COSMIC STRINGS: TOPOLOGICAL FOSSILS OF THE HOT BIG BANG

Some grand unified theories require that stringy exotic matter from the very early universe persist to the present, with astrophysically observable consequences.

William H. Press and David N. Spergel

Was the physics of the early universe like the physics of a rotating bucket of superfluid helium? Condensed matter physicists know that phase transitions can form exotic topological objects, such as quantized vortices in superfluid helium or vortex lines of magnetic field in superconductors. In recent years cosmologists have been exploring whether analogous phenomena occurred in the early universe. Cosmic strings, remnants of an ultrahigh-temperature phase transition at 10^{29} K, may have seeded galaxy formation and could have given rise to other effects observable in the universe today.

Cosmic strings—if they exist—are the stable remains of an ultradense state of matter, completely different from any state that exists in bulk today. This state of matter may have existed in the very early universe. The reason cosmic strings can persist long after the bulk form of their type of matter has decayed away is that they are "topologically" conserved. The idea of topological conservation is a powerful one and worth some detailed explanation here.

Topological conservation

Modern field theories represent matter by a set of quantum fields or operators. In the semiclassical limit, these operators can be considered to be ordinary numbers, representing the wave-amplitude or probability density of each field—each type of material "particle"—at every space–time point. These amplitudes evolve by the equations of wave mechanics: bosons (integer-spin fields) by the Klein–Gordon equation, and fermions (half-integer-spin fields) by the Dirac equation.

One key notion of field theory is that of the "potential" in a field's dynamic equation. In the Lagrangian density that underlies the Klein–Gordon equation for a free particle of mass m,

$$\mathscr{L} = \frac{1}{2}\left[(\nabla\phi)^2 - \frac{1}{c^2}(\dot{\phi})^2 + m^2c^2\phi^2\right] \tag{1}$$

the potential $U(\phi)$ is the term $m^2c^2\phi^2$. When the Lagrangian is varied with respect to ϕ to get the dynamic equation, the time derivative (dot) term and the potential combine to give the ϕ field a harmonic-oscillator-like behavior at each point in space (see figure 1a). The spatial gradient term tries to bring nearby points to the same value of ϕ—which, incidentally, also makes for traveling waves and wave packets.

Fields in realistic particle theories are not free particles. Interaction terms are needed. We will focus on a simple phenomenological model for the particle self-interaction and for its interaction with a thermal bath of other particles. In this model, the particle self-interaction can be described by a potential

$$U(\phi) = \lambda(\phi^2 - \eta^2)^2 \tag{2}$$

where λ and η are constants (see figure 1b). The background particles provide a thermal bath that excites the ϕ field. When the ϕ field is in a state of thermal excitement at high temperatures, it populates energy levels high up in the $U(\phi)$ potential, and ϕ can range freely between positive and negative values. If we cool a region of the ϕ field, however, as in an expanding Big Bang universe, then ϕ becomes trapped at low energy levels in one or the other minimum (as shown in figure 1b).

Because the spatial gradient term enforces some continuity on ϕ, nearby points in space cool to the same minimum. Points that are not causally connected—that is, points that are farther apart than a signal can have propagated in the time since the Big Bang—obviously must end up with uncorrelated values of ϕ. The universe, as it cools, thus breaks up into domains of positive and negative ϕ. Domain walls arise from giving the ϕ field and its effective potential a discrete symmetry consisting of two symmetrical minima and termed Z_2.

Cosmic strings occur when there is a slightly more

William H. Press is a professor of astronomy and physics at Harvard University, Cambridge, Massachusetts.
David N. Spergel is an assistant professor of astrophysical sciences at Princeton University, Princeton, New Jersey.

complicated symmetry, termed U(1)—that of a circle (or of a phase in the complex plane). To exemplify U(1) symmetry, we make the ϕ field complex, so that it has real and imaginary parts. If the effective potential depends only on the absolute value of ϕ, then it will have circular symmetry in the complex plane (see figure 2). At high temperatures, the value of ϕ can range anywhere within some disk centered on complex zero, but at low temperatures the value must cool to the circular minimum of the potential.

