) modes in an 82-cm-diameter bass drum fall surprisingly near a harmonic series, and if their partials were the only ones heard, the bass drum sound would be expected to have a rather definite pitch. At frequencies above 200 Hz, however, there are many inharmonic partials, which sound louder because the ear discriminates against sounds of low frequency. Fletcher and Irwin Bassett[13] found 160 partials between 200 and 1100 Hz. Coupling between the two heads results in a splitting of the (0,1) and (1,1) modes into pairs of modes, as in the snare drum.[14] Because of the bass drum's large area-to-volume ratio, the splitting of the (0,1) mode is especially large.

The average tension of a membrane increases when it vibrates at finite amplitude. The increase in the average tension ΔT is proportional to the square of the displacement amplitude, and the vibrational frequency is proportional to the square root of $T + \Delta T$. Thus, just after the drum is struck, each mode has a higher frequency, which decreases as the amplitude dies down. When a bass drum is struck a full blow, a typical value for the initial drumhead amplitude is 6 mm, which results in an upward frequency shift of about 10%, nearly a whole tone on the musical scale. Of course, this shift is made less noticeable by the perceived downward shift in pitch with increasing sound intensity, a well-known psychoacoustical effect that is especially strong at low frequency.

Indian drums

In India the drum is considered a most important musical instrument. Foremost among the drums of India are the tabla of northern India and the mridanga of southern India. (See figure 6.) One tunes the overtones of both of these drums harmonically by loading the drumhead with a paste of starch, gum, iron oxide, charcoal or other materials.

The tabla has a rather thick head made from three layers of animal skin. (Calf, sheep, goat and buffalo skins are used in different regions.) The shell is traditionally made of wood, but nowadays may be made of metal as well. The innermost and outermost layers of the head are annular, or ring shaped, and the layers are braided together at their outer edges and fastened to a leather hoop. Small straws or strings are placed around the edge between the outer and middle head. Tension is applied to the head by means of a long leather thong that weaves back and forth (normally 16 times) between the top and bottom of the drum. The tension in the thong can be changed by moving small wooden cylinders up or down. To tune the tabla one applies to the center of the head many thin layers of black paste, building up a circular patch. The paste consists of boiled rice and water with heavy particles such as iron oxide or manganese dust added to increase the density. After it is applied, each layer is allowed to dry and is then rubbed with a smooth stone until tiny cracks appear in the surface. The patch ends up with a slightly convex surface. Fine-tuning of the head is accomplished by upward or downward taps on the hoop with a small hammer.

The tabla we have described is usually played along with a larger drum, known by various names: banya, bayan, bhaya, dugga or left-handed tabla. The head of this larger drum is also loaded (slightly off center), and the shell may be of clay, wood or metal. To play the tabla the drummer rests the edge of the palm on the widest unloaded portion of the membrane. This constraint causes the nodal patterns to be quite symmetrical. Releasing the palm pressure produces a sound of different quality.

The mridanga or mridangam is an ancient, two-headed drum that functions, in many respects, as a tabla and banya combined into one. The smaller head, like that of the tabla, is loaded with a patch of dried paste, while the larger head is normally loaded with a paste of wheat and water shortly before playing.

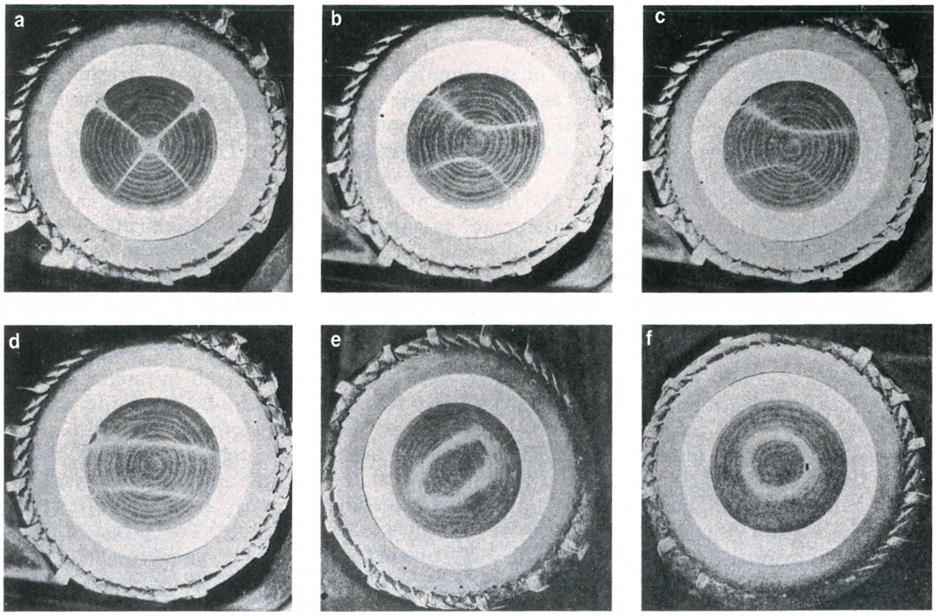
A succession of Indian scientists, beginning with C. V. Raman, have studied the acoustical properties of these drums. Raman and his colleagues recognized that the first four overtones of the tabla are harmonics of the fundamental mode. Later they identified these five harmonics as coming from nine normal modes of vibration, several of which have the same frequencies. The fundamental is from the (0,1) mode; the second harmonic is from the (1,1) mode; the (2,1) and (0,2) modes provide the third harmonic; the (3,1) and (1,2) modes similarly supply the fourth harmonic; and three modes, the (4,1), (0,3) and (2,2), contribute to the fifth harmonic.

