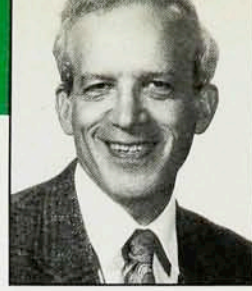


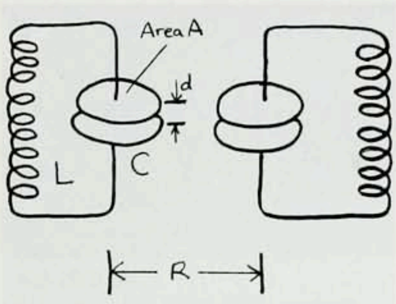
# WITH APOLOGIES TO CASIMIR

Daniel Kleppner

The Casimir effect, in case you have forgotten, is an attraction between conducting planes that arises from the vacuum energy in the intervening space. The force is too weak to command great attention—in fact, it is barely detectable—but Hendrik Casimir's line of reasoning has nevertheless been extraordinarily productive. By encouraging a rather literal view of vacuum fluctuations, his approach unifies and simplifies phenomena such as the van der Waals force, resonance dispersion and the effects quantum electrodynamics produces in a cavity, such as inhibited spontaneous emission and vacuum Rabi oscillations. On the cosmic scale of things these might seem like small potatoes, but Casimir's thinking also underlies the most plausible explanation yet for Creation—namely, generation of the universe from vacuum fluctuations.

Casimir's papers on vacuum forces and the van der Waals interaction[1] are not easy reading because the mode structure of an extended system is a trifle complicated mathematically. However, the physically important results practically pop out if one limits the system to two modes. So, here—with apologies to Casimir for doing crudely today what he did elegantly more than 40 years ago—is a matchbook calculation of the van der Waals force and some of its cousins.

Consider a system composed of two harmonic oscillators—identical $LC$ circuits will do nicely. The energy of each oscillator is given by $(n + \frac{1}{2})\hbar\omega_0$, where $n$ is its occupation number and $\omega_0 = 1/\sqrt{LC}$. The oscillators are in their ground states, located so far apart that any interaction is negligible. Their energy is $E_0 = \frac{1}{2}\hbar\omega_0 + \frac{1}{2}\hbar\omega_0$. Suppose next that the oscillators are arranged as shown in the figure below, separated by some distance $R$ and interacting via their electric dipole fields. The degeneracy




of the two modes is split, and the natural frequencies become $\omega_\pm = \omega_0\sqrt{1 \pm \kappa}$, where a simple calculation shows that the coupling coefficient $\kappa$ has the value $Ad/4\pi R^3$. (Out of respect for Casimir, who does not much care for the SI system, I am using Gaussian units.) The system's energy is now $E' = \frac{1}{2}\hbar\omega_+ + \frac{1}{2}\hbar\omega_-$. The interaction energy is

$$\Delta E = E' - E_0 = \frac{1}{2}\hbar\omega_0(\sqrt{1+\kappa} - 1) + \frac{1}{2}\hbar\omega_0(\sqrt{1-\kappa} - 1) \qquad (1)$$

Evaluating this result to lowest order in $\kappa$ yields

$$\Delta E = -\frac{\hbar\omega_0}{8}\kappa^2 = -\frac{\hbar\omega_0}{8}\frac{\alpha^2}{R^6} \qquad (2)$$

The symbol $\alpha$, whose value equals the polarizability of the capacitor, is introduced to emphasize the similarity of the $LC$ circuit to an atomic system. Equation 2 has the same form as the usual expression for the van der Waals interaction of two atoms.

The van der Waals interaction is generally described in terms of a correlation between the instantaneous dipoles of two atoms or molecules. However, it is evident that one can just as easily portray it as the result of a change in vacuum energy due to an alteration in the mode structure of the system. The two descriptions, though they appear to have nothing in common, are both correct. Incidentally, although we considered identical oscillators, the result also holds for nonidentical oscillators, with suitable modifications of the constants.

