erally adduced in favor of "unwinding the program": Unavoidable energy dissipation occurs only where information is discarded, that is, when resetting the output register (or the measuring apparatus) to the standardized state; "after a long computation the final state of a reversible computer has many more balls [the information-bearing particles in the Fredkin–Toffoli billiard-ball model] whose state depends on the computation"[3]; and after unwinding the program, the input register can be "uncopied," without dissipation, against a copy of the input information preserved there for just this purpose. However, if the second argument were correct (even though every reversible computing step is a 1:1 imaging process!) this computation would reduce entropy (or generate negentropy), evidently by taking heat from the environment. The computer would get colder the longer it ran. During unwinding, this negentropy certainly would get lost again. But with Landauer's dissipationless copying process we can do much better by providing a resetting store at the output (with all bits in the 0 position). Copying this "information" into the output register after computation is completed involves precisely the same, presumedly dissipationless copying steps Landauer described in reference 3. Evidently, his copying process makes it possible to discard information without dissipating energy. (Unwinding the program then is just a waste of time and energy, and there is no need for extra hardware to store all the input information.)

In conclusion, the question arises of whether "reversible computation" is another of those "many episodes" Landauer so correctly describes in his Opinion column, where "the advocates are . . . carried away by their enthusiasm" and the skeptic will not be "invited to the conferences, which the proponents . . . dominate." In all generality, the skeptics are simply those who take the trouble to think a few steps further than the enthusiasts, who try to ignore everything that does not fit into their (frictionless) dreams.

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LANDAUER REPLIES: My Opinion column provided a conservative assessment of some logic technology proposals and undoubtedly appears controversial to their respective advocates. Eginhart Biedermann uses this as an opportunity to return to a different and earlier debate already published in *Nature*. The earlier debate concerned a much more fundamental and conceptual question: What is the minimal energy dissipation requirement imposed on computation by the laws of physics? Biedermann takes issue with Charles Bennett's notion of reversible computation, first published in 1973. Bennett demonstrated that computation can be carried out with an energy dissipation *per step* that can be reduced to any desired extent, if we are willing to compute sufficiently slowly. This concept has been confirmed and elaborated by a great many subsequent investigators with different backgrounds and viewpoints, including the late Richard Feynman.[1] Bennett's work has been labeled "epoch-making."[2] I do not believe that reversible computation requires a detailed defense against *all* of Biedermann's critique, and cite below three recent items to lead to the citation trail.[3]

I do suggest that the reader of Biedermann's critique keep two items in mind. First of all, reversible computation as viewed by Bennett and by me is not *totally* without dissipation, as was claimed for one of the proposals that my Opinion item analyzed. Additionally, Biedermann states that for computation with a roughly predictable execution time, a limited total energy expenditure, say, $100kT$, is required. I do not consider the exact energy expenditure significant; the key point is that the expenditure is not proportional to the number of elementary logic functions carried out during a long computation. But why should we even require a computation to be characterized by a "predictable passage time"? Even for today's practical computers, which have a well-defined execution time per step, the number of successive logic steps required to carry out a program is, in general, not predictable. Finally, Biedermann ignores the fact that some reversible computer proposals, such as that of Konstantin Likharev using Josephson junction circuits,[4] are clocked just as actual current computers are.

The occasional published dissent that, like Biedermann's, still considers reversible computation to be excessively optimistic is balanced on the other side by the proposal of Eiichi Goto and his colleagues asserting that the special precautions invoked in reversible computation are not needed.[5] Reversible computation uses logic functions at every step that are one-to-one and do not discard any information. Goto and his colleagues claim that this is not essential. In a fashion typical of critics of reversible computation, Biedermann analyzes the energy requirements of his own notion of minimally dissipative computation. Once again he tells us that static friction is essential, but does not tell us what is wrong with Likharev's scheme, which clearly avoids static friction.

In his final paragraph, Biedermann suggests that the exponents of reversible computation have been carried along by uncritical enthusiasm. Actually, reversible computation is a somewhat counterintuitive notion on first exposure, as demonstrated by Biedermann's repeated objections. Feynman, at a 1981 workshop, was the only one I have ever seen who caught on immediately. My own history was very different. When I first heard from Bennett about his evolving ideas, in 1971, I was totally skeptical. After all, this was a major departure from my own earlier publications. It took me six months to become convinced. It is now, however, almost two decades and many papers later, and it should no longer be that difficult!

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Wigner Distribution Malfunction

I am writing to point out some minor mathematical inconsistencies in the otherwise interesting and informative article on squeezed and antibunched light by Malvin C. Teich and

Bahaa E. A. Saleh (June, page 26).

