NEUTRINO OSCILLATIONS AND SOLAR NEUTRINOS

The hypothesized quantum mechanical process in which neutrinos transform from one 'flavor' into another can be studied by observing solar neutrinos with a new generation of highly sensitive detectors.

Lincoln Wolfenstein and Eugene W. Beier

The neutrino is the elementary particle postulated by Wolfgang Pauli to account for the apparent nonconservation of energy in nuclear beta decay. Neutrinos are now believed to exist in at least three varieties, or "flavors," labeled ν_e, ν_μ and ν_τ and distinguished by the way they interact. There are also three antineutrinos, $\bar{\nu}_e$, $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$.

The ν_e, which is emitted along with the positron in nuclear beta decay, is the variety expected from the Sun; one detects it by the inverse of the beta-decay process. The ν_μ is emitted in the decays of muons and pions. The ν_τ's, which have never been detected directly, are thought to be products of the decay of the τ lepton.

The question of neutrino mass was raised by Enrico Fermi in the first paper on the theory of beta decay. He pointed out that the shape of the electron spectrum near its high-energy end is sensitive to neutrino mass. In recent years experiments to determine the neutrino mass have concentrated on the $\bar{\nu}_e$ emitted in tritium beta decay,

$$H^3 \to He^3 + e^- + \bar{\nu}_e$$

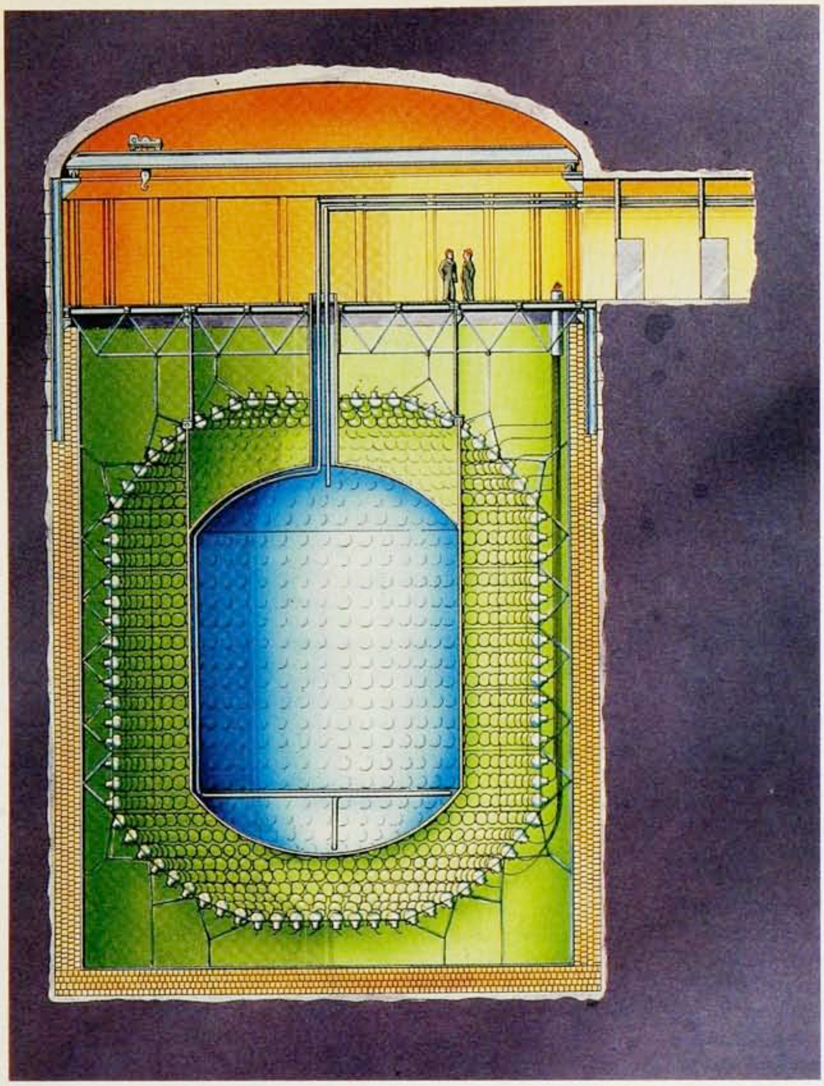
because this decay has a low end-point energy of 18.6 keV. A series of experiments carried out in Moscow by Valentin A. Lubimov and his collaborators[1] indicates a nonzero $\bar{\nu}_e$ mass, on the order of 30 eV. Other experiments have not confirmed this value, however, and the best present result, obtained by a group in Zurich,[2] is the limit $m_{\bar{\nu}_e} \leq 18$ eV. The arrival times of neutrinos from supernova 1987a give a similar limit. The best limits for the ν_μ and ν_τ masses are $m_{\nu_\mu} \leq 250$ keV and $m_{\nu_\tau} \leq 35$ MeV.

The standard electroweak theory of Steven Weinberg, Abdus Salam and Sheldon Glashow is usually presented in a form in which all neutrinos are massless. Most extensions of the theory, however, in particular, grand unified theories, require *nonzero* neutrino masses. The masses suggested by these theories are far too small to be measured by direct kinematic methods. The only possible way to detect such small masses is through the quantum mechanical process known as neutrino oscillation, in which neutrinos of one flavor transform into another—ν_e, for example, becoming ν_μ or ν_τ.

The Sun is expected to be a copious source of neutrinos; about 2% of its energy is thought to be emitted in such particles. It has been believed since the 1930s that the energy emitted by the Sun has its origins in thermonuclear reactions near the solar center. Just a few months ago, results from a directionally sensitive experiment gave clear-cut evidence that the Sun is emitting neutrinos—the first experimental evidence that the Sun's energy indeed originates in nuclear reactions.

Many unsuccessful searches for neutrino oscillations have been carried out with neutrinos from terrestrial accelerators or reactors. If the Sun is used as a source of neutrinos, the search for oscillations is possible over much larger distances, which in turn makes the search sensitive to much smaller neutrino masses. (Figure 1 shows one

Lincoln Wolfenstein is a professor of physics at Carnegie Mellon University, in Pittsburgh. **Eugene Beier** is a professor of physics at the University of Pennsylvania, in Philadelphia. He is a member of the Kamiokande and Sudbury Neutrino Observatory collaborations.



