

TURBULENCE: CHALLENGES FOR THEORY AND EXPERIMENT

High-Reynolds-number flows are ubiquitous. Although many aspects of such flows have been understood phenomenologically, a systematic theory of their detailed properties requires novel experiments.

Uriel Frisch and Steven A. Orszag

Research in macroscopic classical physics, such as fluid dynamics or aspects of condensed matter physics, continues to confront baffling challenges that are by no means less demanding than those at the post-Newtonian frontiers of physics that have been explored since the beginning of this century. This is so even though the basic equations of macroscopic classical physics are known—indeed, have been known for centuries in many cases. Chaos and nonlinear dynamics are examples of the topics that pose new challenges to our understanding of macroscopic classical systems. Turbulence, a phenomenon related to but distinct from chaos, and having strong roots in engineering, has been increasingly in the focus of physics research in recent years.

Turbulence occurs in a very wide variety of flows, ranging from the mixing of cream in a coffee cup to the dispersal of pollutants in the atmosphere, from the formation of galaxies in the early universe to thermal convection in stars, from flows around automobiles, ships and aircraft to combusting flows in turbomachinery. Although there is no unique mathematical model that encompasses all these situations, it is generally recognized that a prototypical case is that of turbulence in an incompressible fluid where the additional complexities due to sound waves, chemical reactions and so on are avoided. We shall concentrate on that prototype in this article.

For incompressible flow, the fundamental dynamical equations are the Navier-Stokes equations:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho_0} \nabla p + \nu_{\text{mol}} \nabla^2 \mathbf{v} \quad (1)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (2)$$

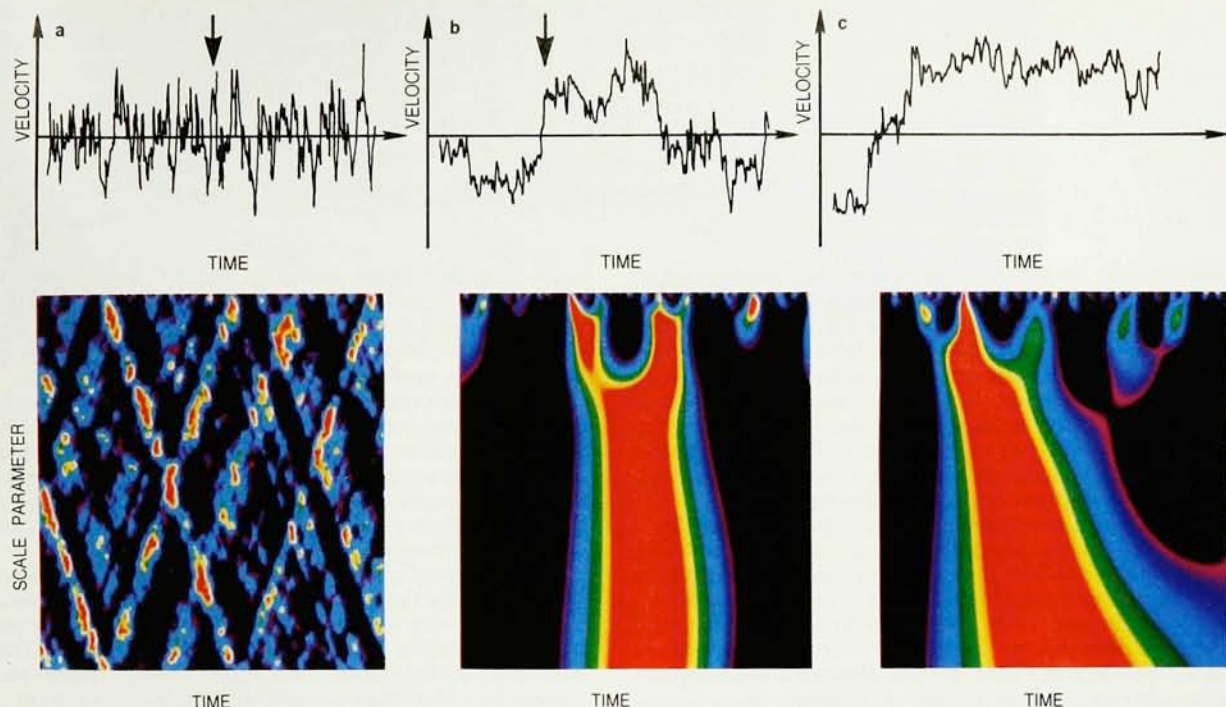
where $\mathbf{v}(\mathbf{x}, t)$ and $p(\mathbf{x}, t)$ are the fluid velocity and pressure, respectively, at point $\mathbf{x}$ and time t ; ρ_0 is the (constant)

density; and ν_{mol} is the kinematic molecular viscosity. This system of coupled partial differential equations must be supplemented by initial and boundary conditions. That the fluid velocity should vanish at a rigid wall is an example of a boundary condition.

It is customary to classify turbulent flows in terms of the so-called Reynolds number, which is a nondimensional measure of the nonlinearity in equation 1 and is defined as $R = UL/\nu_{\text{mol}}$, where U is a typical velocity and L is a typical length scale. If the Reynolds number is not too large, the flow will be *laminar*, in the sense that it will display regular and predictable variations in both space and time. As the Reynolds number increases, flows typically undergo a sequence of instabilities until, at some large enough value of the Reynolds number, they become fully turbulent. In this article, we concentrate on the problem of very-high-Reynolds-number turbulence. High R values tend to characterize real-life problems, because most fluids have $\nu_{\text{mol}} \sim 10^{-2} - 10^{-1} \text{ cm}^2/\text{sec}$, while most flows typically have velocities that are much larger than 1 cm/sec and occur on length scales much larger than 1 cm.

The characteristic of high-Reynolds-number flows that makes their study so difficult is the huge number of dynamically significant scales of motion. The scales vary from the large ones at which turbulent energy is input to small ones where dissipation into heat is effected. Indeed, as we discuss in the next section, the number of active

Uriel Frisch is Directeur de Recherche CNRS at the Observatoire de Nice, France. **Steven Orszag** is Forrest G. Hamrick '31 Professor of Engineering and Professor of Applied and Computational Mathematics at Princeton University.