Figure 7 shows Chladni patterns of six different modes, all of which have frequencies near the third harmonic. These patterns, published by Raman[15] in 1934, were obtained by sprinkling fine sand on the membrane before or immediately after the stroke. The sand gathers along the nodes—the lines of least vibration—forming a map of the vibration pattern excited by that stroke. Figure 7a shows the (2,1) mode and figure 7f the (0,2) mode; the other four modes are combinations of the (2,1) and (0,2) normal modes. Exciting any of the modes shown in figure 7 will result in a third-harmonic partial.

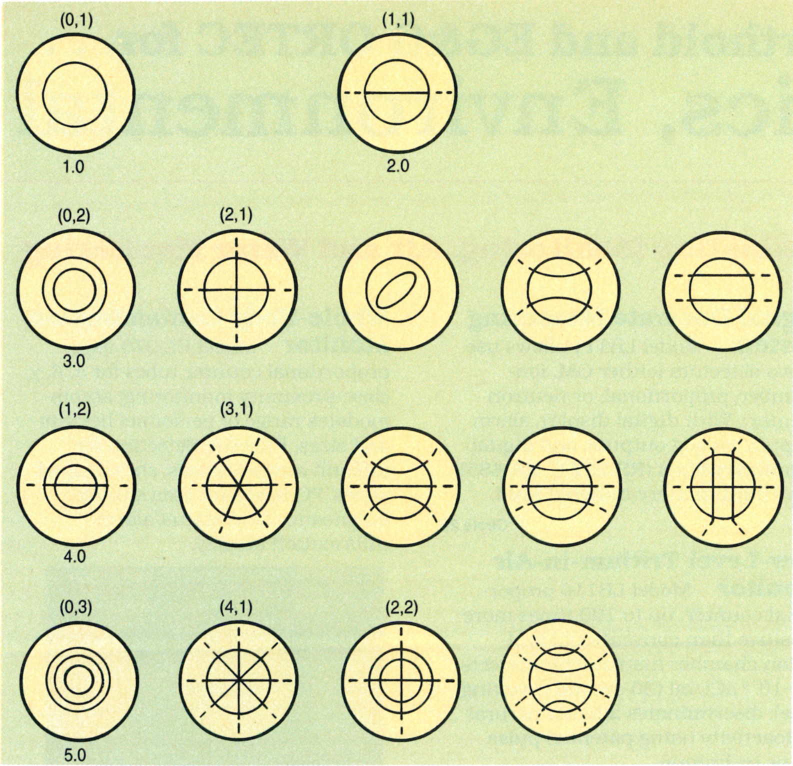
Figure 8 shows the nodal patterns of the nine normal modes corresponding to the five harmonics. Also shown are some of the combination modes whose vibrational frequencies correspond to the five tuned harmonics.

Chladni patterns formed on these Indian drums indicate that most of the vibrational energy is confined to the loaded portion of the drumhead. This confinement is accentuated by the restraining action of a rather stiff annular leather flap in loose contact with the peripheral portion of the drumhead. Thus the head is essentially divided into three concentric regions. The tabla player uses these concentric regions to obtain three distinctly different sounds, which can be described as "tun" (center), "tin" (unloaded portion) and "na" (outermost portion).

To study the effect of the center patch on the modes of vibration and the sound of a mridanga, we measured the sound spectrum at 32 stages (roughly every three layers) during application of a patch to the head.[16] The paste was prepared by kneading together roughly equal volumes of overcooked rice and a black powder of manganese and iron oxide. The area at the center of the head was cleaned, dried and scraped with a knife to raise the nap and provide good adhesion. A thin layer of overcooked rice was first



Chladni patterns of six different modes of the tabla that all have frequencies near the third harmonic. The (2,1) and (0,2) normal modes are shown in **a** and **f**, respectively; **b–e** are combinations of these two modes. (From ref. 15.) **Figure 7**



Nodal patterns of the nine normal modes and seven of the combination modes that correspond to the five harmonics of the tabla or mridanga head. Mode designations are given above the patterns and harmonic numbers are given below. (Adapted from ref. 16.) **Figure 8**

smeared onto this surface as glue, and then a small lump of the paste was applied. It was smeared out evenly with a swirling motion of the thumb. The excess was scraped away with a knife, and a smooth rock was used to pack and polish the mixture by rubbing the surface.

By observing the spectral frequencies, we were able to show how the nine normal modes changed as the patch was gradually built up to its final size. For example, the (0,2) and (2,1) modes gradually approached the frequency of the third harmonic, just as Raman observed in the tabla.

Studies by B. S. Ramakrishna and M. M. Sondhi[17] at the Indian Institute of Science in Bangalore and by S. De[18] in Santiniketan, West Bengal, indicate that the areal density of the loaded portion of the membrane should be approximately ten times as great as that of the unloaded portion, which is typically around 0.02–0.03 g/cm^2. The total mass of the loaded portion is in the range of 9–15 g for different tablas. We estimate that our mridanga patch, which was about 3 mm thick at the center, had a total mass of 29 g and an areal density of 0.8 g/cm^2. The density of the dry paste was about 2.8 g/cm^2.

Although percussion instruments are nearly as old as the human race, their acoustical behavior has been less studied than that of wind and string instruments, partly because the transient character of their sounds has made acoustical measurements more difficult. With the availability of computers and other digital instruments, this is no longer much of a problem, and we have seen a recent growth of interest in the study of drums, gongs and other types of percussion instruments. These studies have led to the development of several interesting new percussion instruments and will probably lead to more. What drums will look like a hundred years from now is anyone's guess.

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