Sudbury Neutrino Observatory. The proposed chain facility, shown here in an artist's conception, would be located 6800 feet under ground in a mine near Sudbury, Ontario. The cavity in the mine is 20 meters in diameter and 30 meters high. The central part of the detector (blue) is 1000 metric tons of heavy water, D_2O, a resource available in quantity only in Canada. Neutrinos emitted by the Sun will interact with the deuterons through three different reactions; in this way the study of solar neutrinos will be made insensitive to details of the calculated flux of neutrinos emitted by the Sun but particularly sensitive to any neutrino oscillations. The heavy water is contained in an acrylic vessel and surrounded by ultrapure ordinary water, H_2O, which serves as a radioactivity shield. Photomultipliers will detect Čerenkov light generated by electrons produced in the neutrino interactions. The yellow bricks are shielding, and the gray represents the rock wall. (Courtesy of Alastair Middleton, Chalk River Nuclear Laboratory, Canada.) **Figure 1**

proposed facility for detecting solar neutrinos.) Furthermore, the material medium of the Sun may enhance the oscillations of neutrinos on their way out. In general, solar neutrinos enable one to study neutrino properties that are difficult or impossible to study with terrestrial sources.

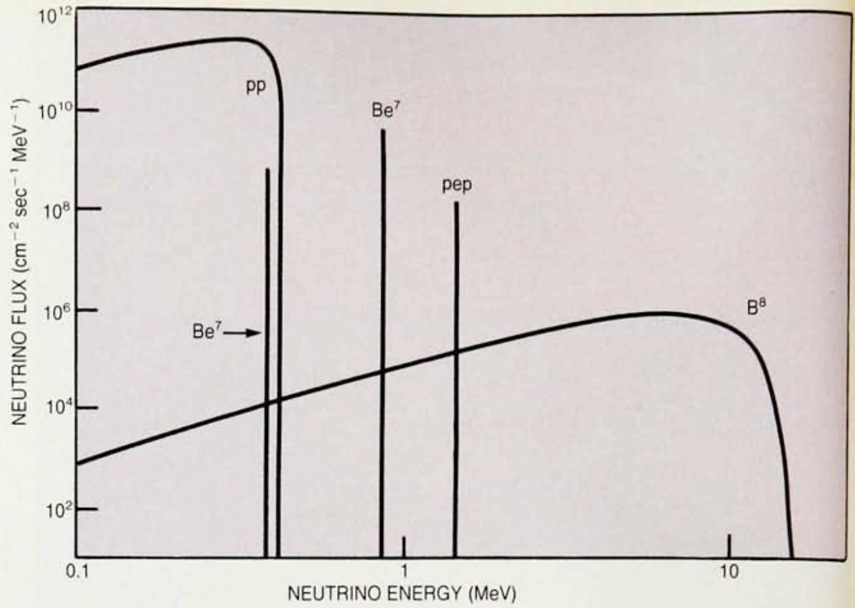
Until recently the only attempt to detect solar neutrinos was Raymond Davis's effort to monitor a large Cl^{37} target deep under ground in the Homestake gold mine in South Dakota.[3] Davis's experiment, which has been running for 20 years, has detected some neutrinos, but the number appears to be significantly lower than what is expected, based on the standard theoretical model of the Sun.[4] This discrepancy is often called the problem of the "missing" solar neutrinos.

Many solutions to this problem have been proposed, even though there was until recently only one experimental result. These solutions divide into two broad classes: those that put the blame on the Sun, and those that put the blame on the neutrino. Those that put the blame on the Sun point out that the Cl^{37} detector is sensitive only to a small part of the solar neutrino spectrum, a part that is itself very sensitive to the temperature of the solar interior. Alternative solar models involving unorthodox ideas about the composition of the Sun or the convection of matter in the Sun reduce the flux expected in the Cl^{37} detector without seriously changing our basic ideas about the source of stellar energy. On the other hand it is possible that the standard solar model is indeed correct but that neutrinos are transformed on their way to Earth.

The solar neutrino flux

In the standard solar model, energy is generated in the interior of the Sun by nuclear processes that in effect fuse four protons into a helium nucleus. The table on page 31 shows the chain of reactions, and figure 2 shows the resulting neutrino spectra. The necessary starting point is the weak reaction that fuses two protons to form a deuteron. This reaction generates a large flux of neutrinos with a continuous spectrum ending at 420 keV. The deuteron reacts quickly to produce He^3, after which the chain divides into two parts. The one of interest produces Be^7, which is consumed by two competing reactions. The Be^7 can capture an electron, producing line spectra of neutrinos with 90% of the intensity at 862 keV and the remaining 10% at 384 keV. Still more important for observation is the very rare chain leading to B^8, because the B^8 beta decay yields neutrinos with energies up to 15 MeV, which are easier to detect. Some neutrinos are also expected from nuclear reactions involving the carbon, nitrogen and oxygen in the Sun, but these are not expected to make a major contribution to the data of any proposed detector.

For many years theorists have carried out calculations of the absolute fluxes of neutrinos from these sources. The most recent detailed analysis was completed



Solar neutrino spectrum according to the standard-solar-model calculation of reference 4. The spectra correspond to the reactions listed in the table on page 31. **Figure 2**

last year by John Bahcall of the Institute for Advanced Study and Roger Ulrich of the University of California, Los Angeles.[4] They begin with the assumption that the primordial Sun is spherically symmetric and chemically homogeneous. They then deduce the percentage abundances of heavy elements from recent observations of the solar surface while allowing the helium–hydrogen ratio to vary. Finally, they calculate the evolution of the Sun up to the present day, varying the original conditions until they obtain the observed solar luminosity.

The primary physical assumptions of the standard solar model are the initial conditions described above, hydrostatic equilibrium and energy transport primarily by radiative transfer, that is, by photon diffusion. All the elementary processes should be well understood from terrestrial physics. The major input parameters describing these processes are the nuclear reaction rates, extrapolated from laboratory experiments, and the opacities of solar material determined from detailed atomic physics calculations.