Richardson cascade, in which “big whirls have little whirls which feed on [the big whirls’] velocity,”⁴ visualized using the wavelet transform. Panel **a** shows the time dependence of the fluid velocity at a point in a fully turbulent fluid (top) and the corresponding wavelet transform (bottom). The amplitude of the wavelet transform is color coded: For a given value of the scale parameter a , which is a measure of the “filter width” applied to the data, the transform amplitude is negative or zero in black regions and large and positive in red regions. The parameter a decreases by a factor of 200 from bottom to top. In **b** the data near the arrow in **a** are shown with the time and scale magnified by 20, and in **c** there has been a further 20-fold magnification near the arrow in **b**. Under the conditions of the experiment, the time variation of the velocity is equivalent to its spatial variation, so the successive forkings seen in the lower panels as the scale parameter a is varied are suggestive of a fractal branching in the Richardson cascade. (Courtesy of Emanuel Bacry, Gilles Grasseau, Alain Arnéodo, Centre de Recherche Paul Pascal, Talence, France.⁶ The data were obtained at the Modane, France, S1 wind tunnel). **Figure 1**

degrees of freedom in a turbulent flow is of order $R^{9/4}$ per unit volume L^3 . Therefore at a Reynolds number of 10^8 (which is modest by geophysical standards), there are on the order of 10^{18} active degrees of freedom per L^3 . This enormous complexity is compounded by the absence of a distinct separation between scales, in the sense that turbulent flows exhibit excitations from their largest scales down to the smallest.

There is no consensus on what will constitute a “solution” to the turbulence problem. On the one hand, engineers are mostly interested in mean (average) properties of these random flows, such as mean velocity profiles, mean wall stresses and mean pressure gradients. On the other hand, from a basic physical point of view, it is important that we understand the nonlinear physical processes responsible for those mean properties as well as the details of motions across the broad range of excited scales. For example, in tornadoes it is necessary to understand the details of locally intense circulations. But such an understanding may not be possible to achieve using traditional statistical turbulence theory, which regards the space and time dependence of, say, the velocity as a random field and deals only with its average properties.

In this article, we survey some of the key ideas about

turbulent flows that have evolved over the last century, and we shall describe some of the challenges for the present and future (see reference 1 for background and further details). As we shall see, there have been substantial advances in our understanding and we are now capable of making significant and reliable predictions of various characteristics of turbulent flows. Yet fundamental challenges remain for both theory and experiment that will require new, and probably quite novel, ideas for their solution. We believe that the effective interplay among theory, computation and experiment is the key to clarifying the physics of turbulence.

Four basic concepts

Randomness, eddy viscosity, cascade and scaling are four of the basic concepts that are at the root of early attempts to understand turbulence.

Randomness. Turbulent flows usually appear extremely complex and unpredictable. In the late 19th century, Osborne Reynolds suggested that such flows might best be described in terms of their average, rather than their detailed, properties. The further development of this idea led to the statistical theory of turbulence.

Turbulent flows become random, according to the classical explanation, because instabilities intrinsic to the

flow amplify the fluctuations in, say, the forces acting on the flow, the conditions at the boundary, the thermal noise and so on. The idea that sensitive dependence on initial data might give rise to chaotic behavior was already familiar to Henri Poincaré nearly a century ago.² There has been much research in recent years on how purely deterministic and causal systems can have chaotic solutions, and we now have a mathematical understanding of how minute inaccuracies in the initial conditions can make a flow unpredictable in the distant future.

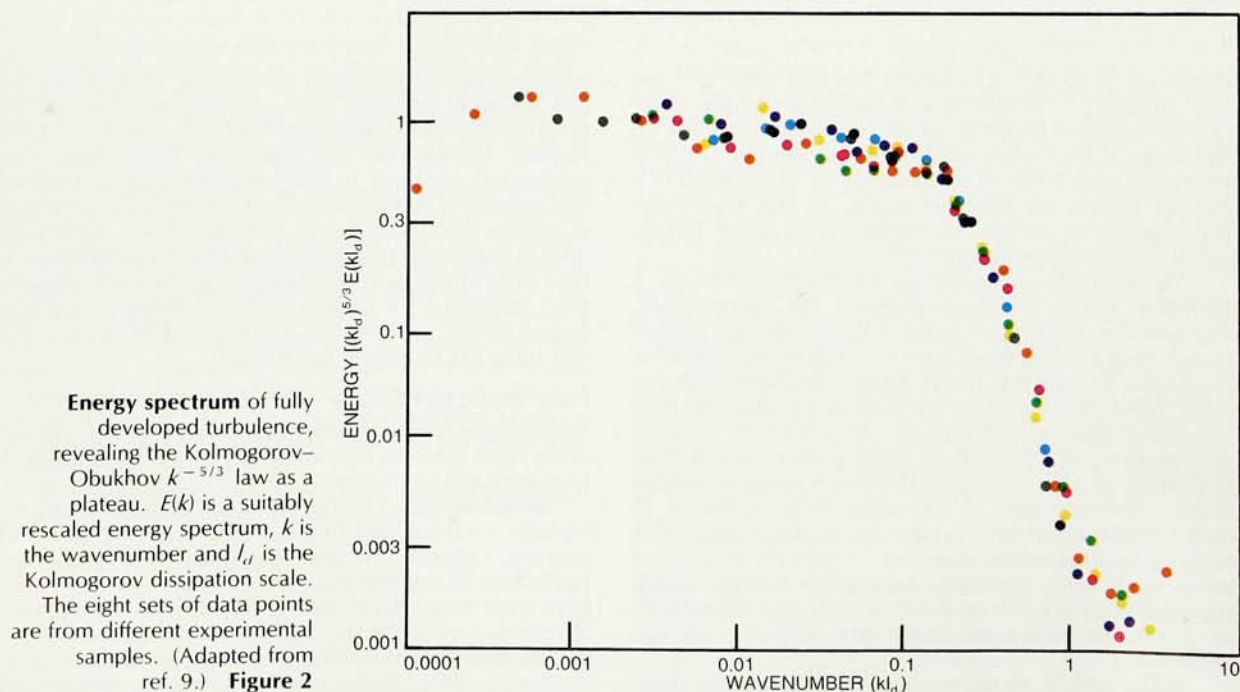
For many years it was thought that a picture proposed by Lev Landau was generic for turbulent flows. According to that picture, a flow would undergo an infinite sequence of bifurcations (instabilities) before it became unpredictable and chaotic. For chaotic behavior in dynamical systems, however, we know, since the seminal work of David Ruelle and Floris Takens, that such behavior can arise after only a finite number of bifurcations.

