$$P_W(x,p) = (2\pi\hbar)^{-1}\int_{-\infty}^{\infty} \sigma(x,\xi) \times \exp(-i\xi p/\hbar)\,d\xi$$

$$= (2\pi\hbar)^{-1}\int_{-\infty}^{\infty} \psi(x+\xi/2)\,\psi^*(x-\xi/2) \times \exp(-i\xi p/\hbar)\,d\xi$$

With the integration variable $y \equiv -\xi/2$ this expression becomes identical with the standard definition of the Wigner quasidistribution,[1] which is a real-valued function that may take on negative values. Finally we compute

$$P(x) := \int_{-\infty}^{\infty} P_W(x,p)\,dp$$

and

$$\tilde{P}(p) := \int_{-\infty}^{\infty} P_W(x,p)\,dx$$

The results, $P(x) = |\psi(x)|^2$ and $\tilde{P}(p) = |\tilde{\psi}(p)|^2$, with

$$\tilde{\psi}(p) = (2\pi\hbar)^{-1/2}\int_{-\infty}^{\infty} \psi(x) \times \exp(-ipx/\hbar)\,dx$$

are indeed the genuine probability distributions, thus verifying the proper normalization and scaling of the standard P_W.

Reference

1. M. Hillery, R. F. O'Connell, M. O. Scully, E. P. Wigner, Phys. Rep. **106**, 121 (1984).

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PHILPOTT REPLIES: My comment on the article by Malvin C. Teich and Bahaa E. A. Saleh was simply that the expression they used for the Wigner distribution function is inconsistent with the commutation rule $[x,p] = i/2$, which they adopted in their paper. Wolfgang P. Schleich and Georg Süssmann do not address this point.

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2/91

TEICH AND SALEH REPLY: The standard definition of the Wigner distribution function can indeed be written as

$$W(x,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} \psi\left(x+\frac{\xi}{2}\right) \times \psi^*\left(x-\frac{\xi}{2}\right)\exp\left(-i\xi\frac{p}{\hbar}\right)d\xi \qquad (1)$$

$$= \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \psi(x-y)\,\psi^*(x+y) \times \exp\left(i2y\frac{p}{\hbar}\right)dy \qquad (2)$$

as Wolfgang Schleich and Georg Süssmann point out. In our article we had intended to take $\hbar = 1$ and obtain

$$W(x,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} \psi(x-y)\,\psi^*(x+y) \times \exp(i2yp)\,dy \qquad (3)$$

Unfortunately, a factor of 2 in the exponent of equation 3 was missing, and the normalization factor $1/\pi$ was ignored. The results presented in the article are not affected by these errors, however.

If, instead, we substitute $\hbar = \frac{1}{2}$ in equation 1 we obtain

$$W(x,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} \psi\left(x+\frac{\xi}{2}\right)\psi^*\left(x-\frac{\xi}{2}\right) \times \exp(-i2\xi p)\,d\xi \qquad (4)$$

This is equivalent to the definition suggested by John Philpott. Thus equations 3 and 4 are special cases of the definition (equation 1) advocated by Schleich and Süssmann, with the units defined such that $\hbar = 1$ and $\frac{1}{2}$, respectively. It is unfortunate that our missing factor has stirred a discussion on a subject where no real differences exist.

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The Meter's Origins: A Clockwork Conspiracy?

Why, in SI units, is $g \cong \pi^2$? Coincidences are only occasionally accidental. I wish to suggest that this one is evidence for a conspiracy to obscure the origins of the metric unit of length.

The lore is that the meter was originally defined in 1791 as one ten-millionth of the distance from the North Pole to the equator.[1] It is taken as a remarkable coincidence that the period of a 1-meter simple pendulum is almost exactly 2 seconds. (For $g = 9.81$ m/sec^2, the period is 2.006 sec.) If the period were 2 seconds, then from $2\text{ sec} = 2\pi\sqrt{1\text{ m}/g}$ we would get $g = \pi^2$ m/sec^2.

I believe that this coincidence arises because there existed an older standard of length, defined by the group of skilled instrument makers who manufactured pendulum clocks. It was the length of a pendulum for a clock with a 1-second tick. (That is a 2-second period.) The escapement had been invented long before 1791, so that this standard length was well established. It could be reproduced with high accuracy by any skilled instrument maker anywhere on Earth.

It appears as if the new French Republic chose a rational standard of length and then confounded rationality by linking it to one-quarter the circumference of the Earth. The recipe for the meter became truly grand, and difficult to implement. To reproduce the French standard, one had either to remeasure the circumference of the Earth or to make a pilgrimage to the location of the standard meter. The conspiracy? There can be little doubt that the French Board of Tourism was behind the 1791 standard length.

I have enjoyed using this speculation to engage the attention of my students. I am writing to ask if there is any real evidence for the use of clock pendulums as length standards.

Reference

1. F. W. Sears, M. W. Zemansky, H. D. Young, *University Physics*, 7th ed., Addison–Wesley, Reading, Mass. (1987), p. 5.

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3/91

Honor Compton with a Postage Stamp

As a philatelist and scientist I note with dismay the dearth of American scientists pictured on US postage stamps. The 100th anniversary of the birth of Arthur Holly Compton is an opportunity to address that problem. Compton was born on 10 September 1892 and received the Nobel Physics Prize in 1927 for his discovery of the Compton effect. Accordingly, I have urged the US Postal Service to consider issuing a commemorative postage stamp to honor this outstanding physicist.