It is of great importance to evaluate the uncertainties in the solar model fluxes. By far the most uncertain flux is that associated with B^8, both because it depends on the uncertain $p + Be^7$ cross section and because the reaction rate is extremely sensitive to the calculated central temperature of the Sun. Compounding the uncertainties of the input parameters, Bahcall and Ulrich find with better than 99% confidence that the value of the calculated flux from B^8 is accurate to $\pm 37\%$. It is difficult to make a quantitative evaluation of the uncertainty associated with the simplifying assumptions of the solar model.

To explain the smaller flux from the B^8 reaction observed in the Cl^{37} detector, theorists have proposed various nonstandard solar models.[5] All of these models require changing the assumptions of the standard solar model in an *ad hoc* fashion with no real motivation other than solving the solar neutrino problem.

In contrast to the calculated flux of neutrinos from the beta decay of B^8, the calculated flux of neutrinos from the $p + p$ reaction is almost independent of the input parameters and has nearly the same value for all solar models, including the nonstandard ones.

The search so far

In 1970 Davis and his collaborators began operating their experiment designed to detect neutrinos from the Sun. The primary detector is a 380 000–liter tank of perchlorethylene (C_2Cl_4), located 4850 feet under ground in the Homestake mine. The idea is to detect solar neutrinos through the inverse beta-decay reaction

$$\nu_e + Cl^{37} \rightarrow Ar^{37} + e^- \tag{1}$$

The energy threshold for the reaction is 0.81 MeV; the target Cl^{37} nucleus has a natural abundance of 24% of all chlorine. The Ar^{37} nucleus produced in the reaction is radioactive and decays with a halflife of 31 days. Detection of the Ar^{37} decay is evidence that reaction 1 has occurred.

In a passive radiochemical experiment such as this one, the detector is exposed for one to two halflives of the Ar^{37} reaction product and is then purged with helium gas to collect the few Ar^{37} atoms produced by the reaction. The efficiency of the collection and detection of Ar^{37}, and the production of Ar^{37} by other processes, must be measured in detail. Many auxiliary experiments have been performed to determine these factors. The detection efficiencies are large and well determined, and the backgrounds, most of which are induced by cosmic-ray muons, have been measured as a function of depth in the Homestake mine. The result of the experiment, averaging all runs from 1970 to 1985, is that 0.472 ± 0.037 Ar^{37} atoms are produced in the detector each day, after subtraction of a background of about 20%.

The importance of this result, as we mentioned at the beginning of this article, is that it is significantly below estimates based on conventional models of stellar evolution. In terms of solar neutrino units, where 1 SNU $= 10^{-36}$ captures/atom/day, the experimental rate is 2.1 ± 0.3 SNU, and the predicted rate is 5.3–10.5 SNU. These predicted values are the extremes from the standard solar model discussed above.[4] The quoted experimental error is one standard deviation.

From February 1985 until October 1986 data from the Cl^{37} experiment could not be analyzed at routine intervals, because of the failure of certain circulation pumps. Davis recently reported the results[6] of runs for the period October 1986 through early 1988. These runs give a Cl^{37} capture rate of 4.2 ± 0.8 SNU. This result by itself is only a factor of two smaller than the central value of the theoretical prediction and, given the theoretical and experimental

Proton–proton chains of nuclear reactions in the Sun

Chain I

$p + p \rightarrow d + e^+ + \nu_e$ (99.75%)

or

$p + e^- + p \rightarrow d + \nu_e$ (0.25%)

$\downarrow$

$d + p \rightarrow He^3 + \gamma$

$\downarrow$

$He^3 + He^3 \rightarrow He^4 + p + p$ (85%) or

Chain II

$He^3 + He^4 \rightarrow Be^7 + \gamma$ (15%)

$\downarrow$

$Be^7 + e^- \rightarrow Li^7 + \nu_e$ (99.9%) or

$\downarrow$

$Li^7 + p \rightarrow He^4 + He^4$

Chain III

$Be^7 + p \rightarrow B^8 + \gamma$ (0.1%)

$\downarrow$

$B^8 \rightarrow Be^{8*} + e^+ + \nu_e$

$\downarrow$

$Be^{8*} \rightarrow He^4 + He^4$

uncertainties, would not constitute a serious problem.

Whether this new result represents a statistical fluke or has some other explanation is not yet clear. The skeptic's first reaction is that the experimental conditions must have changed, and therefore one of the series of measurements, probably those made before the 1986 pump replacement, is wrong. The experimenters are confident that conditions have not changed and that the results since 1986 represent either a statistical effect or a real time dependence of the signal.

An intriguing possibility is that the solar neutrino flux is anticorrelated with solar activity. As one can see in figure 3, the Cl^{37} data up to 1985 show a minimum at the height of the sunspot cycle in 1979–80. A statistical analysis of those data suggested that the effect was not very significant. The 1987 data were taken during a solar activity minimum, and reinforce the idea of the anticorrelation. Finding a minimum in the Cl^{37} capture rate during the 1990–91 maximum of the solar cycle would make it difficult to escape the conclusion that the Cl^{37} capture rate is anticorrelated with solar activity. We can only wait and see.