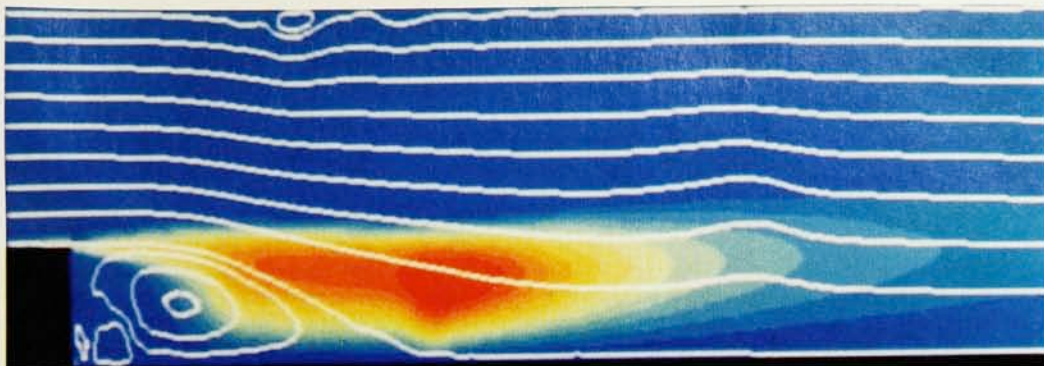
From a practical point of view, the impact of the modern theory of dynamical systems has been greatest in problems in which there is only temporal chaos and the dimension of the attractor is not too large (of order 10 or less). (An attractor is the set of points in phase space that the system approaches at large times.) With higher-dimension attractors, it is extremely difficult both to measure the attractor dimension and to use the concepts of dynamical systems theory to make predictions. However, semi-empirical methods have been used to study dynamical systems with moderately many (of order 10^2) degrees of freedom and to model aspects of turbulence

close to walls.³ More generally, though, high-Reynolds-number flows are believed to have attractors with large dimensions, and it seems probable that present dynamical systems concepts may have to be extended significantly before we can predict the properties of turbulent flows.

Eddy viscosity. Turbulent flows are both strongly constrained and dominated by other, perhaps considerably simpler, phenomena. Indeed, many important features of turbulence dynamics can be described in terms of eddy viscosity, a concept introduced over a century ago by Joseph Boussinesq and developed later by Geoffrey Taylor and Ludwig Prandtl.

The concept is based on an analogy with statistical mechanics. By the late 19th century, it was understood, through the work of Maxwell, Boltzmann and others, that molecular motion has macroscopic consequences. The existence of transport coefficients, which relate the transport, or flux, of some quantity such as heat to the gradient of the mean value of that quantity, is an example of the macroscopic manifestation of molecular motion. The work of Sydney Chapman, David Enskog and others showed that when there is a separation of scale between molecular motions (which typically are characterized by the mean free path) and hydrodynamic motions (which typically occur on macroscopic scales), the principal effect of the molecular motions on the large hydrodynamic scales is to cause the dynamics to be diffusive. An example is momentum diffusion, which smooths velocity gradients on the hydrodynamic scales. Transport coefficients such as ν_{mol} are typically on the order of the velocity of thermal





Stationary turbulent flow over a step for a Reynolds number R of 40 000. The step (black) extends infinitely to the left. The flow was calculated using the renormalization group ideas developed in reference 12. The curves are mean flow streamlines, while color indicates turbulence intensity. (In a stationary turbulent flow the mean properties are time independent.) The calculation used no *ad hoc* parameters or experimental input and is a prototypical example of the successful use of theory and robust computer codes to make predictions of large-scale turbulent flows in complicated geometries. Such predictions are needed in engineering design. The small vortices to the immediate right of the step have been observed experimentally. (Courtesy of George Karniadakis, Princeton University, and Alexander Yakhot, Ben-Gurion University.) **Figure 3**

molecular motion multiplied by the mean free path.

Prandtl had the inspired idea that turbulent dynamics might be regarded in an analogous way to molecular motion, in the sense that small-scale eddies could be thought to act on large-scale eddies in a diffusive manner. He argued that the appropriate diffusion coefficient, called the eddy viscosity, would be on the order of the root-mean-square value v_{rms} of the fluctuating velocity times a length scale l , called the mixing length. The eddy viscosity ν_{eddy} is then responsible for smoothing gradients in the mean velocity.

The ratio $\nu_{eddy}/\nu_{mol} \approx v_{rms}l/\nu_{mol} \equiv R$, so that the molecular diffusion rate is enhanced by a factor on the order of the Reynolds number to give the eddy diffusion rate in a turbulent flow. As a consequence, the transport of momentum, heat, particles and so on is enhanced in turbulent flows. The enhancement of, for example, momentum transport implies that velocity gradients are smoothed out much more rapidly in a turbulent flow than in a laminar flow. Similarly, particles or some similar passive scalar contaminant such as a droplet of a dye introduced into a turbulent flow will diffuse at rates that may be orders of magnitude larger than molecular rates. Indeed, Taylor in 1915 observed markedly enhanced transfer rates in turbulent flows, which he was able to explain in terms of eddy diffusion coefficients.

The enhanced momentum transport in turbulence also leads to enhanced kinetic energy dissipation. Eddy viscosity ideas are useful in describing this effect. The local rate of viscous dissipation of energy per unit mass in an incompressible flow is given, on average, by $\mathcal{E} = \nu_{mol} |\nabla \mathbf{v}|^2$. To evaluate this formula properly, it is necessary to estimate accurately $|\nabla \mathbf{v}|$, which, as we shall see below, is dominated by the very smallest scales of turbulent motion. If, however, the details of the small-scale motions are not observable, as for flows on distant planets, or are not available, as would be the case if an experiment recorded only the mean velocity, the use of an eddy viscosity together with estimates of $|\nabla \mathbf{v}|$ on the largest scales of motion allows us to get a good estimate of the average dissipation rate. If we consider $\mathbf{v}$ on a typical scale of an energetic eddying motion, then $|\nabla \mathbf{v}|$ is of order v_{rms}/L , where L is the scale of the eddying motion. But

dissipation on these scales occurs through the eddy viscosity $\nu_{eddy} \approx v_{rms} \cdot L$. So we may estimate the local rate of energy dissipation as $\mathcal{E} \approx v_{rms} L (v_{rms}/L)^2 \approx v_{rms}^3/L$. Observe that the molecular viscosity does not appear in this estimate of $\mathcal{E}$. Such estimates of the local rate of energy dissipation in turbulence, and their approximate independence of the molecular viscosity, are reasonably well supported by experiment and computer simulations. Our estimate for $\mathcal{E}$ can be reinterpreted if we rewrite it as $\mathcal{E} \approx v_{rms}^2/(L/v_{rms})$. Thus the energetic eddying motions, which have kinetic energy per unit mass of order v_{rms}^2 , are dissipated in a time L/v_{rms} , which is the time it takes for an eddy of size L with typical velocity v_{rms} to flip over.