Pencil forest

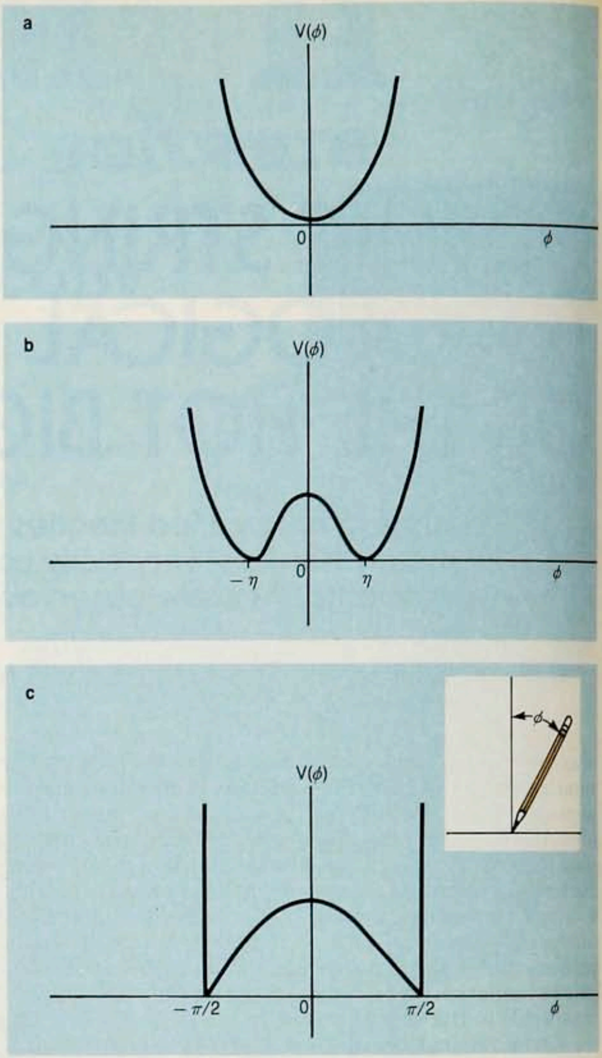
Figure 3 shows a mechanical analogy.[1] Imagine an infinite forest of pencils, with each pencil initially balanced upright on its point. The potential energy of each pencil as a function of its angle ϕ from the vertical is graphed in figure 1c. The energy decreases as $\cos\phi$ until ϕ reaches 90°, then shoots off to positive infinity (representing the constraint force of the hard tabletop). We also want to imagine that each pencil is connected to a few of its nearest neighbors, at the eraser ends, by weak springs, so that when one pencil falls, its neighbors will tend to fall in the same direction—this exemplifies the role of the spatial gradient terms already mentioned.

What will happen if at time zero we let all the pencils fall? They will not all fall in the same direction, because distant parts of the pencil forest have no way of communicating a common direction (except by very slow elastic waves through the weak springs). Rather, the direction of fall will be a continuously varying parameter on the now-felled forest. This direction of fall must have singular points, around which the direction of fall goes through 360° continuously. An amazing, but now obvious, corollary is that at these singular points isolated pencils will still be upright, stably balanced in place by the symmetrical force of the springs connected to their felled neighbors, like the center pole of a tent held by ropes. Even though a horizontal position is energetically favorable locally, an occasional pencil gets topologically "locked" into a high-energy state from which it could escape only by a gross global—and energetically unfavorable—rearrangement of the whole forest.

Our analogy puts a field with U(1) symmetry—the pencils—onto a two-dimensional surface—the forest floor. We have seen that this construction gives one-dimensional regions, points or *monopoles*, where a previously high-energy state is "frozen in." We now want to see what happens if we put the same U(1) symmetry into a three-dimensional space. The answer is that the singular points are not isolated, but form continuous one-dimensional curves, or "strings."

Gauged strings and false vacuum

Robert Brandenberger[2] has given a simple proof of why this is so: Consider a closed curve. If the argument (complex phase) of ϕ around a path changes by 2π, then there must be a singular point, a point of "false vacuum," somewhere on the surface enclosed by the curve. But we can deform this surface and find yet another point of false



Potentials for the Lagrangian of a particle or field. **a:** The harmonic oscillator potential, which describes a free particle of nonzero rest mass. **b:** A potential with two degenerate minima, representing two different vacuum (ground) states. **c:** An idealization of **b**—the potential of a pencil resting on its point at some angle to the vertical. **Figure 1**

vacuum. These points thus connect together to form a string. In three-dimensional space, monopoles are produced by the breaking of a different symmetry—SU(2).

In the grand unified theories the symmetries that can produce strings are "gauged." The ϕ field couples to a gauge field like the electromagnetic field. Because of the presence of the gauge field, cosmological strings differ from the vortex lines one sees in the laboratory. The energy density in the vortex line is distributed equally in logarithmic intervals of distance from the string. The energy density of a cosmic string is confined to its core. Vortex lines have long-range interactions, as do so-called nongauged, or global, strings. The only long-range interaction among gauged strings is gravitational.