If we had considered magnetic, rather than electric, coupling, the natural frequencies $\omega_\pm$ would have been $\omega_0/\sqrt{1 \pm \kappa}$ instead of $\omega_0\sqrt{1 \pm \kappa}$. The result would have been a van der Waals *repulsion*. Such a repulsion is not generally observed in atomic and molecular systems because magnetic dipoles are smaller than electric dipoles by roughly $\alpha = \frac{1}{137}$. However, in other coupled systems, such as an atom and a cavity, either sign of interaction can occur. If one wants to flog the argument to the point of diminishing returns, one can include retardation, terms due to higher-order multipoles, and angular effects.

The first excited state of the system, that is, a state in which one of the occupation numbers is unity, is a good deal more interesting than the ground state because it displays a variety of effects that are first order in $\kappa$. If the excitation is in the $\omega_+$ mode, for instance, equation 1 becomes

$$\Delta E = E' - E_0 = \frac{3}{2}\hbar\omega_0(\sqrt{1+\kappa} - 1) + \frac{1}{2}\hbar\omega_0(\sqrt{1-\kappa} - 1) \qquad (3)$$

Depending on which mode is excited, the two possibilities are

$$\Delta E_\pm = \pm\frac{1}{2}\hbar\omega_0\kappa = \pm\frac{1}{2}\hbar\omega_0\frac{\alpha}{R^3} \qquad (4)$$

In molecular physics this interaction goes by the name of resonance dispersion energy, "dispersion energy" being the generic term for all such two-body electrostatic couplings. Because it is first order, resonance dispersion energy can be large, large enough to bind atoms into long-range molecules—molecular states with radii of tens or even hundreds of angstroms.

**Daniel Kleppner** is the Lester Wolfe Professor of Physics and associate director of the Research Laboratory of Electronics at MIT.

The system becomes more colorful if we consider a two-level atom interacting with a resonant cavity. Provided that the excitation number is limited to unity, this system is formally identical to a pair of harmonic oscillators. In its excited state, the atom and cavity attract or repel depending on whether the system is in its symmetric or antisymmetric state. If the excitation is initially localized on one oscillator, for instance by introducing an excited atom into the cavity, the atom and cavity will spontaneously exchange energy at the rate $\Delta E/\hbar = \kappa\omega_0$. This resembles classical beating, except that one is hard pressed to identify any classical coupling. For example, dipole coupling is by no means evident, because an excited atom has no dipole moment. A number of groups are exploring the behavior of atom–cavity systems using the wizardry of laser spectroscopy, superconducting cavities and modern optics, and this spontaneous oscillation will probably be observed before too long. It already has a name—in fact, two names. Some call it a vacuum Rabi oscillation; others call it a Jaynes–Cummings oscillation. Recent research along these lines has been described by Serge Haroche and myself (PHYSICS TODAY, January 1989, page 24).

It is not difficult to imagine the effect of coupling more cavities to the atom–cavity system. As the number of normal modes increases, the initial excitation takes longer and longer to return to the atom. In the limit of a continuum of modes, the excitation never returns. Instead, we observe the ubiquitous phenomenon of spontaneous emission. From this point of view, the two-mode atom–cavity system executes what might be called reversible spontaneous emission. If the cavity is mistuned, the beat period lengthens and the energy transfer becomes less and less complete, until the atom resides permanently in its excited state. This phenomenon, which has been witnessed in the laboratory, has acquired the name of inhibited spontaneous emission.

So what have we learned? First, that one must take the vacuum seriously; second, that intermolecular forces and a number of slightly bizarre radiative phenomena, and possibly even Creation, have something in common; and third, that Casimir is pretty smart.

## Reference


1. H. B. G. Casimir, D. Polder, Phys. Rev. **73**, 360 (1948). H. B. G. Casimir, J. Chim. Phys. **47**, 407 (1949). ■