Ideas about viscosity can be systematically justified only when there is motion on widely separated scales, whereas in turbulent flows it is generally believed that the dominant interactions are between contiguous, rather than widely separated, scales. So it is at first somewhat surprising that the idea of eddy viscosity turns out to be so useful. Amazingly, however, eddy viscosity concepts underlie the entire range of models for turbulence, from engineering approximations to the most sophisticated analytical theories. Many of these models and theories can be viewed in terms of generalized eddy viscosity coefficients that may be nonlocal in space and time. (See the section below on statistical theories.)

Cascade. The notion that turbulent flows are hierarchical and involve entities, usually loosely referred to as eddies, of varying sizes is a common idea that has been captured by artists over the centuries, most notably Leonardo da Vinci. This common notion underlies the concept of cascade, the third key element of turbulence theory. The modern concept of cascade probably owes its origins to Lewis Richardson,⁴ who took inspiration from observations of clouds and from Jonathan Swift's verse⁵:

So, nat'ralists observe, a flea
Hath smaller fleas that on him prey;
And these have smaller yet to bite 'em,
And so proceed *ad infinitum*.

In Richardson's hierarchical model of turbulence, the largest eddies are produced by the forces driving the flow. The large eddies are unstable and produce eddies of a somewhat smaller size, which themselves become unstable

and generate eddies of even smaller size. This process continues until eventually molecular viscosity is able to suppress further cascading (see figure 1). Richardson's qualitative picture of turbulent flows has dominated the thinking among fluid mechanics experimenters and theorists for over half a century.

Scaling. The hydrodynamic equations retain the basic invariance and conservation principles of mechanics. In fact, the equations represent the fundamental physical laws of conservation of mass, momentum and energy, and they maintain the basic symmetries of Newtonian physics, including translation, rotation and Galilean invariance. The corresponding symmetries of the macroscopic equations have interesting consequences, which will not be fully discussed here. Rather, we will focus on scale invariance, a remarkable new symmetry that emerges macroscopically in the limit of infinite Reynolds number and which is the basis of Andrei Kolmogorov's scaling theory.

Indeed, if we ignore viscosity, the basic incompressible Navier-Stokes equations are invariant if we simultaneously scale the distance by λ , velocity by λ^h , and time by λ^{1-h} , where h is an arbitrary scaling exponent. Kolmogorov's 1941 theory of scaling in turbulence⁷ rests on three assumptions. First, the scale invariance of the zero-viscosity Navier-Stokes equations is assumed to hold in the statistical sense—that is, average quantities are assumed to be scale invariant, whereas detailed structures need not be. Second, a *finite* flux of energy $\mathcal{E}$ is assumed to flow from large scales, where the turbulence is produced, to small scales, where it is dissipated, in the limit of R tending to infinity. Third, the energy flux $\mathcal{E}_l$ through scale l is assumed to depend only on flow quantities local to the scale l , in particular on l and the velocity v_l of eddies of size l . Therefore, since $\mathcal{E}_l$ has the dimensions of energy per unit mass per unit time, dimensional analysis gives $\mathcal{E}_l \approx v_l^3/l$, from which it follows that $\mathcal{E}$ scales as λ^{3h-1} . Then scale invariance of $\mathcal{E}$ implies that $h = 1/3$.

Several immediate consequences of Kolmogorov's 1941 theory are:

▷ $v_l \approx \mathcal{E}^{1/3} l^{1/3}$, where $\mathcal{E} \equiv \mathcal{E}_l$ also equals the rate of viscous dissipation of energy per unit mass. Thus $\nabla v_l \approx \mathcal{E}^{1/3} l^{-2/3}$ is dominated by small scales and would be infinite if the cascading were not cut off at some suitably chosen small scale.⁸

▷ The structure function of order p , defined as the average of the p th power of velocity increments δv_l measured over distances l , scales as $(\delta v_l)^p \approx \mathcal{E}^{p/3} l^{p/3}$. We say that the scaling exponent is $\zeta_p \equiv p/3$.

▷ The energy spectrum, which is directly related to the Fourier transform of the second-order structure function, satisfies the Kolmogorov-Obukhov law $E(k) = C_{KO} \mathcal{E}^{2/3} k^{-5/3}$, where k is the wavenumber and C_{KO} is called the Kolmogorov-Obukhov constant. Scaling does not predict a value for this constant.

▷ The eddy viscosity at scale l is given by $\nu_l \approx l v_l \approx \mathcal{E}^{1/3} l^{4/3}$.

Obviously, basic scaling symmetry, which is assumed to derive the above results, is broken at large and small

scales. At large scales, the mechanisms producing the turbulence—for example, boundaries or external forces—generally single out a scale L . For Kolmogorov's scaling to hold at some scale l , it is therefore necessary that $l \ll L$. At small scales, molecular viscosity ν_{mol} can be ignored only if $\nu_l \gg \nu_{mol}$. Therefore, scaling requires that $l \gg l_d$, where $l_d \approx (\nu_{mol}^3 / \mathcal{E})^{1/4}$ is called the Kolmogorov dissipation scale. For $l \lesssim l_d$, molecular viscosity is important; it is at these scales that viscous dissipation occurs. The range $l_d \ll l \ll L$, in which scaling arguments apply, is called the Kolmogorov inertial range. The predictions of the Kolmogorov theory mentioned above are expected to hold *universally* in the inertial range for all high-Reynolds-number flows. Using our estimates of $\mathcal{E}$, we can evaluate the extent L/l_d of the inertial range in terms of R , namely, $L/l_d \approx \nu_{rms}^{3/4} L^{3/4} / \nu_{mol}^{3/4} = R^{3/4}$. Thus, in the Kolmogorov theory of three-dimensional turbulent flow, there are at least on the order of $(R^{3/4})^3 = R^{9/4}$ dynamically active degrees of freedom per volume L^3 .

It took nearly 20 years for the predictions of Kolmogorov's 1941 theory to be tested convincingly. In 1962, Grant, Stewart and Moilliet⁹ reported analyses of data from a very-high-Reynolds-number flow in a tidal channel in the wake of an island—since obliterated for navigational purposes—near Vancouver. They measured an energy spectrum approximating the Kolmogorov-Obukhov $k^{-5/3}$ law for over three decades of wavenumber (see figure 2). Over the years, similar results have been observed in a large number of high- R flows.