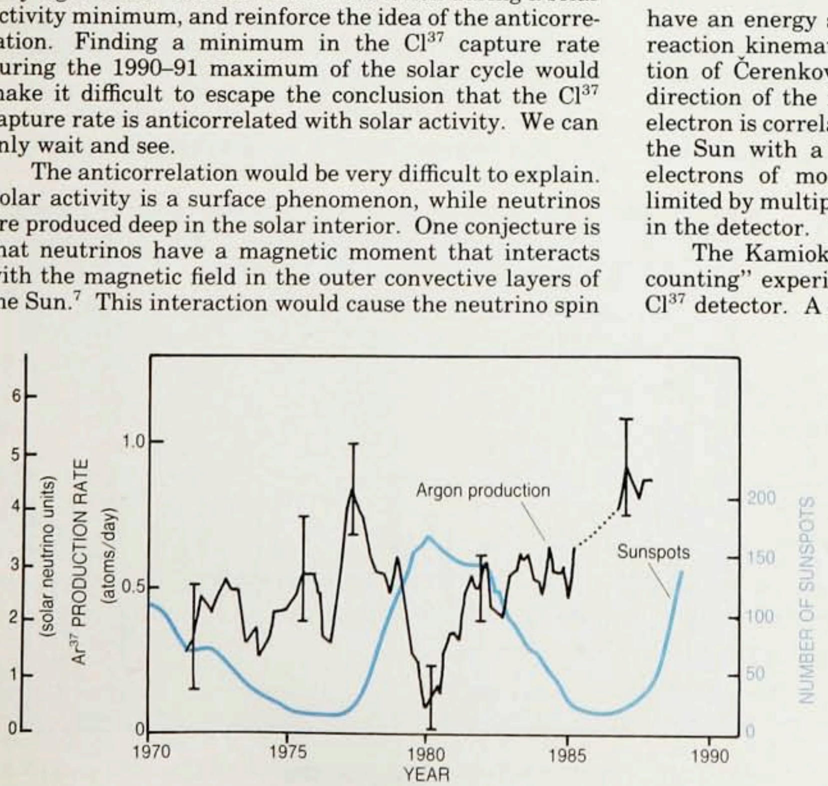
The anticorrelation would be very difficult to explain. Solar activity is a surface phenomenon, while neutrinos are produced deep in the solar interior. One conjecture is that neutrinos have a magnetic moment that interacts with the magnetic field in the outer convective layers of the Sun.[7] This interaction would cause the neutrino spin to precess from left-handed to right-handed helicity. Right-handed neutrinos would not be seen by the Cl^{37} detector. The magnetic moment required to produce this effect is greater than 10^{-11} Bohr magnetons, which is much larger than expected on theoretical grounds given the existing upper limit on the ν_e mass.

Kamiokande II. For 15 years no experiment confirmed or extended the result of the Cl^{37} experiment. At the beginning of 1986 the upgraded Kamiokande II nucleon-decay detector, a water Čerenkov device 300 km west of Tokyo, began operation with an energy threshold that made it sensitive to the B^8 solar neutrinos. Recoil electrons from the elastic scattering of B^8 neutrinos by electrons,

$$\nu + e^- \rightarrow \nu + e^- \qquad (2)$$

have an energy spectrum that extends to 15 MeV. The reaction kinematics and the detection through observation of Čerenkov light preserve information about the direction of the incident neutrino. The direction of the electron is correlated with the direction from the Earth to the Sun with a root-mean-square resolution of 28° for electrons of momentum 10 MeV/c; the resolution is limited by multiple scattering of the electron by the water in the detector.

The Kamiokande II detector is a real-time, "direct counting" experiment, unlike the passive radiochemical Cl^{37} detector. A solar neutrino signal must be extracted



Neutrino detections vs sunspots. The black curve (left-hand scales) shows the running five-point average results of the Cl^{37} solar neutrino experiment as a function of time. No data were analyzed in 1985 and 1986. The blue curve (right-hand scale) shows the average sunspot numbers. If solar neutrinos are anticorrelated with sunspots, the Cl^{37} data should decrease at the next solar maximum, in 1990–91. The tick marks on the horizontal axis represent the beginnings of the indicated years. (Adapted from a figure provided courtesy of Raymond Davis Jr) **Figure 3**

from radioactivity backgrounds from three sources: backgrounds induced by cosmic-ray muons, backgrounds induced by gamma rays from the walls of the underground cavity and the detector materials, and radioactivity from trace-element contamination of the detector water. These backgrounds have been reduced a thousandfold since the initial operation of the upgraded detector.

In a recent *Physical Review Letter*[8] the Kamiokande II group presented its initial results, which are reproduced in figure 4. For 450 days of operation, from January 1987 through May 1988, the number of detected electrons above a measured kinetic energy of 8.8 MeV is $0.46 \pm 0.13 \pm 0.08$ times the number expected from the B^8 flux of the solar model of reference 4. (The first uncertainty is statistical; the second, systematic.) The result is expressed as a ratio of the measured data to a simulated data set analyzed with the same criteria in order to cancel out many systematic uncertainties.

There is no unique way to compare this result with that from the Cl^{37} detector. Assuming that only the B^8 neutrinos are suppressed relative to the fluxes predicted by the solar model, or that all neutrinos detected by the Cl^{37} experiment are suppressed by the same factor, the Cl^{37} experiment and the Kamiokande experiment are in agreement for the 1987–88 period of operation. The Kamiokande experiment demonstrates for the first time, through the directional property of the signal, that neutrinos with energies above 9 MeV are emitted by the Sun.

The Kamiokande experiment is sensitive to only a few percent of the neutrinos expected from the B^8 reaction in the Sun, so one must be careful not to overinterpret the result. The Cl^{37} and Kamiokande experiments are both low-counting-rate experiments with serious statistical limitations. Higher-rate experiments sensitive to different parts of the solar neutrino energy spectrum are required to determine the energies and temporal structure of neutrinos from the Sun. We discuss progress in this direction at the end of this article.

Neutrino mass and oscillations

While it is possible that all neutrinos are massless, no compelling symmetry requires it. In the standard electroweak theory all fermions are introduced as massless particles, and all except the neutrinos acquire a mass as a result of symmetry breaking. In most grand unified theories the up and down quarks, the electron and the ν_e are considered to be states of a single fermion field, and all acquire a mass. Murray Gell-Mann, Pierre Ramond and Richard Slansky have noted[9] that in the simplest grand unified theory of this sort, based on the symmetry group SO(10), neutrinos should have a mass given by a so-called seesaw formula:

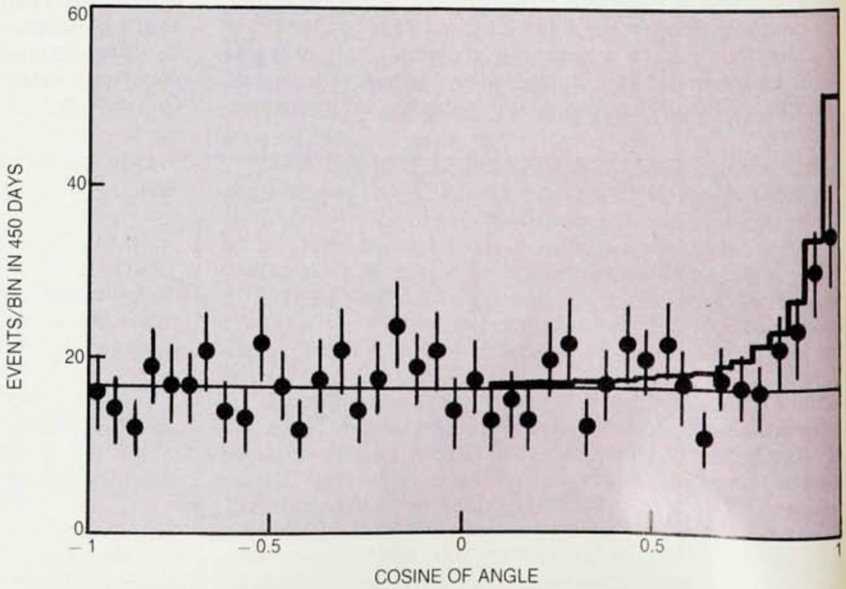
$$m_\nu \simeq \frac{(\text{normal mass})^2}{M}$$

"Normal mass" refers to the mass of the quark or electron; for the ν_μ or ν_τ particles, the "normal mass" is that of the second or third family of fermions, respectively. The seesaw formula thus establishes a hierarchy for the masses of neutrinos of different families: $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$. The value of the mass M is related to the very large mass scale at which the strong and electroweak interactions are unified in grand unified theories. M might be as large as 10^{15} GeV/c^2, in which case even the heaviest neutrino, ν_τ, would have a mass no larger than 10^{-2} eV/c^2.

An important feature of all theories of neutrino mass is neutrino mixing. The neutrino ν_e emerging in nuclear beta decay is expected to be a quantum mechanical mixture of mass eigenstates ν_i:

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

In many theories $|U_{e1}|^2$ is close to unity, so that ν_e is



Directional distribution of neutrinos. The cosine plotted is that of the angle between the Sun and electrons with kinetic energies above 9.6 MeV, as measured in the Kamiokande II detector. A cosine of 1 corresponds to the direction of the Sun. The isotropic component of the distribution is background. The peak in the forward direction is produced by solar neutrinos that scatter elastically from electrons in the 680-metric-ton water target. The horizontal line represents the fit to the isotropic background. The histogram on the right side of the figure represents the solar-model prediction for the Kamiokande II experiment. **Figure 4**

primarily the lightest mass eigenstate. However, one expects a significant mixing of the second mass eigenstate ν_2 (typically $U_{e2} \approx 0.2$) and a smaller mixing of the heaviest state ν_3. The mixing is analogous to the well-known quark mixing originally discussed by Nicola Cabibbo to explain the strength of the weak interaction responsible for strange-particle decays.

As a consequence of neutrino mixing there is a possibility of neutrino oscillations in which a pure beam of ν_e particles is partially transformed into ν_μ or ν_τ particles as it propagates through the vacuum. Considering only two types of neutrino,

$$|\nu_e\rangle = \cos\theta_V |\nu_1\rangle + \sin\theta_V |\nu_2\rangle \quad (3)$$

$$|\nu_\mu\rangle = -\sin\theta_V |\nu_1\rangle + \cos\theta_V |\nu_2\rangle \quad (4)$$

The parameter θ_V is known as the vacuum mixing angle. As a function of time, or of distance along the path of the moving neutrino, the state that is ν_e at $t=0$ becomes

$$|\nu_e(t)\rangle = e^{-iE_1t}\cos\theta_V |\nu_1\rangle + e^{-iE_2t}\sin\theta_V |\nu_2\rangle \quad (5)$$

Here we have used units with $\hbar = c = 1$. Setting $E_i = (p_i^2 + m_i^2)^{1/2}$ we find, as a result of the mass difference $m_2 - m_1$, that the relative phase of the two components changes. Therefore the quantum mechanical state now contains a component of ν_μ, given by

$$\langle\nu_\mu|\nu_e(t)\rangle = \sin\theta_V \cos\theta_V (e^{-iE_2t} - e^{-iE_1t})$$

$$|\langle\nu_\mu|\nu_e(t)\rangle|^2 = \sin^2 2\theta_V \sin^2\frac{(E_2 - E_1)t}{2} \quad (6)$$

This quantum mechanical description is identical to that for the precession in a magnetic field of a spin-½ particle with its spin at an angle of $2\theta_V$ with respect to the field. Thus neutrino oscillations may be viewed as precession in flavor space. If the neutrino's mass is much less than its energy, then the time dependence in equation 6 can be replaced by

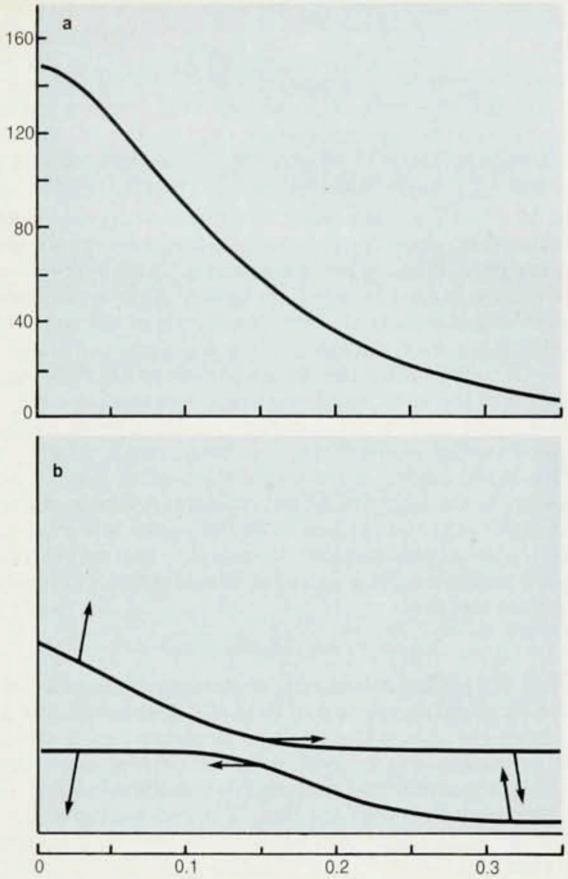
$$\sin^2\left(\frac{\pi l}{l_V}\right)$$

