

EVIDENCE ACCUMULATES, AT LAST, FOR THE WIGNER CRYSTAL

Experimenters may finally have observed the crystalline phase of a degenerate electron gas. Electrons in such a phase do not move freely through the available space; they are confined to the sites of a lattice and their quantum mechanical zero-point motions are limited to small oscillations about their mean positions. Eugene Wigner had proposed this phase based on his theoretical analyses in the 1930s. The "Wigner crystal" phase has been the focus of much discussion for twenty years, since experimenters have been able to fabricate a variety of systems where the phase might be realized.

The new experiments are not the first to observe an electron crystal. In 1979 Charles Grimes and Gregory Adams at AT&T Bell Labs obtained evidence for the crystalline phase of a two-dimensional electron gas formed above the surface of liquid helium. But the experiment by Grimes and Adams, and further ones on electrons at the helium surface, probed the electron crystal and its melting in the classical limit because, at the electron

densities available in that case, even the lowest attainable melting temperature is about an order of magnitude higher than the Fermi temperature, at which effects due to quantum statistics set in. (See the article by Arnold J. Dahm and W. F. Vinen in *PHYSICS TODAY* February 1987, page 43.) The new experiments, by contrast, are thought to be in the quantum regime. Experimenters and theorists hope that further investigations of the newly discovered phase will shed much-needed light on the nature of a quantum crystal and on the mechanisms by which it melts.

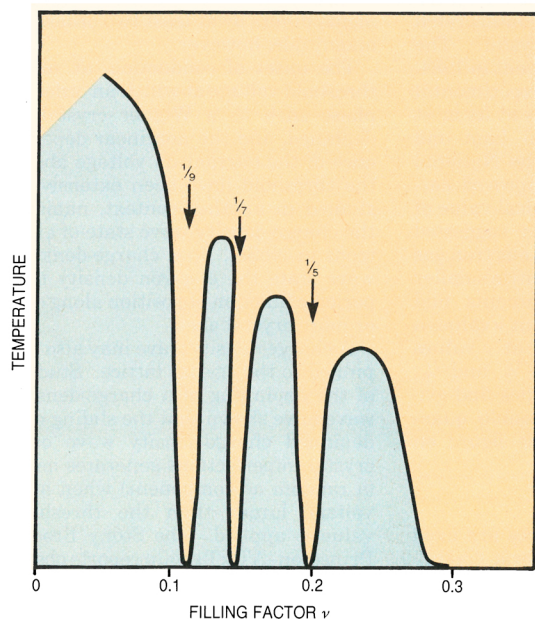
Kinetic vs potential energies

A fairly accurate explanation of many metallic properties became available in the early 1930s. It was based on the notion of bands in the energy spectrum of an electron moving through the periodic potential due to the ions. Wigner, in his two landmark papers of the 1930s, went beyond the simple band theory; he studied correlations among electrons due to the Coulomb repulsions between them. Band theo-

ry ignores such correlations.

Wigner explored the consequences of the correlations for metallic properties, but he also brought out a very interesting feature of a homogeneous electron gas. A homogeneous electron gas is a collection of electrons somewhat unlike that in a metal because the neutralizing positive charge is assumed to be spread uniformly throughout the volume available to the electrons, instead of being lumped at the (periodic) positions of the ions. The behavior of such a gas as a function of the particle density is very different from that of a gas of classical particles, Wigner pointed out. In a classical gas, one expects that the lower the particle density is, the less important will be interparticle interactions for the properties of the gas. By contrast, in the electron gas the interparticle Coulomb interaction becomes more important the smaller the density is, so that a high-density electron gas behaves almost like an ideal gas of fermions.

The behavior of the electron gas therefore depends on the ratio of its kinetic to its potential energy. The kinetic energy at zero temperature may be estimated from the Heisenberg uncertainty relation: The mean kinetic energy per particle decreases with decreasing density, because the uncertainty in the momentum, and therefore the maximum momentum value, is smaller the larger the inter-