In summary, whenever a field theory, a description of matter, contains both a potential with a U(1) symmetry, like that shown in figure 2, and a field that starts out uncorrelated on large scales, it *cannot* cool every-where to a value on the circular minimum. It must be left with one-dimensional loci of ϕ equal to zero, where $U(\phi)$ is nonzero—fossilized one-dimensional remnants of an earlier, high-temperature phase. The state inside the core of the string is called the false vacuum—it appears the same to any observer moving along the string. It has an enormous energy density, $U(0) = \lambda\eta^4$. Thus despite the string's small width—the Compton length of the ϕ field—the string has an enormous mass per unit length, η^2. In typical GUTs, $\eta \approx 10^{16}$ GeV, implying a mass density of 10^{22} g/cm, or 10^7 solar masses per parsec. Although massive, GUT string flux tubes are very narrow—each string has a diameter of only 10^{-30} cm! We will discuss later how these massive strings may seed galaxy formation.

Dynamical strings in action

Having seen a string's topological origin, we now want to investigate its dynamics. Again an analogy is in order. Consider the dynamics of a perfect rubber band. The variable s labels points along the rubber band. The function $\mathbf{x}(s,t)$ describes the positions of these points at all times. The rubber band's equation of motion can be derived from a Lagrangian density containing the usual kinetic energy and potential energy contributions:

$$\mathscr{L} = \tfrac{1}{2}\,\mu\dot{x}^2 - \tfrac{1}{2}\,Tx'^2 \tag{3}$$

Here a dot denotes a time derivative and a prime denotes a derivative with respect to s.

When one varies the Euler–Lagrange equation

$$-\frac{\delta\mathscr{L}}{\delta\mathbf{x}} + \frac{\partial}{\partial t}\frac{\delta\mathscr{L}}{\delta\dot{\mathbf{x}}} + \frac{\partial}{\partial s}\frac{\delta\mathscr{L}}{\delta x'} = 0 \tag{4}$$

one obtains the equation

$$\ddot{\mathbf{x}} = \frac{T}{\mu}\mathbf{x}'' \tag{5}$$

which is instantly recognizable as a wave equation that separates for each Cartesian component of $\mathbf{x}$.

So much for a rubber band. The equation of motion for a cosmic string is almost exactly the relativistic generalization of equation 5, but it comes about from a much prettier action principle: For a gauged string,[3] the Lagrangian density is simply the proper area of the world sheet swept out by the string:

$$\mathscr{L} = |\mathbf{x}'|\,|\dot{\mathbf{x}}|\sin\theta = \sqrt{\dot{\mathbf{x}}^2\mathbf{x}'^2 + (\dot{\mathbf{x}}\cdot\mathbf{x}')^2} \tag{6}$$

(Recall from relativistic mechanics that the action of a free point particle is the length of its world line.) This action associated with the area of a world sheet is called the Nambu action, after Yoichiro Nambu. It has attracted much attention in recent years as the action for superstrings. (Superstring theory is an attempt to find a fundamental theory that unifies gravity with the other interactions. Superstrings are fundamental objects moving in a ten-dimensional space–time; cosmic strings, which are composite particles or topological defects, move in a four-dimensional space–time.) This action has a property called Weyl invariance: The action is invariant under any conformal transformation, a symmetry that has powerful mathematical and physical implications.

Variation of equation 6 (in flat Minkowski space, as a start) gives the relativistic wave equation

$$\ddot{\mathbf{x}} = c^2\,\mathbf{x}'' \tag{7}$$

(We have chosen to work in conformal gauge, where $\dot{\mathbf{x}}\cdot\mathbf{x}' = 0$ and $|\mathbf{x}'|^2 + |\dot{\mathbf{x}}|^2 = 1$.) A string behaves exactly like a relativistic rubber band, with the ratio of the tension to the mass per unit length, μ, being the speed of light squared!

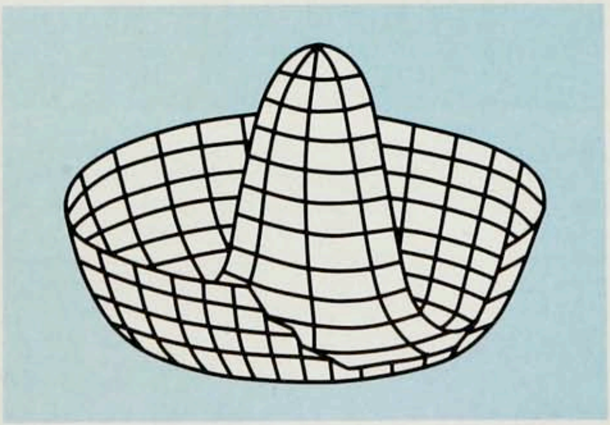
Crossing strings

Equation 7 is simple enough that (as Neil Turok and T. W. B. Kibble[4] have noted) we can write down its general solution, a sum of left- and right-moving waves propagating at the speed of light on the string:

$$\mathbf{x} = \tfrac{1}{2}\,[\mathbf{a}(s+ct) + \mathbf{b}(s-ct)] \tag{8}$$

where $\mathbf{a}$ and $\mathbf{b}$ are periodic functions (for a closed loop of string). Our gauge choices enforce the additional condition $|\dot{\mathbf{a}}| = |\dot{\mathbf{b}}| = 1$, which means that the 3-vectors $\dot{\mathbf{a}}$ and $\dot{\mathbf{b}}$ lie on the unit sphere. Because $\dot{\mathbf{a}}$ and $\dot{\mathbf{b}}$ are derivatives of periodic functions, their mean must be zero. Thus they will visit every hemisphere, and they will usually cross. When they cross, $\dot{\mathbf{a}} = \dot{\mathbf{b}}$ and a point on the string instantaneously reaches the speed of light. This transient point is called a *cusp*. Because of their high velocities, cusps are the dominant source of gravitational and electromagnetic radiation from the string loop.

Not all string loops must have cusps. There are special solutions without crossings. For example, $\mathbf{a}$ and $\mathbf{b}$ can follow a pattern like the seams on a baseball and avoid cusps. Alternatively, there can be discontinuities in either curve. An oscillating string loop can cross itself, reconnect and split into smaller loops (see figure 4). Reconnections naturally produce discontinuities that allow one curve to



Generalization of the potential in figure 1b to a complex scalar field ϕ. In the complex ϕ plane, there is a circle of degenerate minima, in the rim of the "hat," termed U(1) symmetry. **Figure 2**

"jump" across the other and avoid forming a cusp. These discontinuities result in kinks that propagate along the string. The string's velocity changes discontinuously at kinks. Recent articles by Tanmay Vachaspati and David Garfinkle,[5] Christopher Thompson,[6] and Robert Scherrer and Press[7] provide discussions of kinks and cusps.

Oscillating loops of string have varying mass quadrupole moments. They are thus a source of gravitational radiation. The gravitational radiation power from an oscillating mass is on the order of

$$P_{\rm rad} \sim \frac{P_{\rm int}^2}{c^5/G} \tag{9}$$

where $P_{\rm int}$ is the internal oscillating "power flow" of the string. A string of length R has a mass of roughly μR. Every oscillation period, $R/2c$, the string moves $\mu c^2 R$ of energy through space. The string's internal power flow is thus $E/t = \mu c^3$, and it radiates

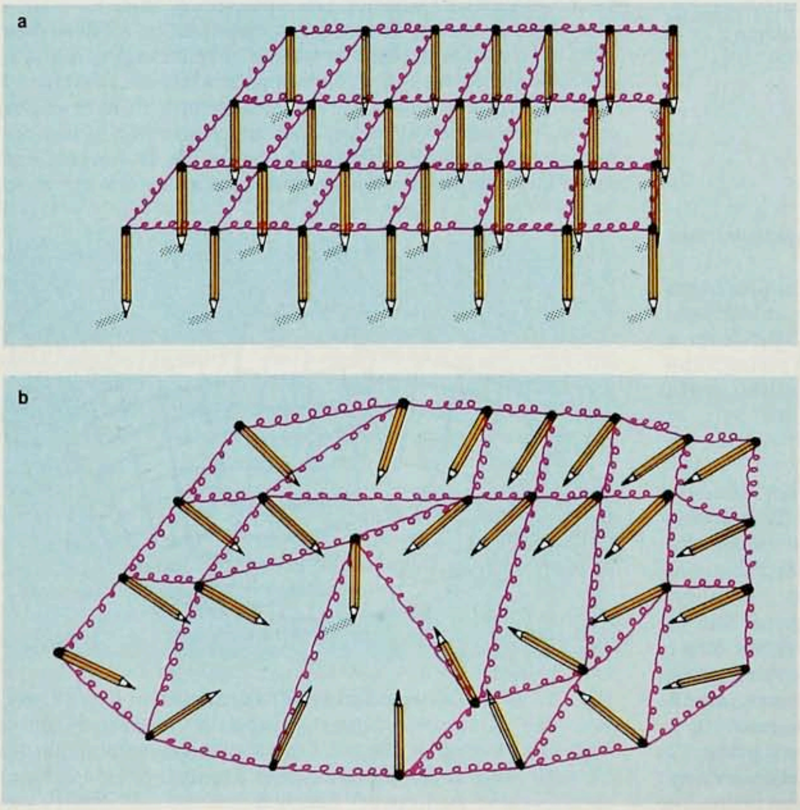
$$P_{\rm grav} \sim \left(\frac{G\mu}{c^2}\right)\mu c^3 \tag{10}$$

in gravitational radiation. The term $G\mu/c^2$, a dimensionless measure of a string's mass per length, indicates how important the gravitational effects of strings are. Equation 10 implies that a string loses $G\mu/c^2$ of its length every oscillation time. Because the oscillation period decreases as the string shrinks, a loop of string shrinks to zero size in a finite time.