The coherent state localized at $\langle x\rangle$ and $\langle p\rangle$ is properly described by the (normalized) wavefunction

$$\psi(x) = (2/\pi)^{1/4}\ \exp(2i\langle p\rangle x) \times \exp[-(x-\langle x\rangle)^2]$$

When this wavefunction is inserted into the Wigner phase-space distribution function, defined as

$$W(x,p) = \frac{1}{\pi}\int \psi^*(x+\frac{1}{2}y)\ \psi(x-\frac{1}{2}y)\ \exp(2ipy)\ \mathrm{d}y$$

the result given by Teich and Saleh is obtained, namely

$$W(x,p) = (2/\pi)\exp[-2(x-\langle x\rangle)^2] \times \exp[-2(p-\langle p\rangle)^2]$$

It is easily shown that the above definition of $W(x,p)$ properly yields $|\psi(x)|^2$ when integrated with respect to p, and $|\varphi(p)|^2$ when integrated with respect to x. The "momentum" wavefunction corresponding to $\psi(x)$ is defined here by

$$\varphi(p) = \left(\frac{1}{\pi}\right)^{1/2}\int \exp(-2ipx)\ \psi(x)\ \mathrm{d}x$$

The extra factors of 2 that appear in the above formulas can be traced back to the commutation rule $[\hat{x},\hat{p}] = i/2$, from which it follows that an appropriate representation of the "momentum" operator is $\hat{p} = (i/2)\,\partial/\partial x$, and the wavefunction of a momentum eigenstate with momentum p is

$$\psi_p(x) = (1/\pi)^{1/2}\ \exp(2ipx)$$

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TEICH AND SALEH REPLY: The definition of the Wigner distribution function used in our article should indeed be modified, as John Philpott points out. The results presented in the article are not affected by this error, however.

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Angular Momentum Quantization Qualm

In his news story about "anyons" (November 1989, page 17) Anil Khurana apparently makes the *general* statement that angular momentum is not quantized in two spatial dimensions. In the *absence* of electromagnetic fields like flux lines, I find this hard to reconcile with the superposition principle and the probability interpretation of quantum mechanics. If one writes the wavefunction of a single spinless particle in polar coordinates ρ and φ, an arbitrary normalizable function $f(\rho)$ is an eigenfunction with angular momentum zero, while $f(\rho)\exp(im\varphi)$ has angular momentum $\hbar m$. If one considers a linear superposition of the two wavefunctions, the corresponding probability density is given by $2|f(\rho)|^2\,[1+\cos(m\varphi)]$. As probabilities should be single-valued, the quantization of angular momentum follows without invoking the single-valued-ness of the wavefunction as the starting point. This argument holds in two as well in higher spatial dimensions. The reasoning given for the quantization of angular momentum in integer units for "normal" (non-fractional-statistic) particles shows that the description of anyons has to involve a superselection rule for states of different orbital angular momentum.

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'Doc' Draper Praised; A-Bomb Reappraised

It is unfortunate that Brian Reid (December 1989, page 101) was troubled by the fact that the National Academy of Engineering decided to name an award honoring engineers and technologists for "contributing to the advancement of human welfare and freedom" after Charles Stark Draper. It is even more unfortunate that Reid did not know "Doc" Draper.

The Charles Stark Draper Prize was established and endowed at the request of the Draper Laboratory because we think it a fitting tribute to Doc's memory and his contributions to engineering and technology. We intend that the prize will focus world attention on the important work of engineers in the same way that the Nobel Prize now focuses attention on accomplishments of scientists.

It is perhaps tragic that Reid does not recognize the contributions to "the advancement of human welfare and freedom" of technologically superior weapons developed to deter war. One of the important lessons of history is that the scourge of war is most likely to occur if free nations are *not* adequately prepared for it. We at Draper Laboratory are proud of our contributions to national defense and consider that work among the most noble in the engineering profession. So did Doc Draper.

It is also unfortunate that Reid apparently does not recognize how useful some engineering achievements initially developed for defense have been for society at large. Mechanical heart valves, silicon carbide ceramics, Mylar, flameproof epoxy paint, cordless tools, graphite composite materials, self-contained breathing apparatus, freeze-dried food, microwave technology, nuclear power, pacemakers, helicopters, electric analog computers and nuclear medicine are just some examples.

Ironically, Reid feels the Greek mathematician, physicist and inventor Archimedes would be a much worthier person for the academy to name a prize after. I say "ironically" because while Archimedes made original contributions in geometry and mathematics and founded the fields of statics, hydrostatics and mathematical physics, he also invented mechanical devices useful both in peace and in war and the defense of his society—just as Doc did.

In 214 BC, when Archimedes's native city of Syracuse was besieged by the Roman general Marcus Claudius Marcellus, the defense of the city was aided by military machines designed by Archimedes—including catapults, missile throwers and grappling hooks (*Encyclopedia Americana*, 1986). Legend has it Archimedes also devised concave mirrors that burned Roman ships by concentrating the Sun's rays on them.

Thus Archimedes made significant contributions to the advancement of human welfare and freedom, at least from the perspective of the Greeks, as Doc Draper did through his numerous engineering developments for his own nation. The achievements of both men had far-reaching effects on all aspects of their respective societies. I think Doc would be quite pleased with the parallel, and to be in such rich company.

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Contrary to Brian Reid, I feel that the citation "contributing to the advancement of human welfare and freedom" precisely describes the career of my late friend Charles Stark Draper.

Most of today's airline passengers are guided to their destinations by his Inertial Navigation System, which also took the Apollo astronauts to the Moon. As the NASA history reports, Charlie volunteered to operate it himself if the astronauts couldn't be taught to do so!

The last time we met—here in Sri