Phase diagram of a two-dimensional electron gas in a strong magnetic field. The filling factor is the number of electrons per magnetic flux line. The fractional quantum Hall effect occurs at fractional filling factors ν with odd denominators (arrows). At those filling factors the ground state (the state at 0 K) is an incompressible quantum liquid. Since 1984 it has been expected that the ground state changes to a Wigner crystal at $\nu = 1/5$ but definitive evidence for such a phase was lacking. Recently several experimenters may have seen a Wigner crystal-like phase. One of these groups has conjectured this phase diagram on the basis of its optical studies of GaAs-AlGaAs heterojunctions.⁴ In this diagram the incompressible quantum-liquid state occurs even at $\nu = 1/5$, but for $\nu < 1/5$ all such states are surrounded by the Wigner-crystal-like state (blue regions). (Adapted from ref. 4.)

particle separation is. In fact, Wigner argued, below a certain critical density the kinetic energy will be negligible and the behavior of the gas will be determined by which electron configuration minimizes the potential energy due to the Coulomb repulsion. Since the potential energy of a random array is higher than that of an ordered array, Wigner concluded that at densities smaller than some critical value the electron gas will form a crystal. In three dimensions, the case Wigner considered, the lowest potential energy is obtained for a body-centered cubic crystal.

In the vast literature on the electron gas, the density is usually measured in terms of the parameter r_s , which is the radius, in units of the Bohr radius, of a sphere whose volume equals the average volume per particle in the gas. This parameter is a decreasing function of the particle density. Wigner estimated that an electron gas for which r_s is at least 10 or so should crystallize. Electrons in alkali metals have r_s values between 4 and 6. Most of the vast literature on the electron gas was generated in the 1960s, following the work done by David Bohm, David Pines and the late John Hubbard, who introduced new ideas to describe the properties of the gas. In 1971 R. S. Crandall and R. Williams applied Wigner's argument to a two-dimensional electron gas and concluded that such a gas will also crystallize in the low-density limit. The crystallization of a degenerate electron gas—an electron gas below its Fermi temperature—has never been observed experimentally in either two or three dimensions.

A further extension of Wigner's argument, by Yu. E. Lozovik and V. I. Yudson (Spectroscopy Institute, USSR Academy of Sciences) in 1975, showed that a two-dimensional electron gas will have a crystalline phase also when the gas is subjected to a strong magnetic field normal to the plane of the gas. The behavior of such a gas becomes classical in the limit of an infinitely strong magnetic field.

'Where is the Wigner crystal?'

Recent evidence for the Wigner crystal comes from experiments on the two-dimensional electron gas in the presence of a strong magnetic field. The experiments were done on the electron gas formed at the interface of GaAs and AlGaAs layers of a modulation-doped heterojunction.

The density parameter for a two-dimensional gas in the presence of a magnetic field is the filling factor ν , which is the number of electrons per flux quantum. The number of elec-

trons per unit area is usually fixed in heterojunctions, but changing the magnitude of the magnetic field changes ν , which is the relevant density parameter.

Studies of the electron gas in heterojunctions in the presence of a magnetic field led Daniel Tsui, Horst L. Stormer and Arthur Gossard, all then at AT&T Bell Labs, to the discovery of the fractional quantum Hall effect in 1983. According to a theory of the effect by Robert Laughlin (Stanford) and the extension of that theory by Duncan Haldane (Princeton), the effect arises because the ground state of a two-dimensional electron gas in a strong magnetic field has special features when the filling factor is of the form p/q , where p and q are integers and q is odd. The special ground states are said to be incompressible quantum liquids: They are liquids because there are no long-range correlations among the positions of the electrons, and they are incompressible because it costs energy to change the density (or the filling factor) away from the special values. (For a report on the most recent of the several interesting discoveries related to the fractional quantum Hall effect, see *PHYSICS TODAY*, January 1990, page 19.) Experimenters have observed the fractional quantum Hall effect at several filling-factor fractions of the form p/q . In the low-density limit, there has been some evidence for the effect at $\nu = 1/5$ and $1/7$ but no definitive evidence at $\nu = 1/9$.

One of the enduring questions in the colorful history of the fractional quantum Hall effect has been "Where is the Wigner crystal?" That is, why had experimenters not seen any feature that might be interpreted as evidence for the crystal? The question became quite urgent in 1984, when theoretical and experimental research on the effect was at its most feverish. That year Pui Lam and Steven Girvin, then at the National Bureau of Standards, improved earlier estimates for the critical value of ν for crystallization and suggested that the two-dimensional electron gas should crystallize for ν a little smaller than $1/5$. Recent experiments find evidence for a solid phase at ν values a little larger than $1/5$ and at ν values below $1/5$, except at $\nu = 1/7$ and $1/9$, where luminescence studies find evidence for the incompressible quantum liquid state (see the figure on page 17).