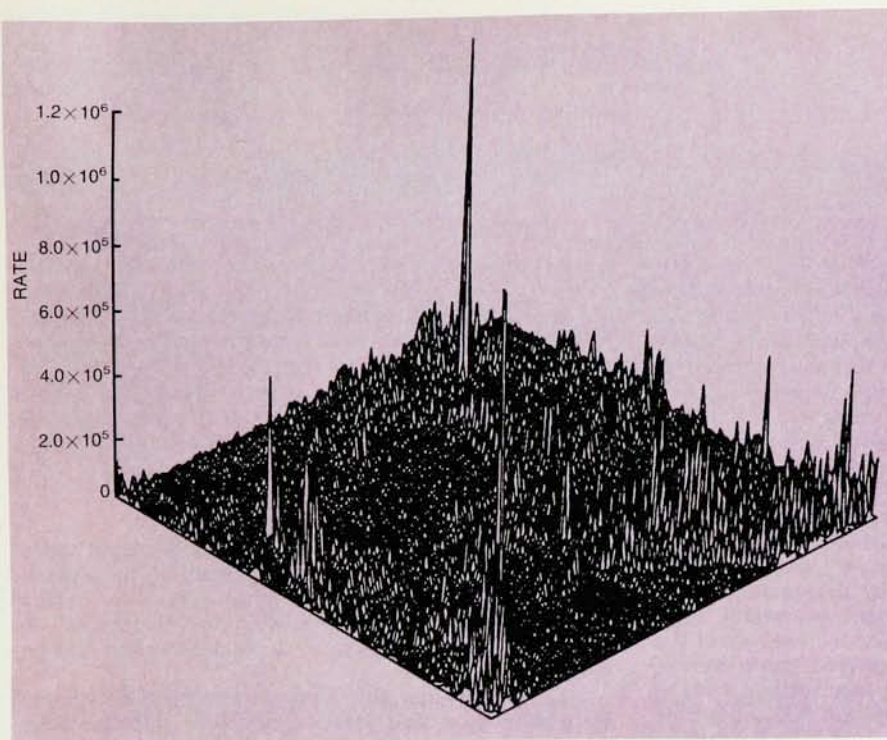
The Kolmogorov scaling theory predicts scaling exponents but not amplitudes. It must be supplemented by a more quantitative theory or by input from measurements. Kolmogorov's theory has other limitations of a more basic nature. We address these questions in the following sections.

Statistical theories

An early goal of the statistical theory of turbulence was to obtain a finite, closed set of equations for average quantities, including the mean velocity and the energy spectrum. That goal is now viewed to be unrealistic. The goal nowadays is to reduce to a manageable number the many degrees of freedom necessary to describe the flow, to determine the equations governing the dynamics of the reduced degrees of freedom, and to solve those equations analytically or numerically to calculate fundamental quantities that characterize the flow. Thus a theory may treat all or only some of the degrees of freedom statistically.

The validity of earlier statistical theories was often tested by their ability to predict Kolmogorov scaling. It is now recognized that this approach also is unrealistic. While it is very reasonable to demand that the scale invariance possessed by the basic Navier-Stokes equations should survive the (severe) approximations involved in any closure process for the equations for average quantities, that scale invariance should be viewed as an input to, rather than as an outcome of, the theory.

Closure. Modern formulations of analytic turbulence theory are mostly based on field-theoretic ideas. In



Rate of dissipation of concentration fluctuations of a scalar contaminant in a turbulent water jet. The rate across a square cross section of the jet illuminated by a sheet of laser light is shown. The extremely spiky character of the rate is evidence of the intermittency of the flow. (Adapted from R. R. Prasad, C. Meneveau, K. R. Sreenivasan, *Phys. Rev. Lett.* **61**, 74, 1989.) **Figure 4**

1958, Robert Kraichnan¹⁰ pioneered renormalized perturbation techniques for statistical turbulence theory. He showed that diagrammatic perturbation methods of the sort developed in quantum electrodynamics are directly applicable to the statistical Navier-Stokes problem.

Kraichnan showed that the analog of mass renormalization in quantum electrodynamics is the renormalization of viscosity to a spatially and temporally nonlocal eddy viscosity. Using this concept, he proposed a field-theoretic closure called the direct-interaction approximation. This was the first model for turbulence consistent with basic probabilistic requirements. In fact, the direct-interaction approximation gives the exact statistical solution to a stochastic model having many properties in common with the Navier-Stokes equations. However, solutions to the direct-interaction approximation violate the third assumption necessary for the Kolmogorov-Obukhov law in the inertial range: Instead of a $k^{-5/3}$ behavior, the direct-interaction approximation leads to a $k^{-3/2}$ inertial range spectrum. Indeed, the direct-interaction approximation misrepresents the convection of small eddies by large ones. Later, Kraichnan circumvented this difficulty by using Lagrangian-coordinate formulations. In this way he obtained very good quantitative agreement with data such as those in figure 2 over the full range of experimentally accessible scales. Without further simplifications, however, it is hard to apply theories such as Kraichnan's to complex flows such as the inhomogeneous shear flows, because their solution may then involve many more dependent and independent variables than occur in the original Navier-Stokes equations.

Renormalization group. A technically related but conceptually rather different approach is based on renormalization group techniques.^{11,12} The RG method for the Navier-Stokes equations is an adaptation of the RG methods developed in critical dynamics. When an incompressible fluid is subjected to a random force in such a way that the energy input per unit mass per unit wavenumber

is $F(k) \approx Dk^{3-\epsilon}$, with $0 \leq \epsilon \leq 1$, the dominant nonlinear interactions are between widely separated scales; actually for $\epsilon = 0$ the eddy viscosity has an ultraviolet logarithmic divergence.¹¹ (D is the amplitude of the random force.) In that case, the asymptotic leading-order Fourier-space dynamics is given by a Langevin equation $\partial u(k,t)/\partial t = -\nu(k,t)k^2 u(k,t) + f(k,t)$ with an eddy viscosity $\nu(k)$ that increases slowly with scale $l = 1/k$. The effective Reynolds number at scale l then becomes sufficiently small to allow a perturbative determination of the eddy viscosity produced on even larger scales, thus "bootstrapping" the solution. Explicitly, this bootstrap is expressed through the differential recursion relation

$$\frac{d\nu}{dk} = -\frac{AD}{\nu^2 k^{1+\epsilon}} \quad (3)$$

with an explicitly evaluated constant A . Equation 3, apart from the constant A , follows from dimensional analysis and the condition that $d\nu/dk$ is evaluated to leading (linear) order in the forcing spectrum $F(k)$. The solution constructed in this way inherits the scale invariance of the force; the scaling exponent for the physical-space velocity is $h = \epsilon/3 - 1$. It may be shown that this scaling relation, originally due to Sam Edwards, is true to all orders in ϵ , in contrast to equation 3, which is valid only to leading order in ϵ for ϵ small and positive. Thus, formally, Kolmogorov scaling is obtained by setting $\epsilon = 4$. It must be emphasized that the Kolmogorov-Obukhov inertial range that differs sharply from the regime $\epsilon \ll 1$ originally described by these RG techniques. For small ϵ , the statistical equilibrium results from a balance between the input due to the assumed forcing and a drain due to eddy viscosity. In the inertial range, both the input and the output originate from nonlinear interactions.

