book that should prove useful to researchers for many years to come.

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Mathematical Analysis and Numerical Methods for Science and Technology

Robert Dautray and Jacques-Louis Lions [Translated from the French by Ian N. Sneddon (Vols. 1 and 2) and by John C. Anson (Vols. 3 and 4)]
Springer-Verlag, New York.
Volume 1: Physical Origins and Classical Methods
1990. 695 pp. $89.90 hc
ISBN 0-387-50207-6
Volume 2: Functional and Variational Methods
1988. 561 pp. $89.90 hc
ISBN 0-387-19045-7
Volume 3: Spectral Theory and Applications
1990. 515 pp. $98.00 hc
ISBN 0-387-50208-4
Volume 4: Integral Equations and Numerical Methods
1990. 465 pp. $89.00 hc
ISBN 0-387-50209-2
Volume 5: Evolution Problems I
Due December 1991. 728 pp. $120.00 hc ISBN 0-387-50205-X
Volume 6: Evolution Problems II: The Navier–Stokes and Transport Equations, and Numerical Methods
Due spring 1992.
ISBN 0-387-50206-8

Physicist Robert Dautray and mathematician Jacques-Louis Lions, with the assistance of several mathematicians and engineers of the French Atomic Energy Commission, have written a monumental and remarkable treatise. To serve physicists, chemists and engineers, the authors have combined the power of modern mathematical analysis and numerical analysis to bear on a wide range of physical problems.

This work is remarkable for three reasons.

▷ *The work comprises 3774 pages, yet it is a well-coordinated treatise.* Although many have contributed to it, the series reads like a two-author book. This illustates teamwork at its best: Outstanding leaders gather first-class collaborators, organize and coordinate their work and take responsibility for the final product. (However, the contribution of each collaborator is more clearly identified in the French edition than in the English one.)

▷ *This is a treatise for engineers that is based on sophisticated mathematics.* Most engineers are trained in 19th-century mathematics, and books on numerical analysis rely on the same mathematics. This book is based on the functional methods of the 20th century. In France elite engineers trained by the most prestigious mathematicians will enjoy this book readily, but it will require a serious effort on the part of most scientists with a US college degree. (Recall the creation of the Ecole Polytechnique in 1794. To meet the needs of industrial development, its first directors, Monge and Lazare Carnot, created an institution based on the best possible mathematics and physics expertise. To name but a few of its faculty: Fourier, Berthollet, Vauquelin, Lagrange, Laplace, Arago, Cauchy, Navier, Ampère, Gay-Lussac, Chasles, Hermite, Painlevé, Paul Lévy and Poincaré. The tradition continues, and many engineers at the Commissariat à l'Energie Atomique had the competence to contribute to the present treatise.)

In their preface, Dautray and Lions express the hope that a background in mathematics at the level of the Lebesgue integral and some familiarity with distribution theory are adequate preparation for using this treatise, but sophistication is also needed to translate properties of function spaces into the properties of the solutions of an engineering problem: For example, sophistication is needed to translate "X is dense in Y" into "the limit of the approximate solutions (points in X) is the solution (a point in Y) of the exact equation."

Function spaces, which play a dominant role in this treatise, are seen in basic US college science curricula, but science undergraduates do not think in terms of function spaces. They do not appreciate the distinctive personality of a space characterized by its axioms. They do not marvel at the possibility of finding Banach spaces in the space of distributions. But these are precisely the Sobolev spaces that are indispensable for the theory of partial differential equations: As Laurent Schwartz once said, "Distributions reestablish differentiation as a simple operation of analysis." But the basic theorems of calculus on $\mathbb{R}^n$ generalize easily on function spaces only if they are Banach spaces.

Simple, strategically placed examples would have helped the reader think in terms of function spaces. Two examples (there are many others) come to mind: Given an action S on the space X of paths F with given boundary conditions, say fixed initial and final points, the solutions of the Euler–Lagrange equation (a second-order ordinary differential equation with a boundary) are the functions in X that minimize $S(F)$, a variational problem on X stated in terms of functions and their first derivatives. Another example could be used to relate the difficulties of some boundary-value problems to the difference between the Sobolev spaces H^1 and H^1_0. Without such introductory examples, readers unfamiliar with function spaces can look at the applications (of which there are many—in fluid mechanics, elasticity, electrodynamics, quantum mechanics and neutron kinetics) to find the motivation for functional methods. They will then appreciate that linear algebra and Sobolev spaces form the foundation for the numerical analysis of problems cast in differential or integral equations.