The evidence

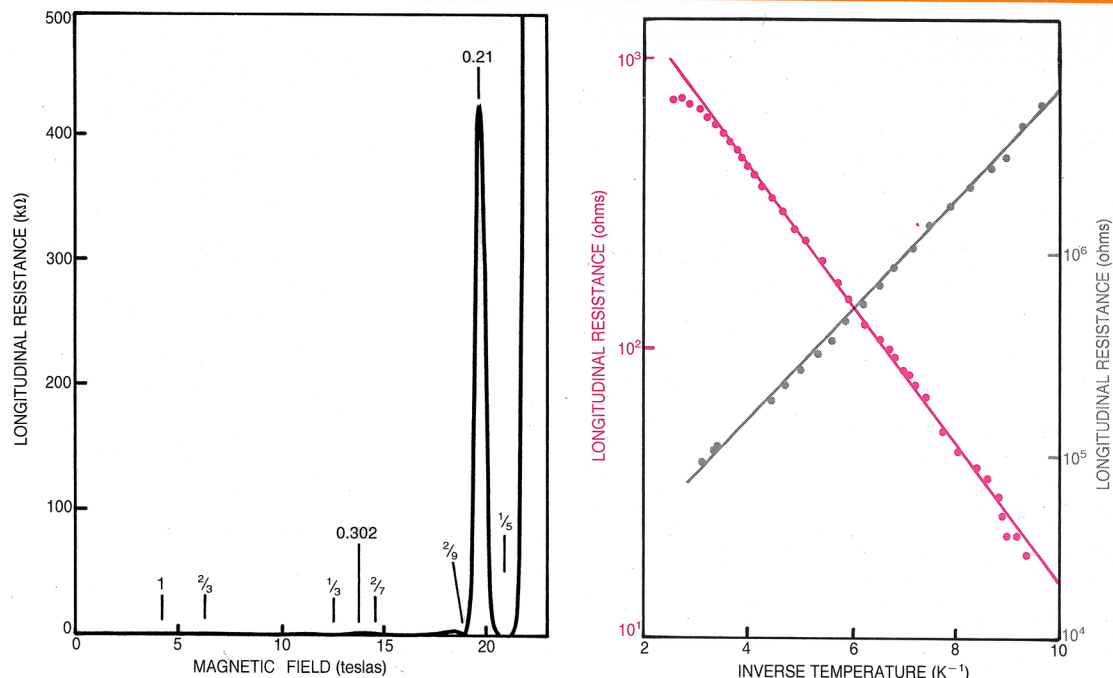
The interparticle distances are fixed at some constant value in a crystalline phase. Consequently, the whole crystal will be pinned to the substrate

even if only a few particles in it prefer to occupy certain sites on the substrate. A crystal that is pinned to the substrate cannot move until the electric field exceeds a threshold value, which is a measure of the force responsible for the pinning. When the crystal is not pinned to the substrate, however, all electrons in it will slide coherently on the substrate on application of an electric field, so the resistivity in that case will be vanishingly small at low temperatures. The current-voltage characteristics of an electron gas in the Wigner crystal phase are therefore expected to be nonohmic: The conductivity is expected to be very small below a certain threshold voltage, and it is expected to rise rapidly when the voltage exceeds the threshold value. Two groups have reported seeing such behavior. One of these groups is a collaboration among researchers from the State University of New York at Stony Brook, Princeton and AT&T Bell Laboratories¹; the other group is a collaboration among researchers from Service de Physique du Solide et de Résonance Magnétique at Saclay, Rutgers University, CNRS at Bagnex and Philips Laboratories (Redhill, UK).² They find a similar threshold behavior and nonlinear conductivity in their studies of the longitudinal resistivity of high-mobility heterojunctions. (Longitudinal resistivity is the resistivity in the direction of the applied electric field.)

The features reported by these two groups occur both at ν values a little larger than $1/5$ and at ν values smaller than $1/5$. The value of the threshold voltage observed in the two experiments differs by at least two orders of magnitude, however.