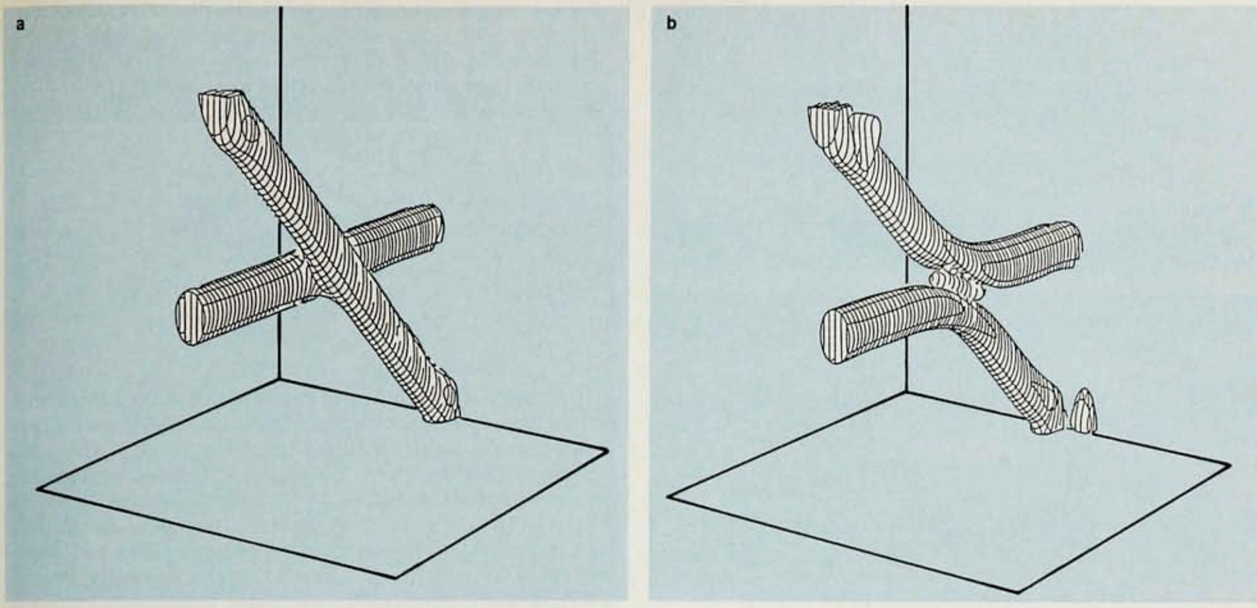
String evolution

Having described the basic mechanical behavior of a string, let us turn to the evolution of cosmic strings in the early universe. In the standard hot Big Bang, the universe is very hot at early times. At those high temperatures, many of the symmetries broken in today's universe (such as the electroweak and grand unification symmetries) are restored. As the universe expands and cools, these symmetries become broken, and if any of the symmetries had a topology similar to that of the U(1) field, cosmic strings form. Because the Higgs field cannot be coherent on a scale larger than the horizon size that represents the light travel distance since time zero, the initial string network has the topology of a self-avoiding random walk, with a correlation length smaller than the horizon size.[8]

As the universe continues to cool, its energy density drops. Cosmic strings decouple from the surrounding matter and begin to oscillate freely. These freely oscillating strings often cross. When two segments of string cross, they usually intercommute,[9,10] as shown in figure 4. And



Forest of pencils interconnected by springs. In **a** the pencils are balanced on their points. In **b** almost all have fallen down flat on the table. However, pencils at the singular points of the U(1) field remain stably standing, with their potential energy in the central peak (false vacuum) of figure 2. Strings result from an analogous process in three dimensions. **Figure 3**



Crossing strings. When strings cross, they generally intercommute, that is, reconnect in a sense that will result in an eventual shortening of their length. (From numerical calculations of Richard Matzner.[9]) **Figure 4**

as figure 5 indicates, this reconnection process can either spawn new, small string loops or append small string loops to a large string. Numerical simulations by Andreas Albrecht and Turok[11] and by David Bennett and François Bouchet[12] suggest that in an expanding universe, the generation of small loops is the dominant process. Figure 6 shows a "snapshot" of the evolved string network from a simulation by Bennett and Bouchet. Most of the energy density of this network is in strings that meander off to infinity in both directions, but one also sees a large number of small, disconnected loops. The network is thought to evolve toward a similarity solution in which the ratio of finite to infinite strings is constant.

