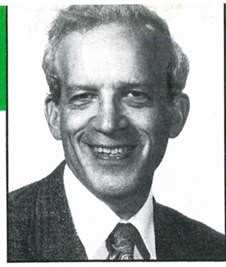

QUANTUM CHAOS AND THE BOW–STERN ENIGMA

Daniel Kleppner

Leo Kadanoff's essay on complexity and chaos in this column four years ago (March 1987, page 7) was, at least for me, a revelation, for new fields of physics don't pop up every day. Since then, research on complexity has mushroomed. Kadanoff recently described a few of the advances (March, page 9), and just last month Philip Anderson recounted how the science of complexity originated and sketched some of the new subjects it has spawned (July, page 9). But if complexity has mushroomed, chaos has exploded. Conferences on chaos proliferate, new journals abound, and nonlinear dynamics has elbowed its way into the mechanics curriculum. A book on chaos has made the *New York Times* best-seller list, and hardly a week passes when that arbiter of scientific good taste, *Physical Review Letters*, does not present us with a few papers on the subject.

Some of chaos's newfound popularity is obviously aesthetic. The phantasmagoria of the Mandelbrot set, with its fractal visions of solar eruptions and sea horses, is truly spectacular. (Actually, the connection between fractals and chaos is not so easy to understand, but hardly anything about chaos is easy to understand.) Some of the popularity must also be due to the exotic vocabulary of chaos. The very word "chaos" quivers with connotations, evoking the state of the universe before God switched on the lights, the state of society today, the state of most American families while the children are coming of age, and the erratic motions of those kinetic toys that one can see in expensive gift shops or in lectures on nonlinear dynamics. Can one think of more seductive terms than "strange attractor" or "the butterfly effect"?

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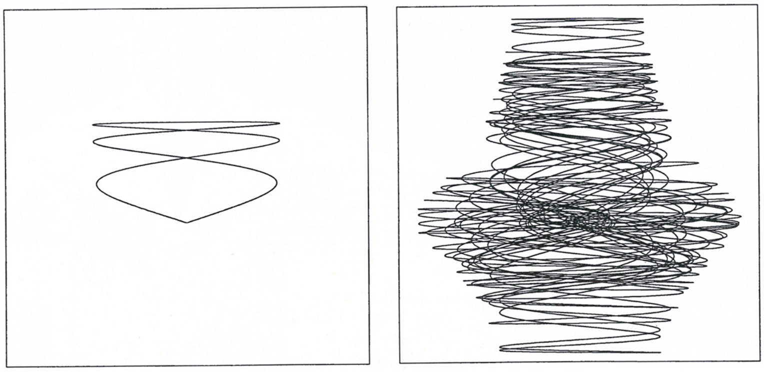

'Unusual sensitivity to initial conditions.' Both of these trajectories represent particles moving in the *xz* plane under the potential $V = A/r + B(x^2 + y^2)$. The initial directions of the two trajectories differ by only 1 microradian. The *z* axis is vertical. (Courtesy of R. V. Jensen, Wesleyan University.)

To this list I would add "quantum chaos," a phrase that juxtaposes two of the glitzier words of modern physics, but two that do not get on happily together. The problem is that Schrödinger's equation is linear and its solutions are fundamentally periodic or quasiperiodic. Quantum mechanics leaves no room for irregular motion. Chaos is a strictly classical concept; "quantum chaos" is an oxymoron.

Oxymoron or not, the subject has attracted lots of interest. Usually the phrase is interpreted to mean the study of those particular features of the quantum mechanical behavior of a system that occur when the corresponding classical motion is chaotic. Michael V. Berry's 1987 Bakerian Lecture provides a fascinating overview of quantum chaos.[1] Berry described the subject as "the study of semiclassical, but nonclassical, behavior characteristic of systems whose classical motions exhibit chaos," and suggested that it be called "quantum chaology." Unfortunately, his title has not taken hold—physicists appear to shy away from words that end in "ology," like "astrology" and "phrenology" (though not, of course, "cosmology"). Experts are never confused by the term "quantum chaos," though they often go out of their way to refrain from using it. I attended one workshop in which the words were scrupulously avoided by every speaker but one, who summoned the courage to refer to "the phrase that all of us think but none of us dares utter."

Chaos is frequently defined as motion that displays "extreme sensitivity to initial conditions." Because there is no way to specify initial conditions for a many-particle system, much less map the individual trajectories, chaos is not a property of complex systems but of simple systems. The figure above illustrates a case of extreme sensitivity to initial conditions. Shown are two trajectories of a particle moving in the *xz* plane under the potential $V = A/r + B(x^2 + y^2)$, where A and B are constants. The left-hand trajectory returns to the origin: The particle's motion reverses, and the particle traverses the same path again and again—clearly a case of periodic motion. In the right-hand plot the initial angle of the trajectory is changed by about 1 part in 10^6. The particle misses the origin and takes off on an erratic path, apparently never to retrace itself.