Victor Yakhot and Orszag¹² have recently extended RG ideas to make quantitative predictions about amplitudes in the Kolmogorov scaling regimes. Their model has a random force with $\epsilon = 4$, and they calculate the

amplitude D of the force from the energy flux $\mathcal{E}$ by a self-consistency argument. Among the results obtained to date are explicit evaluations of the Kolmogorov-Obukhov constant $C_{KO} \approx 1.6$, the von Kármán constant for turbulent boundary layers $\kappa = 0.37$ and turbulence transport coefficients for engineering modeling. Figure 3 shows a massively separated turbulent flow over a step, which was calculated using the ideas of Yakhot and Orszag—in particular, an RG modification of a so-called $K\text{-}\mathcal{E}$ transport approximation.

In contrast to many other field-theoretic models, RG models, because they involve a Langevin description with an easily evaluated eddy viscosity, are easy to use in complex flow problems. They thereby allow direct contact with the early, heuristic, but rather successful, eddy viscosity modeling of turbulence. The RG analysis of the inertial range does involve bold steps that ignore some of the subtleties of turbulence (see the next section), but the method does not have the usual *ad hoc* adjustable constants, and it may provide a rational basis for further progress.

Intermittency and fractals

Kolmogorov scaling is the backbone for the theories presented above. The scaling has received experimental verification for low-order statistical averages, as already discussed, but there is also experimental evidence suggesting that exact solutions to the Navier-Stokes equations will weakly break the scaling invariance. For example, while the Kolmogorov theory predicts that the p th-order structure function scales with exponent $\zeta_p = p/3$, experimental evidence^{1,13} incorporating data from a variety of flows suggests that ζ_p is appreciably smaller than $p/3$ for $p \geq 4$. This implies that the statistics are increasingly non-Gaussian at small scales, a phenomenon referred to as inertial-range intermittency.

One gets a more dramatic visualization of intermittency by simultaneously displaying the turbulent signals on many scales,⁶ as in the wavelet transform. (See figure 1.) If the behavior in time is representative of the full space-time structure of $\mathbf{v}(\mathbf{x}, t)$, and if the cascade of branching processes persists to the smallest scales, then in the limit $R \rightarrow \infty$ the fine scales of turbulence form a fractal set. This was first realized by Benoit Mandelbrot¹⁴ in the 1960s. He noticed that earlier work of Kolmogorov and of Evgeny Novikov and Robert Stewart had hidden geometrical content that is consistent with fractal behavior. Since then many kinematical and dynamical fractal models have been constructed.

Satisfactory geometrical models of small-scale turbulence must account for the paradox, however, that neglecting intermittency and assuring scale invariance gives a very satisfactory picture of turbulence energetics (which involves only second-order moments) and that significant corrections are needed mostly for higher-order statistics.

The multifractal model has been suggested to resolve this paradox.¹⁶ In this model, it is assumed that the flow has structures whose velocities scale as λ^h for a variety of scaling exponents h , with each structure residing on a

fractal set of dimension $D(h)$. Then the structure function of order p scales with exponent $\zeta_p = \min_h [ph + 3 - D(h)]$, which is just the Legendre transform of $3 - D(h)$, the codimension. A multifractal description of small scales of a scalar passively convected by turbulence seems to be consistent with some recent experiments (see figure 4). Multifractality also provides a useful characterization of phase-space singularities of attractors for dynamical systems.¹⁷

Vortex dynamics

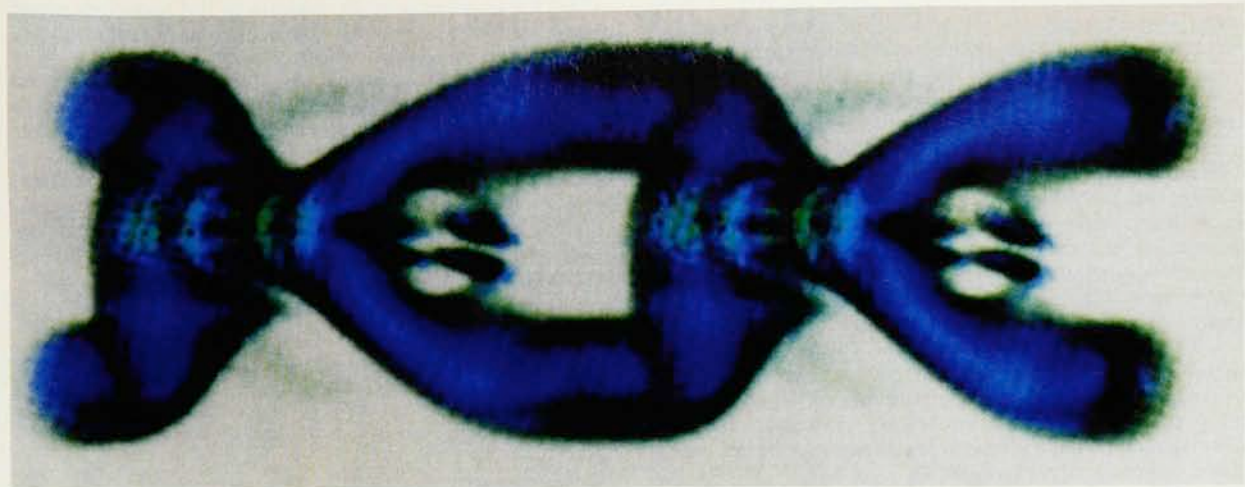
The geometrical models of intermittency discussed above successfully describe observed deviations from Kolmogorov 1941 scaling, but the dynamical mechanism for this putative symmetry breaking is still poorly understood. A key mechanistic ingredient for its understanding may be vorticity dynamics.