In fact, if strings did not rapidly convert most of their energy density into loops, they would have dominated the energy density of the universe long ago. The energy density in infinite strings declines as only the inverse square of the expansion factor, a, while the energy density in radiation drops as a^{-4}. The formation of loops, however, is not enough to save the universe from string domination: The energy density in string loops decays away as a^{-3}. Fortunately, the rapidly oscillating loops decay into gravitational radiation, whose energy density also scales as a^{-4}; the similarity solution "saves" our universe from string domination. The similarity solution suggests that most of the energy density in the original string network was long ago radiated as gravitational radiation. However, some energy density remains in large loops (whose radii exceed $G\mu/c^2$ multiplied by the age of the universe) and in infinite strings, whose characteristic radii of curvature are comparable to the horizon size.

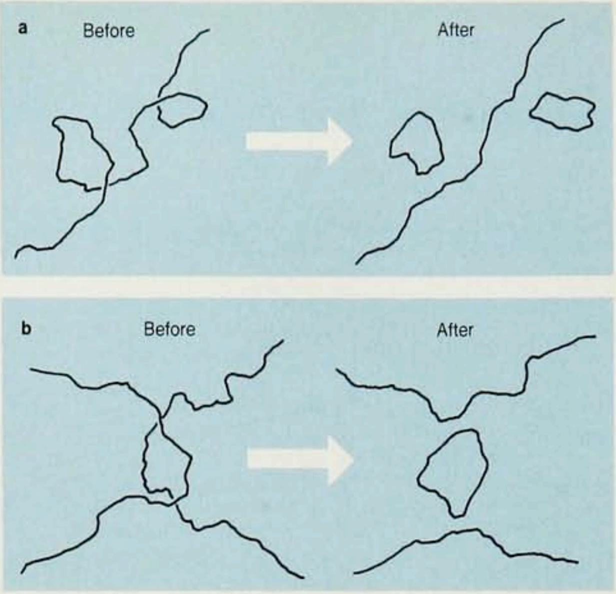
Large-scale structure

Observations of the cosmic background radiation reveal that the early universe was highly homogeneous at a redshift z of about 1300. The existence of galaxies and the clustering of galaxies on large scales suggest that the universe has become rather inhomogeneous on scales as large as 1% of the horizon size, and perhaps on even larger scales. Astrophysicists studying galaxy formation must attempt to explain this structure. Their effort is handicapped by our ignorance of the nature of the "dark matter" that constitutes about 90% of the matter in galaxies and by our ignorance of the initial perturbations that stimulated the growth of the inhomogeneities that formed galaxies.

In the most popular model of galaxy formation,[13] perturbations generated during an early inflationary epoch produce a scale-invariant Gaussian fluctuation spectrum. These fluctuations grow in a universe consisting mostly of nonrelativistic nonbaryonic particles ("cold dark matter") to form the halos of galaxies. Baryons fall into these halos, cool and form the stars in galactic disks and spheroids. This picture explains both the flat rotation curves and the angular momenta that are observed in galaxies; however, it is *not* consistent with the reported large-scale streaming velocities and the strong two-point correlations between clusters of galaxies. [See PHYSICS TODAY, April 1987, page 28, and October 1987, page 19.] And because galaxy formation is a relatively recent phenomenon according to this scenario, recent observations of high-redshift galaxies[14] are also difficult to explain.

Cosmic strings provide an alternative scenario for the formation of large-scale structure.[15] According to this picture, the surviving cosmic-string loops serve as nonlinear seeds and accrete both dark matter and baryons. Because the seeds start out as nonlinear—that is, already very dense—perturbations, galaxy formation starts at a very high redshift. High-redshift quasars and galaxies are not problematic for the cosmic-string-seeded galaxy formation scenario. If the "missing mass" is cold dark matter, however, the early accretion around the nonlinear perturbations generated by cosmic-string loops produces halos that are too dense.