The onset of conduction, at some threshold voltage at which an ordered phase such as a Wigner crystal is depinned, and the nonlinear dependence of the current on voltage above the threshold have been extensively studied in another context, namely, the charge-density-wave state of electrons in NbSe₂. In a charge-density-wave state the electron density is a periodic function of position along one or more crystal axes.

A charge-density wave may also get pinned to the crystal lattice. Studies of the depinning of a charge-density wave have shown that the sliding of a depinned charge-density wave over crystal imperfections generates noise (a random ac component) when a dc voltage larger than the threshold value is applied. The Stony Brook-Princeton-AT&T group reports observation of similar behavior in heterojunctions near $\nu = 1/5$.



Longitudinal resistance of a two-dimensional electron gas in a strong magnetic field at 90 mK (left panel). The arrows indicate the positions of the minima observed in the experiment and the corresponding values of the filling factor ν . The temperature dependences of the resistance for two ν values— $1/5$ (red) and 0.21 (grey)—are shown in the right panel. At $\nu = 1/5$ the resistance data fit the form $e^{-\Delta/2T}$, the shape expected for an incompressible quantum liquid. At $\nu = 0.21$, by contrast, the temperature dependence fits the form expected from an insulating phase such as a pinned Wigner crystal. (Adapted from ref. 3.)

In the quantum Hall effect the longitudinal resistivity as a function of the filling factor ν has minima at the ν values corresponding to the incompressible quantum liquid states; the Hall resistivity at those ν values is independent of small variations in ν . The longitudinal resistivity minima are deeper the lower the temperature is, and the resistivity is expected to vanish as $T \rightarrow 0$. The figure on page 19 shows the longitudinal resistance of a modulation-doped high-mobility heterojunction at $T \sim 90$ mK. The data were obtained by a collaboration among physicists from the Francis Bitter National Magnet Laboratory, Princeton and AT&T Bell Labs.³ Many of the minima at low fields are invisible on the scale chosen for the figure, but there is clear evidence of a minimum and therefore an incompressible quantum liquid state at $\nu = 1/5$. On either side of $\nu = 1/5$, however, the resistance is very large, and it increases exponentially with decreasing temperature (right panel in the figure). Such a temperature dependence of the resistivity is unlike that expected for an incompressible quantum liquid state (red curve in the right panel) or observed at any other ν

values intermediate between the quantum liquid states. Rather, it is indicative of an insulating phase, such as a Wigner crystal pinned to defects in the substrate.

And the new signature

The minima in the longitudinal resistivity and the plateaus in the Hall resistivity have been the most prominent experimental signatures of the quantum Hall effect since Klaus von Klitzing (Max-Planck-Institut für Festkörper Physik, Stuttgart) discovered the effect in 1980. Since 1986, however, experimenters have determined another good signature of the effect, in luminescence studies.

The luminescence experiment studies the spectrum of light emitted by a heterojunction sample after it is illuminated by light of energy greater than the bandgap of GaAs (1.5 eV). The illumination produces electron-hole pairs, and the electric field at the junction sweeps the holes away from the two-dimensional electron gas and into the GaAs layer. Because the wavefunction of the electrons forming the two-dimensional gas does not fall abruptly to zero normal to the interface, the probability that those elec-

trons will recombine with the holes also is nonzero. The recombination produces light whose wavelength is a measure of the energy of the electrons in the two-dimensional electron gas.

The new signature for the quantum Hall effect that has been discussed and developed since 1986 is seen when one plots the position of the luminescence line (that is, the wavelength of the peak intensity) as a function of the magnetic field: At field values corresponding to the incompressible quantum liquid states, the position changes discontinuously. But the discontinuous change is thought to occur because electrons in those states have lower energies than those in the conventional electron gas.

Two collaborations this summer reported seeing new line (besides the ones due to the incompressible liquid states) in the luminescence spectra from heterojunctions. One collaboration involved researchers from the Max Planck Institute in Stuttgart, the Max Planck Institute-High Magnetic Field Laboratory (Grenoble), the Institute for Solid State Physics (Chernogolovka, USSR) and CNRS (Grenoble)*; the other involved researchers from Oxford University and Philips Re-