The behavior in the drawing is of more than formal interest because the potential describes a real system: a hydrogen atom in a magnetic field. (Spin, relativity and other such effects are neglected, for they play no important role here.) I have a personal interest in this problem, often called the diamagnetic hydrogen problem, because my students and I happened to be studying the system experimentally when we were told that at certain combinations of fields and energies the classical motion undergoes a textbook transition to chaos. Being naive, we had no idea what might happen to the spectrum: The quantum analog of extreme sensitivity to initial conditions could be a nightmare; the spectrum might be unstable, never to be reproduced because we could never exactly reproduce the experimental conditions, for instance the magnetic field.

Much of the picture we should have expected is well known. As the classical motion becomes chaotic, the energy levels repel one another and the spectrum undergoes a transformation whose signature is in its fluctuations. The distribution of separations between energy levels becomes the Wigner distribution, $p(s) = (\pi/2)s \exp(-\pi/4s^2)$, where s is the separation between adjacent levels in units of the average separation.

An impressive body of theory on quantum chaos has been developed, and there is a healthy assortment of spectral data that all display the ubiquitous Wigner distribution. However, experiments in which one studies flesh-and-blood quantum systems and manipulates them throughout various regimes of classical motion are about as scarce as slide rules at a computer convention. Martin Gutzwiller's recent book *Chaos in Classical and Quantum Mechanics* (Springer-Verlag, New York, 1990), a fascinating study of modern mechanics, has over 800 references, but only about a dozen of these are to experiments on quantum chaos. It is not that the theorists are ignoring the experiments—on the contrary, anyone carrying out such an experiment will find an appreciative audience of theorists.

Being among the few experimenters in a field dominated by theorists gives one license to ask naive questions, somewhat as a foreigner can be excused for behavior that would otherwise be a gaffe. I propose two questions. The first is this: Since quantum mechanics is more fundamental than classical mechanics, shouldn't the aim of quantum chaos be to *predict* classical behavior from quantum theory?

In some cases classical motion has been deduced from quantum behavior. For example, Karl Welge and his colleagues studied the photoabsorption spectrum of the diamagnetic hydrogen atom, or, more specifically, the Fourier transform of the spectrum.[2] They found peaks that should correspond to the periods of various periodic orbits, as predicted by the work of Gutzwiller, William P. Reinhardt and others. Knowing the periods, Welge and his collaborators went on to discover a group of orbits whose existence was previously unsuspected. (One of these is shown in the figure.) It is not a trivial matter to find such orbits, since they are generally unstable. Our own data suggest the existence of orbits with extremely long periods, though these have yet to be found. Long-period unstable orbits are notoriously difficult to find classically, though quantum mechanics can give some insight into classical motion.

The second question is, Does the first question matter? Is it essential for classical physics to have a solid underpinning of quantum mechanics? In some cases the answer is certainly yes. Classical statistical mechanics, for example, had fundamental problems that could only be resolved by quantum theory. In nonlinear dynamics and chaos, however, there are so many conceptual troubles at the classical level that it is not evident that one can look to quantum mechanics for insight.

As Gutzwiller's book explains, classical mechanics is by no means well understood: It is a far more complicated and subtle subject than most physicists realize. For example, no way has yet been devised to predict whether a given system will exhibit orderly or chaotic motion. It would be pleasing to derive chaos from quantum mechanics, though this seems like an unrealistic agenda as long as we cannot even predict chaos from classical mechanics. So choosing the direction in which to proceed to connect quantum mechanics and chaos presents something of a dilemma.

This dilemma is an example of what I call the bow–stern enigma. The term has its origin in a canal voyage I made with some friends. Upon arriving at our boat we were confronted with an embarrassing but apparently fundamental problem: deciding which end of the boat was the bow and which the stern. The problem, however, turned out to be merely superficial. The countryside was so delightful that it made little difference which way we traveled: The scenery was interesting in either direction.

References

1. M. V. Berry, in *Dynamical Chaos*, M. V. Berry, I. C. Percival, N. O. Weiss, eds., Princeton U. P., Princeton, N. J. (1987), p. 183.
2. J. Main, G. Wiebusch, K. H. Welge, Comments At. Mol. Phys. **25**, 251 (1991). This special issue, subtitled "Irregular Atomic Systems and Quantum Chaos," was edited by J. C. Gay and provides an excellent review of atomic studies of quantum chaos. ■