Equipped with the Dautray–Lions treatise, readers will have a tool to move into the 21st century. Which other tools are needed? An obvious one is stochastic calculus. A remark in Volume 1 of the English translation led us to expect later chapters based on stochastic calculus. Not finding them, we checked the original French edition and found that stochastic calculus is mentioned but as a project for the indefinite future. We do hope that this project will be completed and will include Monte Carlo numerical methods, giving readers another equally important but often neglected tool.

A book as sophisticated as this needs a user's manual. The "user's manual" for this treatise is made up of sections scattered throughout the text that deal with current usage of the analytic tools described in the main text and prepare the ground for other possible uses in the decades to come. These include fast Fourier transforms, finite elements (a subject often omitted from texts on numerical methods) and continuous spectrum analysis. The series contains no examples of working computer codes. Some sophistication is needed to locate the numerical techniques; for example, the chapter heading "stationary problems" means to a large extent "eigenvalue–eigenvector problems"; "evolution problems" means "hyperbolic partial differential equations, stability conditions and so on"; and "transport" means "diffusion equations and their application to relaxation techniques for elliptic partial differential equations."

▷ *This series serves as both a reference book and a textbook.* However, it lacks a cumulative index. One has to consult the six indexes of the English translation (one per volume), which are excellent, or the three indexes of the original French version (one per tome), which are so skimpy as to be useless. On the other hand, several excellent features help the reader find answers to his or her questions, both formulated and unformulated. For one, enough explanation is given in each section to make each sufficiently self-contained. Also, the "reviews" (*bilan* in the French version) scattered throughout the book are in fact retrospective commentaries examining previously solved problems from new perspectives: Problems are regrouped so that they enhance each other. The table of notations, repeated at the end of each volume includes brief definitions.

For a textbook, the book is pitched at a rather advanced level. It is entirely proper that the French paperbacks appear in the "Collection Enseignement" of the Institut National des Sciences et Techniques Nucléaires. Scientists nowadays cannot expect to learn in college all the basic material they will need in a lifetime—whether in an academic or an industrial career. They will therefore have to rely on books such as the present treatise. Appendixes, short and long, and various sections (for example, 33 pages to introduce the mathematical problems of quantum physics) bring back to the reader's mind topics seen in college but not really absorbed.

The typesetting in the English edition is more elegant than in the French original but the English version is harder to read for two reasons: The translation is often awkward—particularly in the first two volumes—because it follows the French too closely (at press time the translations of the last two volumes were not available); and the indentation of the paragraphs, so useful in studying a difficult text, has been eliminated in the English text. [The French original is available from SMPF, 100 East 42nd Street, Suite 1002, New York, NY 10017, (212) 983-6287. The French Tome I (or equivalently the French paperback volumes 1–4) has been translated into the English volumes 1–2, Tome II (paperback volumes 5–6) into the English volumes 3–4 and Tome III (paperback volumes 7–9) into the English volumes 5–6.]

This treatise is desirable for individual researchers and indispensable for libraries, where it will be a basic tool for many years to come.

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Elementary Fluid Dynamics

D. J. Acheson
Clarendon (Oxford), New York, 1990. 397 pp. $85.00 hc
ISBN 0-19-859660-X

A Mathematical Introduction to Fluid Mechanics

A. J. Chorin and J. E. Marsden
Second edition.
Springer-Verlag, New York, 1990 [1979]. 168 pp. $29.00 hc
ISBN 0-387-97300-1

The National Science Foundation has relegated fluid mechanics to the catch-all desk, which oversees fluid, particulate and hydraulic systems. Some say that since the equations are known, fluid mechanics is a problem in applied mathematics. Physicists generally spurn it, except for certain areas of turbulence. Yet it is an exceptionally difficult subject that continues to present phenomena that are not well understood and often nearly impossible to calculate, attracting the attention of experimenters and mathematicians alike. These phenomena have enormous influence on every phase of existence, determining rates of transport and reaction in biology, the environment and technology and ultimately controlling much in the processes of manufacturing, energy consumption, pollution and life itself.

These two books are intended to introduce students to a subject they nearly always find difficult. A. J. Chorin and J. E. Marsden are distinguished mathematicians at Berkeley; this is the second edition of their book, originally published in 1979. D. J. Acheson is at Jesus College, Oxford, where he teaches fluid mechanics. Although probably less well-known, he has to his credit a non-negligible body of work in the field.

Despite the title, Chorin and Marsden's book is not in substance particularly mathematical—not much more so than similar courses we teach at Cornell. What does give it a slightly mathematical flavor is the style, which has a certain purity and economy, a perhaps deceptive simplicity.

The book has interesting exercises and good drawings throughout. The