

EXPLORING CHAOTIC DYNAMICS THROUGH MODEL SYSTEMS

Chaotic Dynamics: An Introduction

Gregory L. Baker
and Jerry P. Gollub

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Reviewed by Ronald Fox

This book was a pleasure to review. The authors' express purpose was to fill the void in the literature on chaotic dynamics between the readable and enjoyable but technically light books, best exemplified by James Gleick's *Chaos* (Viking, New York, 1987), and the technically difficult books, such as Michael Lichtenberg and Michael Liebermann's *Regular and Stochastic Motion* (Springer-Verlag, New York, 1983). Gregory Baker and Jerry Gollub have accomplished this superbly and provided us with a text, or text supplement, suitable for the advanced undergraduate.

The mode of presentation in *Chaotic Dynamics* is very pedagogical. Baker and Gollub have chosen one particular dynamical system as the vehicle for presenting nearly all of the useful concepts in chaotic dynamics. That paradigm is the damped, periodically driven pendulum. They successfully use this paradigm to discuss phase-space trajectories, Poincaré sections, time series, Fourier power-spectrum analysis, sensitivity to initial conditions, bifurcations, attractors, basins of attraction, period doubling, Lyapunov exponents, Kolmogorov entropy, mode locking, the Devil's staircase, intermittency, fractal dimension, information measures, Lyapunov-exponent connec-

tions to fractal dimension and more. Computer exercises are provided that demonstrate all these features for the damped, driven pendulum. (Appendix B contains 11 programs written in the readily accessible TRUE BASIC language.) The reader can learn much of the material in this book by executing these programs. In addition, each chapter ends with several informative problems to be solved by the reader.

In Chapter 4 even simpler paradigms are used to develop some of the concepts. The logistic map, the circle map and the horseshoe map are briefly introduced and discussed. Mitchell Feigenbaum's universal parameter δ is mentioned, and Vladimir Arnold's "tongues" are illustrated. Even Steven Smale's "heteroclinic tangle" is illuminated. The fractal nature of chaotic attractors is motivated by a presentation of the Cantor set. All of this gives the student a taste of some of the deeper aspects of chaotic dynamics.

My pleasure in reading this book was dampened just a little by two things. First the material in the final chapter, which addresses chaos in real physical systems, was far too brief. Gollub is one of the pioneers in research on chaos in real fluids, and I think students would benefit from his insights into the actual performance of chaos experiments—especially his expertise on the special problems such experiments pose. The coverage of the other experiments could have been fuller. As it stands, the reader will have to go to the cited literature for more.

The second problem is that the authors omitted one property of chaotic dynamics that I always like to emphasize: its connection to number theory and other areas of pure mathematics. Baker and Gollub could have included this connection without going into any more sophisticated mathematics. For example, they could have raised questions like: Why are the Arnold tongues on rational numbers and not on irrational

numbers? Why is the golden mean of the Greeks significant in mode locking? They could also have made observations such as the fact that the Devil's staircase contains steps of different sizes that can be described by Farey addition. Although the authors do show that a Cantor set has a fractal dimension less than one, they do not emphasize the equally significant fact that it is greater than zero. They could have easily included the ternary representation of the Cantor set and its isomorphism to the binary representation of the real numbers on the unit interval to show that the Cantor set is uncountable, is nowhere dense, has measure zero and yet has a fractal dimension greater than zero. This information could have been used to introduce the reader to symbolic dynamics (which is closely related to the Smale horseshoe).

These are just a few examples of the relationship between pure mathematics and chaotic dynamics that has surprised so many researchers. Fortunately, the references include some of the literature on this side of the story, and any student who becomes intrigued by this excellent book will naturally want to explore more.

Neutron Optics

Varley F. Sears

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In the mid-1930s Enrico Fermi realized that thermal neutrons, which he discovered, are slow enough to be subject to the basic optical phenomena so well known for light. Yet at that time neutron sources were far too feeble to use in the experiments. Therefore, the experimental science of neutron optics had to wait for the advent of nuclear reactors, at which time sufficiently strong neutron beams became available. Eventually, as pointed out by Varley Sears in *Neutron Optics*, Fermi and Walter

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