Here l is the distance between the source and the detector, and l_V, which is called the vacuum oscillation length, is given by

$$l_V = \frac{4\pi p_\nu}{m_2^2 - m_1^2}$$

Given fixed neutrino momenta from a source, larger distances l give sensitivities to smaller values of the difference $m_2^2 - m_1^2$, or Δm^2. For a neutrino momentum p_ν of 1 MeV/c and a mass difference Δm^2 of 1 eV^2 the oscillation length l_V is 2.5 meters.

Many experiments have searched unsuccessfully for oscillations of neutrinos coming from reactors or accelerators. The reactor experiments search for the disappearance of $\bar{\nu}_e$'s with energies of a few MeV at distances up to 50 meters. Accelerator experiments typically search for the disappearance of ν_μ's or the appearance of ν_e's or ν_τ's in a beam that is initially made up of ν_μ's, at energies of a few GeV and at distances up to a kilometer. Assuming the parameter θ_V is at least 0.1, both types of experiment provide upper limits on $m_2^2 - m_1^2$ of about 0.2 eV^2/c^4. If $m_2 \gg m_1$, this corresponds to a limit of 0.5 eV/c^2 on m_{ν_2}, where a natural assumption is that ν_2 is mainly ν_μ. While this limit is more than an order of magnitude better than direct limits, it is still much higher than the values suggested by many theories. To improve these limits we must study oscillations over much larger distances. The Sun is a unique source of neutrinos for such a study.



Density profile of the Sun, and eigenvectors that govern the mixing of neutrino flavors. **a:** Density of the Sun as a function of distance from its center.[4] **b:** Schematic picture of the eigenvalues and eigenvectors of the matrix that describes the propagation of neutrinos generated near the center of the Sun. In the vacuum (the far right of the figure) the eigenvectors represent the mass eigenstates. If the eigenvectors are represented by two-component Pauli spinors, then the arrows represent the direction of the spin. Spin up is pure ν_e, and spin down is pure ν_μ. Spin vectors at an angle represent mixtures of the two flavor eigenstates. **Figure 5**

The MSW effect

The possibility that vacuum oscillations might change the observed flux of solar neutrinos was suggested even before Davis's experiment. It is possible to explain the Cl^{37} result in terms of vacuum oscillations for values of Δm^2 as small as 10^{-10} eV$^2/c^4$. However, to explain a large suppression factor it is necessary to imagine a large amount of mixing among the three types of neutrinos. Such large amounts of mixing cannot be ruled out by any other experiment for such small values of Δm^2, but they are not expected in most theoretical pictures.

An alternative possibility that does not require large values of the vacuum mixing angle was pointed out by S. P. Mikheyev and A. Yu. Smirnov building on a formalism developed by Wolfenstein.[10] In their model, usually called the MSW model, a transformation of ν_e into ν_μ or ν_τ occurs as the neutrino travels from its source in the center of the Sun to the surface. The basic idea is that neutrino oscillations are modified in matter. Neutrinos passing through matter have an index of refraction n given by the optical theorem

$$p_\nu^2(n-1) = 2\pi N f(0)$$

Here $f(0)$ is the forward elastic scattering amplitude due to the weak interaction, and N is the number of scattering targets per unit volume. The imaginary part of the index of refraction n is related to the absorption cross section, which is so small that a negligible fraction of neutrinos are lost on the way out of the Sun. The real part of n is important for oscillations because we must include in equation 5 a phase factor

$$\exp(ip(n-1)x)$$

We are interested, of course, in differences in the phases for different components of the wavefunction. Such a difference arises because ν_e has a larger elastic scattering amplitude from the electrons in ordinary matter than does ν_μ or ν_τ. All neutrino flavors have the same neutral-current (Z^0 exchange) amplitudes from all targets, but electron neutrinos have an additional charged-current ($W^\pm$ exchange) amplitude for elastic scattering from electrons.

Considering only two types of neutrino, the neutrino mass can be described by a 2×2 matrix with eigenvalues m_1 and m_2 and eigenvectors given by equations 3 and 4. The effect of the index of refraction on the phases is equivalent to adding to ν_e a mass that depends on the electron density N_e. Thus, in the medium, we may view the masses m_1 and m_2 and the mixing angle θ as functions of the electron density. For the large densities characteristic of the solar center the index-of-refraction effect dominates in the MSW model, so that ν_e is primarily in the upper state ν_2 with $\theta(N_e)$ close to 90°. At the solar surface, where the electron density is zero, ν_e is mainly in the lower state ν_1, and the mixing angle θ is equal to its normal vacuum value θ_V, which is assumed to be fairly small.

Figure 5 illustrates the mass eigenvalues and eigenvectors as a function of the distance from the center of the Sun. The neutrinos produced at the center of the Sun are the ν_e type and are primarily in the upper state ν_2; there is only a small probability, given by $\cos^2\theta(N_e)$, of their being in the ν_1 state. As the neutrinos move outward from the center of the Sun there is some probability of a transition to the state ν_1, particularly in the region where ν_1 and ν_2 are close together. If the states do not come too close together, this probability stays very small, corresponding to the well-known adiabatic approximation. In this case, when the neutrinos reach the solar surface, they are still primarily in the mass eigenstate ν_2, which is now mainly composed of the flavor eigenstate ν_μ. (As the box on page 35 explains, this is analogous to the situation of a spin-½ particle in a magnetic field that gradually reverses direction but never vanishes.) This behavior is an example of the quantum mechanical phenomenon of level crossing. The two levels would cross for that value of N_e for which the diagonal values of the effective neutrino mass matrix are equal, but they are kept apart by the off-diagonal terms proportional to $\sin 2\theta_V$.