For inviscid flow, the classical theorems of Kelvin and Helmholtz show that vortex lines move with the fluid. (Vortex lines are curves whose tangents are parallel to the vorticity field $\boldsymbol{\omega} = \nabla \times \mathbf{v}$.) Infinitesimal segments of vortex lines expand or shrink in time; the vorticity itself changes in direct proportion to the changes in the infinitesimal vortex-line elements. In three dimensions, vorticity amplification by stretching of vortex lines and vorticity deformation by folding of vortex lines can lead to highly intermittent vortical fine-scale structures. Such structures appear as vortex tubes, sheets and blobs in numerical simulations.¹⁸ Detailed visualizations of such structures—the picture on the cover of this issue is an example—are essential for the development of a dynamical theory. We stress that a nonzero value of viscosity is necessary for the emergence of nontrivial vortex-line topologies by viscous reconnection (see figure 5).¹⁹

Vortex stretching amplifies vorticity but does not generate it. Vorticity is produced by viscosity near rigid, no-slip walls and by buoyancy effects in the interior of flows, for example. These vorticity-producing mechanisms typically control the large-scale structure of turbulent flows: The large-scale structure can be particularly prominent near walls and laminar-turbulent interfaces, and it may appear in a spatially intermittent—that is, statistically spotty—way.

In an inviscid three-dimensional incompressible flow, it has been speculated, vortex stretching may proceed catastrophically even when the flow is smooth and bounded, leading to vorticity singularities in finite time. Although such issues have been studied intensively for a number of years, they are not settled, and we cannot yet rule out the possibility of solutions to Navier-Stokes equations that are smooth and bounded for all time.

Vortex singularities, if they exist and are distributed spottily, would provide a convenient mechanism for breaking the scaling symmetry. On the other hand, scaling symmetry can also be broken if there are long-lasting organized (coherent) structures on all scales. The existence in turbulence of persistent, large-scale coherent structures is well documented. It is seen, for example, in jets, mixing layers, boundary layers, and many other three-dimensional flows. The existence of such structures in two-



Vorticity in a three-dimensional flow during topological reconnection. The vortex pairing was induced by perturbing two antiparallel vortex tubes in a low-viscosity fluid. (The low viscosity enables reconnection.) The color intensity indicates the vorticity magnitude; there is significant turbulent activity in the region where the vortex tubes merge. (Courtesy of Daniel I. Meiron, Caltech, and Michael J. Shelley, University of Chicago. Graphic simulation done using software provided by Vital Images Inc.) **Figure 5**

dimensional flows has also been documented, even though vortex stretching is absent in those flows and the physics differs dramatically from the three-dimensional case.

Persistence of coherent structures requires that, in some reference frame, the nonlinear term in the Navier-Stokes equations be smaller than the order-of-magnitude estimates of it based on the Reynolds number. For example, the effect of the nonlinear term in the Navier-Stokes equations decreases when the fluid vorticity is nearly aligned with the fluid velocity or when there is a pressure gradient: From the identity $\mathbf{v} \cdot \nabla \mathbf{v} + \nabla p / \rho_0 = \boldsymbol{\omega} \times \mathbf{v} + \nabla(p/\rho_0 + v^2/2)$, it follows that the nonlinear term can be partially subsumed into the pressure gradient and that the nonlinearity is depleted if $\boldsymbol{\omega}$ tends to align with $\mathbf{v}$. Some coherent structures, like rapidly rotating atmospheric storms, have been observed to have this geometric character. Small-scale coherent structures with significantly depleted nonlinearity have also been found in computer simulations.

A hotly debated issue in turbulence theory is whether a purely statistical description that does not explicitly account for structures is sufficient to capture the universal features of turbulence. In critical phenomena, Kenneth Wilson's renormalization group is often successful even though it, too, does not explicitly invoke structures. There are instances, however, where information about the structure of the excitation is essential. The spin vortex structure in a two-dimensional spin model, the X - Y model, explained by the Kosterlitz-Thouless theory,²⁰ is an example. The limitations of purely statistical descriptions of turbulence must be further explored.

What next?

Over the past two decades, there have been major advances in the use of numerical simulations to compute turbulent flows. It is now widely accepted that moderate-Reynolds-number turbulent flows in simple geometries can be as well analyzed by advanced computer simulations as by laboratory experiments. This rich and rapidly developing field deserves a separate review. We focus here instead on challenges for theory and experiment.

Challenges for theory. However hard it may be to make forecasts, we nevertheless think it useful to assess

what now seem to be reasonable—and unreasonable—goals of turbulence theory. Among the distant yet reasonable goals are:

- ▷ The complete description of turbulence as a state of matter.
- ▷ The derivation of the observed universal features, such as scaling and intermittency, from a well-defined set of assumptions.
- ▷ The application of our present and future understanding of turbulence to predict and control its effects, even in complex situations such as those involving contaminants, combustion, free surfaces and magnetic fields.

We may also add to the list two more immediate goals: the qualitative understanding of the origin of spatial chaos, and the mathematical description of coherent structures.

Some other goals for turbulence theory seem to us somewhat unreasonable, but we think it instructive to mention them here. For example, it is well known that detailed properties of turbulent flows at far-off times cannot be predicted. However, even the *statistical* properties of these flows may be “uncomputable.” Indeed, it is known that some of the simplest deterministic dynamical systems have several attractors with highly intertwined basins of attraction, so that the correspondence between their initial conditions and final behavior seems to be “uncomputable” in the sense used in algorithmic theory. Demonstrating this rigorously is a challenge for mathematicians. If true, it would imply, in the context of meteorology for example, that while the weather clearly is not predictable at long times, neither, in fact, is the climate. It is also unreasonable to aim for a grand, exact, eternal turbulence theory; it seems to us more likely that there is an infinite hierarchy of increasingly refined descriptions of turbulence suitable for increasingly refined experimental verification. The asymptotic behavior in the limit of infinite Reynolds number and infinitely small external noise may well be infinitely complex.

Challenges for experiments. We believe that new turbulence experiments using state-of-the-art techniques borrowed from other branches of physics are a prerequisite for basic progress in a field of such tremendous—and apparent—complexity. Many of these experiments should

be designed to address questions posed by our current ideas about turbulence. But experiments conceived and formulated without prejudice to existing ideas are also important, provided that the control, instrumentation, processing and analysis are state of the art. An example of the latter type of experiment is the recent study at the University of Chicago²¹ of high-Rayleigh-number thermal convection in a gaseous helium cell, which revealed new scaling regimes and flow structures. The results from this experiment allowed a refinement of the classical scaling theory of convection. A significant feature of that experiment is that it explored regimes exhibiting a form of turbulence with the control and accuracy nowadays characteristic of experiments on transition to chaos.

The challenging questions for future experiments include:

▷ Are the small-scale properties of very-high- R flows truly universal, or is the breaking of scale invariance (intermittency) dependent on boundary and initial conditions, external forces and so on?