search Laboratories (Redhill).⁵

In the Max Planck-Chernogolovka-CNRS experiment the new luminescence line occurs at ν values smaller than $1/5$ as well as at ν values a little larger than $1/5$. The intensity of the new peak is presumed to be zero at $\nu = 1/5$, but it grows, and that of the peak due to the incompressible state at $\nu = 1/5$ diminishes, as ν changes away from $1/5$. For a given ν value at which the new peak occurs at some low temperature, the peak's intensity decreases to zero at a critical temperature when the sample is warmed. Finally, the wavelength at the peak intensity of this line is longer than that of the line due to the incompressible liquid state at $\nu = 1/5$, which implies that the new peak arises from electrons whose energies are lower than those in the incompressible quantum liquid state. These features of the new luminescence line suggest that electrons in magnetic fields close but not equal to the value for $\nu = 1/5$ form a new ground state that is different both from the incompressible state at $\nu = 1/5$ and from the conventional electron gas in a magnetic field, and that this state has a lower energy than either of those states. The experimenters identify the new state with the Wigner crystal.

The second, Oxford-Philips group finds that the appearance of a new feature in their luminescence studies correlates with the onset of out-of-phase conduction—that is, it first occurs at the same temperature and ν values as that onset. Current in a normal electron system is in phase with the applied voltage, and out-of-phase conduction has not been reported in the quantum Hall effect. The group therefore regards the data as indicative of a new phase of the electron system. Once again, by elimination, the new phase is identified with the Wigner crystal.

Why now?

If the Wigner crystal has at last been observed, one might ask, why did it

take so long? After all, the incompressible quantum liquid state at $\nu = 1/3$ was reported in 1983. An important factor in the success of the recent experiments, Stormer told us, is the sample quality. One measure of the “goodness” of a sample is the electron mobility. For example, the 1983 samples in which the fractional quantum Hall effect was discovered had electron mobilities of $80\text{--}100 \times 10^3 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1}$. Electron mobilities in samples used in the recent experiments were 50–100 times higher. Since inhomogeneities and imperfections in the samples scatter electrons and increase the resistivity, the higher the electron mobility is, the easier it is to unravel new features—in, for example, the resistivity data—that arise from electron-electron interactions.

Hints of an insulating phase around $\nu = 1/5$ began to appear in measurements of the temperature dependence of the resistivity about two years ago.⁶ According to Tsui, “In spite of the evidence for thermally activated conduction indicative of an insulating phase, it has been extremely difficult to tell if the insulating state is indeed due to the pinning of a solid.” A group of experimenters claim, however, that their 1988 experiment gave indirect evidence for the Wigner crystal. Eva Andre (Rutgers University), Gérard Deville, Christian Glatthli, Francis (Tito) Williams (all at Saclay), Etienne Paris and Bernard Etienne (CNRS, Bagneux) measured rf absorption by a two-dimensional electron gas in a strong magnetic field.⁷ They observed strong resonance absorption at some frequencies which they identified with those of the normal modes of the electron solid.

Most experimenters challenge that interpretation even today. However, Robert Clark (now at the University of New South Wales, Sydney) told us that recent experiments show a strong correlation between the onset of threshold behavior in the current-voltage characteristics and the onset

of resonance absorption.⁸ He thinks that both effects can be understood if one assumes that the electron solid is not perfect but is broken into domains by the disorder in the substrate. As true long-range translational order cannot exist at nonzero temperatures in two dimensions, experimenters and theorists are busy trying to understand the new phase—already sometimes referred to as the Wigner glass, in analogy with other phases whose solid-like properties arise from imperfections and disorder. Hopefully any new understanding of the phase will also resolve the two-year-old controversy regarding the significance of the resonance absorption experiment.

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CONSTRUCTION PROPOSALS TAKE AIM AT TOP AND BOTTOM QUARKS

Cornell and Stanford are competing to build a B-meson “factory,” and Fermilab wants to build a new injector ring for its Tevatron Collider. These are the two principal new construction initiatives proposed by the US high-energy-physics community for completion by the middle of this decade. They are intended to address the two

remaining “dangling particples” of the spectacularly successful standard model of the elementary particles: Understanding the violation of *CP* symmetry would seem to require a new source of B mesons much more profuse than the existing electron-positron rings; and determining the properties of the (still missing) top

quark in adequate detail appears to necessitate an order of magnitude increase in the luminosity of the Tevatron proton-antiproton collider.

The assault on the unfinished business of the standard model has been enlivened in recent months by new experimental results. High-statistics studies of the intermediate vector