For this solution to hold it is necessary that the vacuum value of Δm^2 be small enough that the index-of-refraction effect will dominate at the solar center. For B^8 neutrinos this yields the requirement

$$\Delta m^2 \leqslant 10^{-4}\ \mathrm{eV}^2/c^4 \tag{7}$$

There is also the requirement that the adiabatic approximation hold; this yields the result

$$\sin^2\theta_V \geqslant 3\times 10^{-8}\,(\mathrm{eV}^2/c^4)/\Delta m^2 \tag{8}$$

For values of θ_V of about 0.1 and assuming $m_{\nu_2} \gg m_{\nu_1}$, equations 7 and 8 give the result that m_{ν_2} must be between 10^{-2} and 5×10^{-4} eV$/c^2$.

In general ν_2 can be the state that is primarily ν_μ or the state that is primarily ν_τ. The transformation probability for ν_e to become ν_μ or ν_τ due to this MSW effect is dependent on energy. Equations 7 and 8 together define a set of values of Δm^2 and θ_V such that this probability is large for most or all of the neutrino energies detectable by the Cl^{37} experiment. The equality signs in equations 7 and 8 correspond to the case in which (for values of θ_V less than 0.3 or so) there is an overall suppression by a factor of 3 in the Cl^{37} experiment.

Figure 6 shows two examples of numerical solutions for the probability, as a function of neutrino energy, that a ν_e arrives at Earth without transformation.[11] Both examples yield a factor-of-3 suppression in the rate observed in the Cl^{37} experiment. In both, the ν_e spectrum produced by the B^8 reaction is distorted relative to the source spectrum. In the solution marked A, high-energy ν_e's are suppressed, and in the solution marked B, low-energy ν_e's are suppressed. Experiments that measured the ν_e energy spectrum could distinguish between these two solutions. Furthermore, both cases differ from the expectations of nonstandard solar models, in which there are no distortions of the energy spectra. Note that in solution B there would be a large suppression of low-energy ν_e's coming from the p + p reaction, while there would be no such suppression in solution A. Assuming the correctness of the fluxes expected from the standard solar models, the Kamiokande II data, which involve only the high-energy B^8 neutrinos, favor solution B but cannot yet rule out solution A.

Future experiments

Experiments so far have provided only limited information about the energies and numbers of neutrinos emitted by the Sun. A complete set of experiments would measure the entire neutrino spectrum and identify neutrinos from each of the different sources in the Sun. Such experiments could indicate whether flavor oscillations, perhaps en-

Analogy with Spin Precession

There is an analogy between neutrino oscillations and spin precession. Consider a spin-$1/2$ system with magnetic moment μ in a magnetic field with components B_z and B_x, which are given in terms of a fixed field B_0 and a varying field $B_1 f(t)$ by

$$B_z = B_0 \cos 2\theta - B_1 f(t)$$
$$B_x = -B_0 \sin 2\theta$$

Here $B_1 \gg B_0 > 0$ and the function $f(t)$ varies from 1 to 0. The equation of motion is

$$i \frac{d}{dt}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \mu \begin{pmatrix} -B_0 \cos 2\theta + B_1 f(t) & B_0 \sin 2\theta \\ B_0 \sin 2\theta & B_0 \cos 2\theta - B_1 f(t) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

We focus our attention on small values of θ. In the absence of B_1 the eigenvectors are

$$|1\rangle = \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}, \quad |2\rangle = \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}$$

The state $|1\rangle$, with spin parallel to the field, has the lower energy. If in the absence of B_1 we start at $t = 0$ in the state $\binom{1}{0}$, the solution represents the precession of the spin around the field, so that there exists an oscillating probability of being in the state $\binom{0}{1}$.

In the presence of B_1 the field at time $t = 0$ points predominantly downward. The original state $\binom{1}{0}$ is now close to the upper eigenstate. As the function $f(t)$ varies from 1 to 0 the field reverses until it points mainly upward with an angle 2θ with respect to the axis; the field never vanishes, however, because there is always an x component of magnitude $B_0 \sin 2\theta$. If the field varies slowly enough, the adiabatic approximation can be used, indicating that the system always remains close to the upper eigenstate. At the end the upper eigenstate is the state $|2\rangle$, which is predominantly $\binom{0}{1}$, spin down. Thus the spin reverses almost completely as the field reverses.

In the case of neutrinos, spin is replaced by flavor:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to |\nu_e\rangle, \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to |\nu_\mu\rangle$$

The B_0 term now represents the difference in the vacuum energies due to the neutrino mass difference. Thus

$$2\mu B_0 \to (p^2 + m_2^2)^{1/2} - (p^2 + m_1^2)^{1/2} \simeq \frac{m_2^2 - m_1^2}{2p}$$

And the angle θ is replaced by $-\theta_V$.

The vacuum eigenstates correspond to the states ν_1 and ν_2 of equation 4. The analog of the spin precession about the field B_0 is neutrino oscillation in vacuum. The B_1 term represents the effect of the index of refraction of matter for ν_e and ν_μ and is given by

$$2\mu B_1 \to \sqrt{2} G N_e(0)$$

where G is Fermi's constant and $N_e(0)$ is the electron density near the center of the Sun, where the neutrinos originate. As the neutrinos pass through the Sun the electron density goes from this value to 0. The neutrinos are born in the state ν_e, which is close to the upper eigenstate at the center of the Sun, and in the adiabatic approximation they remain in the upper state and so emerge primarily as ν_μ. Thus the flavor is almost completely transformed.

hanced by matter effects, are present. Direct evidence of flavor oscillations can be obtained from experiments that detect ν_μ's and ν_τ's via their neutral-current interactions.