▷ Is there a hierarchy of structures in turbulence, and how can those structures be characterized dynamically or topologically? How does turbulence react to organized and random disturbances? This question is relevant to achieving active and passive control of turbulence for drag reduction, enhanced or diminished mixing, and other effects.

▷ What is the behavior of more complex turbulent flows in which compressibility, sound production and propagation, chemical reactions and combustion, magnetohydrodynamics effects, non-Newtonian behavior, multiphase effects and so on may be present?

Experiments seeking to answer these questions will require tradeoffs between control and accuracy on the one hand and the complexity of the flow situation on the other. It is highly desirable that facilities have very well controlled environments and that data be analyzed with careful error estimates. A high priority should be the development of data acquisition and visualization techniques that will give spatio-temporal information across a broad range of scales. High-repetition-rate laser scanning, holographic methods, and fluorescence and scattering techniques, combined with advanced digital processing, are among promising approaches. The invention of special techniques for strophography (the spatial visualization of vorticity intensity) would have a major impact on the field.

A wide range of new facilities will be required to meet these challenges, including both open- and closed-flow facilities. Among exciting new small-scale devices could be a cryogenic (helium) wind tunnel with innovative probes. Since the viscosity of cold helium gas is on the order of 10^{-3} – 10^{-4} cm²/sec, such a facility should be capable of achieving high Reynolds numbers in a very controlled environment.

A number of large-scale facilities were suggested at a workshop held in December 1987.²² They include:

▷ A water tank with a moving grid generating turbulence with no appreciable mean velocity, thereby simplifying visualization, and with a range of R values that are tunable by adjusting the grid motion

▷ A pipe air-flow facility with large aspect ratio, so that the turbulence is well established and the fluid is relaxed to equilibrium

▷ A circular Couette flow facility in water or air to explore regimes with and without organized structures. These facilities will need to be well instrumented and controlled, and they will need to be integrated with massive computer capability for data acquisition, storage and processing. They will have to be at least one order of magnitude larger than the facilities already available in

order for them to provide answers to the questions posed above. Such experiments will require an internationally coordinated community effort.

Let us conclude by noting that less is known about the fine scale of turbulence—for example, the scale of 1 mm in the atmosphere—than about the structure of atomic nuclei. Lack of basic knowledge about turbulence is holding back progress in fields as diverse as cosmology, meteorology, aeronautics and biomechanics. Understanding the hierarchically organized complexity of turbulence may well provide a paradigm for understanding a variety of problems at the frontiers of physics research.

* * *

Frisch's work is supported in part by the European Economic Community's SCIENCE program and Direction de la Recherche et des Etudes Techniques. Orszag's work is supported in part by the Air Force Office of Scientific Research, the Defense Advanced Research Projects Agency and the Office of Naval Research.

References

1. A. S. Monin, A. M. Yaglom, *Statistical Fluid Mechanics*, vols. 1 and 2, MIT P., Cambridge, Mass. (1971, 1975).
2. See the introduction to H. Poincaré, *Calcul des Probabilités*, Gauthier-Villars, Paris (1912).
3. N. Aubry, P. Holmes, J. L. Lumley, E. Stone, *J. Fluid Mech.* **192**, 115 (1988).
4. See L. F. Richardson, *Weather Prediction by Numerical Process*, Cambridge U. P., Cambridge, England (1922), p. 66.
5. J. Swift, *On Poetry. A Rhapsody*. See A. H. Scouten, ed. *A Bibliography of the Writings of Jonathan Swift*, 2nd ed., U. Pennsylvania P., Philadelphia (1963).
6. F. Argoul, A. Arnéodo, G. Grasseau, Y. Gagne, E. J. Hopfinger, U. Frisch, *Nature* **338**, 51 (1989).
7. A. N. Kolmogorov, *Dokl. Akad. Nauk SSSR* **30**, 299 (1941).
8. L. Onsager, *Nuovo Cimento* **6**, 279 (1949).
9. H. L. Grant, R. W. Stewart, A. Moilliet, *J. Fluid Mech.* **12**, 241 (1962).
10. R. H. Kraichnan, *Phys. Rev.* **109**, 1407 (1958); *J. Fluid Mech.* **5**, 497 (1959).
11. D. Forster, D. Nelson, M. Stephen, *Phys. Rev. A* **16**, 732 (1977). C. de Dominicis, P. C. Martin, *Phys. Rev. A* **19**, 419 (1979).
12. V. Yakhot, S. A. Orszag, *J. Sci. Comp.* **1**, 3 (1986); *Phys. Rev. Lett.* **57**, 1722 (1986). A. Yakhot, S. A. Orszag, V. Yakhot, M. Israeli, *J. Sci. Comp.* **4**, 139 (1989).
13. F. Anselmetti, Y. Gagne, E. J. Hopfinger, R. A. Antonia, *J. Fluid Mech.* **140**, 63 (1984).
14. B. Mandelbrot, *J. Fluid Mech.* **62**, 331 (1974).
15. A. N. Kolmogorov, *J. Fluid Mech.* **13**, 82 (1962). E. A. Novikov, R. W. Stewart, *Izv. Akad. Nauk SSSR, Ser. Geofiz.* **3**, 408 (1964).
16. G. Parisi, U. Frisch, in *Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics*, M. Ghil, R. Benzi, G. Parisi, eds., North-Holland, Amsterdam (1985), p. 71.
17. See the report in *PHYSICS TODAY*, April 1986, p. 17, for further references.
18. E. Siggia, *J. Fluid Mech.* **107**, 375 (1981). Z.-S. She, E. Jackson, S. A. Orszag, in *Proc. Newport Conference on Turbulence*, Springer-Verlag, New York (in press).
19. D. I. Meiron, M. J. Shelley, W. T. Ashurst, S. A. Orszag, in *Mathematical Aspects of Vortex Dynamics*, R. E. Caflisch, ed., Society for Industrial and Applied Mathematics, Philadelphia (1989).
20. J. M. Kosterlitz, D. J. Thouless, *J. Phys. C* **6**, 1181 (1973).
21. B. Castaing, G. Gunaratne, F. Heslot, L. Kadanoff, A. Libchaber, S. Thomae, X.-Z. Wu, S. Zaleski, G. Zanetti, *J. Fluid Mech.* **204**, 1 (1989). See also the report in *PHYSICS TODAY*, June 1988, p. 17.
22. Grenoble Workshop on Experimental Data for Fully Developed Turbulence, Institut Mécanique de Grenoble, December 1987. ■