If the unseen matter is in the form of neutrinos, the high neutrino temperature suppresses matter accretion around loops at high redshift. The spectrum of perturbations generated by strings and hot dark matter is similar to that of cold dark matter and inflation (without strings); these perturbations grow to form galaxies.[16] Hot dark matter plus strings, like cold dark matter plus inflation,



String reconnection can result either in the separation of daughter loops from a mother infinite string, as in **a**, or in the formation of longer mother strings, as in **b**. In an expanding universe, the formation of daughter loops is statistically favored. **Figure 5**

may account for large-scale structures in the universe, except for the very largest (in excess of 10 megaparsecs).[17]

Because cosmic strings generate non-Gaussian fluctuations, the structures that form around cosmic-string loops and in the wakes of oscillating infinite strings are qualitatively different from structures formed from Gaussian initial perturbations. Turok[18] suggested that clusters of galaxies ought to be identified with large string loops—this would explain the observed slope and amplitude of the cluster–cluster two-point correlation function. Recent numerical work,[19] however, casts doubt on the one-to-one identification of clusters with large loops and suggests that large loop velocities may erase initial loop–loop correlations. This work thus may eliminate one of the touted successes of the cosmic-string galaxy formation scenario.

Observable effects of strings

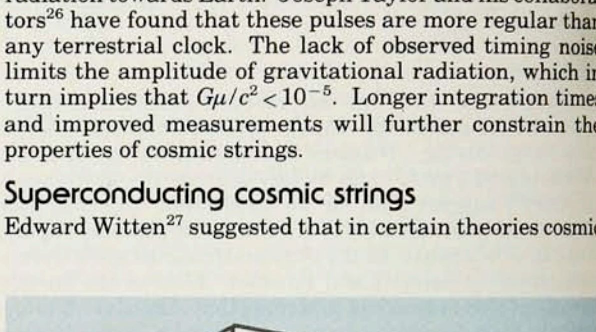
Future observations may reveal the existence of cosmic strings directly. Cosmic strings curve space: While a straight string does not attract matter, it does curve space–time. In effect, it cuts a wedge out of space–time and produces a conical universe.[20,21] (An observer cannot differentiate locally between conical and flat space–time—the straight stationary cosmic string produces only global effects.) Because of this wedge, photons emitted by an object behind a cosmic string can take two possible paths to our telescopes. Thus these objects are doubled into two images of equal brightness. Lennox Cowie and Esther Hu[22] have claimed that they may have observed such a cosmic-string lens: They found several pairs of galaxies with nearly identical redshifts. Future observations will test whether this is indeed evidence for a cosmic string.

Not only will cosmic strings lens distant galaxies and quasars, they also will perturb the cosmic background radiation. J. Richard Gott[21] and Nick Kaiser and Albert Stebbins[23] pointed out that moving strings will induce perturbations in the CBR. Photons in the wake of the string are redshifted; photons in the path of the string are blueshifted. This should create a unique signature in the infrared sky (see figure 7 and the cover of this issue). The failure so far to detect any perturbations argues[24] that $G\mu/c^2 < 10^{-5}$. Improved measurements and reductions in theoretical uncertainties could tighten this limit. (String-seeded galaxy formation scenarios require that $G\mu/c^2 \sim 10^{-6}$.)

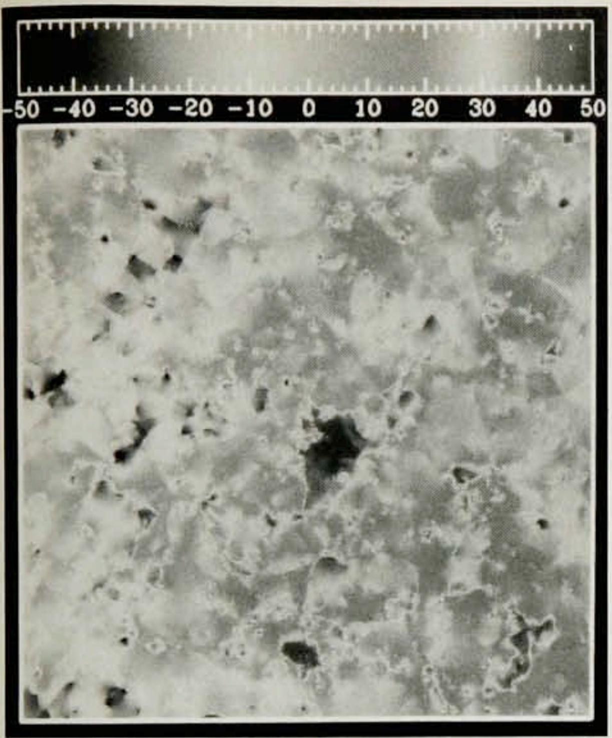
Even the gravitational radiation predicted by cosmic-string theories could be their undoing. The gravitational radiation produced by decaying string loops in the early universe should survive to today and produce fluctuations in the fabric of space–time.[25] This radiation should induce slight variations in the distance between Earth and astronomical objects. Such variations would be undetectable were it not for the existence of extremely precise astronomical clocks—the so-called millisecond pulsars. These rotating neutron stars regularly beam pulses of radiation towards Earth. Joseph Taylor and his collaborators[26] have found that these pulses are more regular than any terrestrial clock. The lack of observed timing noise limits the amplitude of gravitational radiation, which in turn implies that $G\mu/c^2 < 10^{-5}$. Longer integration times and improved measurements will further constrain the properties of cosmic strings.