Possible radiochemical experiments similar to Davis's Cl^{37} experiment have been considered for many years because the background rates are, in general, relatively small. Of particular interest is the use of Ga^{71} as a target, because its very low threshold of 0.23 MeV makes it sensitive to the neutrinos produced in the Sun by the reaction

$$p + p \to d + e^+ + \nu_e$$

The predicted capture rate[4] via the reaction

$$\nu_e + Ga^{71} \to Ge^{71} + e^-$$

is large—132 SNU—and the uncertainty in the calculation is small—less than about 15%. The p + p reaction contributes slightly more than one-half the total capture rate. An observed capture rate significantly below the predicted rate would be a strong indication of some kind of neutrino transformation. On the other hand, if the measured Ga^{71} rate is near the predicted value, the energy-dependent MSW effect might still be operating, but with values of the parameters Δm^2 and $\sin^2 2\theta_V$ such that the lower-energy neutrinos are not transformed.

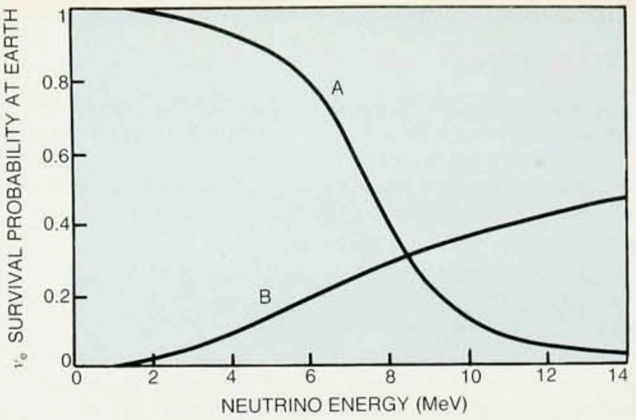
Two experiments to measure the solar neutrino capture rate in Ga^{71} are under construction. A Soviet–US experiment that will eventually exploit 60 tons of metallic gallium has begun taking data with 30 tons of gallium in the Baksan underground laboratory. A second experiment, known as Gallex, is under construction in the Gran Sasso underground laboratory in Italy. This experiment uses 30 tons of natural gallium in an aqueous gallium chloride solution and should be in full operation by the beginning of 1990. Because these experiments are expected to obtain one or two counts per day, statistically significant results from the two experiments should be available for comparison by the end of 1991.

Two second-generation water Čerenkov detectors have been proposed and could be approved for funding within the coming year. These direct-counting experiments have thresholds in the 5–7-MeV energy range and are sensitive only to the B^8 neutrinos, which are produced as shown in the table on page 31.

The Super Kamiokande detector, proposed for a site in the same mine in Japan that the Kamiokande II detector occupies, would have 32 kilotons of water in the sensitive volume as opposed to the 2.1 kilotons in the existing detector. Solar neutrinos would be detected through νe^- elastic scattering (see reaction 2). Because this reaction preserves the directional information of the incident neutrino and has a relatively large rate of 21 per day for an electron threshold energy of 7 MeV, it is an excellent monitor of the time dependence of the higher-energy solar neutrinos.

The Sudbury Neutrino Observatory is a dedicated, high-rate (10–20 events per day) solar neutrino detector proposed for construction at a depth of 6800 feet in the INCO Creighton mine near Sudbury, Ontario (see figure 1). At that depth, cosmic-ray backgrounds are negligible. The unique feature of this detector is the planned use of 1 kiloton of heavy water, D_2O, for a target. The B^8 neutrinos would interact through elastic scattering (reaction 2), through the flavor-specific charged-current reaction

$$\nu_e + d \to p + p + e^- \tag{9}$$



Neutrino propagation. A and B are numerical solutions of the equation describing ν_e propagation through the Sun to the Earth for specific mixing and mass parameters that reduce the response of the Cl^{37} detector by a factor of 3. Both curves correspond to the mixing parameter $\sin^2 2\theta_V = 0.032$. Solution A corresponds to $\Delta m^2 = 1.0 \times 10^{-4}$, and solution B corresponds to $\Delta m^2 = 1.1 \times 10^{-6}$. (Adapted from ref. 14.) **Figure 6**

and through the neutral-current reaction

$$\nu_x + d \rightarrow \nu_x + p + n \tag{10}$$

The signature of reaction 10, which has the same cross section for incident ν_e, ν_μ and ν_τ particles, is the Compton-scattered electron produced by the gamma ray that is emitted when the thermalized neutron is captured.

The electron produced in the charged-current absorption reaction 9 is weakly correlated in direction with the incident neutrino but has an energy equal to the energy of the incident neutrino minus the threshold energy of the reaction. For many of the MSW oscillation parameters consistent with the Cl^{37} experiment, the distortion of the ν_e spectrum produced in the B^8 reaction is directly reflected in a spectral distortion of the electron energy in reaction 9.

The Sudbury Neutrino Observatory experiment would confirm neutrino oscillations independently by comparing reactions 9 and 10, because the neutral-current reaction 10 proceeds with the same cross section for all neutrino flavors, provided the neutrino energy is above the 2.2-MeV threshold for the reaction. In the Sudbury observatory the expected detection rate by reaction 10 is 5–10 events per day. Measurement of both the flavor-specific reaction 9 and the flavor-independent reaction 10 makes the interpretation of this experiment insensitive to the uncertainty in the calculation of the B^8 flux.

Many other experiments are at the research and development stage. Experiments sensitive to the B^8 neutrino flux include a geochemical Mo^{98} experiment at Los Alamos National Laboratory, and direct-counting B^{11} and Ar^{40} experiments[12,13] at Gran Sasso. If the Ga^{71} experiments indicate that the $p + p$ or Be^7 neutrino flux is different from the calculated value, a direct-counting experiment sensitive to the $p + p$ spectrum would be needed. A proposal to use a superfluid He^4 detector is one of the options under development for this purpose.[14]

There are many opportunities to exploit the solar neutrino beam. To move forward we need two pieces of information: the results of the Ga^{71} experiments, to understand the low-energy part of the solar neutrino spectrum; and a measurement of the spectrum of the B^8 neutrinos, to determine whether there is any energy-dependent distortion of the higher-energy neutrinos, as the MSW effect predicts. If these results indicate neutrino flavor oscillations, a measurement of a flavor-independent process would also be required to determine directly the interaction rates of the ν_μ and ν_τ particles. Detailed quantitative studies of the neutrino spectrum provide a unique way of studying neutrino properties such as mass as well as learning more about the interior of the Sun.

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