Superconducting cosmic strings

Edward Witten[27] suggested that in certain theories cosmic



'Snapshot' of an evolved network of strings in an expanding universe. The infinite strings intersect and spawn large numbers of small daughter loops. (From numerical calculations of David Bennett and François Bouchet.[12]) **Figure 6**



Fluctuations expected in the temperature of the cosmic background radiation if galaxy-forming perturbations are due to cosmic strings. These non-Gaussian perturbations could in principle be distinguished from the Gaussian perturbations predicted by other theories, for example, inflationary cosmology. The same image is shown in color on the cover of this issue. (From numerical calculations of Bouchet, Bennett and Albert Stebbins.[24]) **Figure 7**

strings could behave as superconductors and carry large electric currents. The addition of electromagnetism enriches enormously the interaction of a string with its surroundings. If they exist at all, superconducting strings will be much easier to observe than their nonconducting cousins. Oscillating superconducting strings will emit not only gravitational radiation but also copious amounts of electromagnetic radiation. Jeremiah Ostriker, Thompson and Witten[28] have suggested that these strings would blow giant cosmological bubbles, scaled-up analogs of the Crab Nebula. (That Galactic supernova remnant, which consists of gaseous debris from an explosion seen in 1054 AD, is powered by low-frequency radiation from a pulsar within.) According to their scenario, galaxies could form on the surfaces of these bubbles. In this alternative picture of galaxy formation, strings *push* matter rather than attract it!

The superconducting-cosmic-string scenario requires strong magnetic fields to generate the initial currents along the strings. There exists no known mechanism for generating magnetic fields in the standard inflationary universe, hot Big Bang scenario. However, superconducting cosmic strings could behave as dynamos[29] and convert some of their mechanical energy into electromagnetic energy.

Even if superconducting cosmic strings are not powerful enough to power galaxy formation, they might still be detectable through their effects on the surrounding environment. An oscillating string loop in our own Galaxy would generate shocks as it interacted with the Galactic magnetic field. Eugene Chudnofsky, George Field, Spergel and Alexander Vilenkin[30] suggested that these shocks would produce synchrotron radiation. Linear synchrotron sources moving at relativistic speeds would be a noticeable and clear string signature.

The detection of cosmic-string loops, through either their gravitational or their electromagnetic effects, would be a dramatic discovery. It would reveal the existence of a phase transition at scales inaccessible in the laboratory and would provide a window into the very earliest moments of the universe.

References

1. G. F. Mazenko, W. G. Unruh, R. M. Wald, Phys. Rev. D **31**, 273 (1985).
2. R. H. Brandenberger, *Inflationary Universe Models and Cosmic Strings*, World Scientific, Singapore, to be published.
3. R. Gregory, Phys. Lett. B **206**, 199 (1988). K. Maeda, N. Turok, Phys. Lett. B **202**, 376 (1988).
4. N. Turok, T. W. B. Kibble, Phys. Lett. B **116**, 141 (1982).
5. T. Vachaspati, D. Garfinkle, Phys. Rev. D **36**, 2229 (1987).
6. C. Thompson, Phys. Rev. D **37**, 283 (1988).
7. R. J. Scherrer, W. H. Press, Phys. Rev. D **39**, 371 (1989).
8. T. W. B. Kibble, J. Phys. A: Gen. Phys. **9**, 1387 (1986). R. J. Scherrer, J. A. Frieman, Phys. Rev. D **33**, 3356 (1986).
9. R. A. Matzner, Computers in Physics, September–October 1988, p. 51.
10. K. J. M. Moriarty, E. Myers, C. Rebbi, in *Cosmic Strings: The Current Status*, F. Accetta, L. Krauss, eds., World Scientific, Singapore (1989), in press.
11. A. Albrecht, N. Turok, Phys. Rev. Lett. **54**, 1868 (1985).
12. D. Bennett, F. Bouchet, Phys. Rev. Lett. **60**, 257 (1988).
13. G. R. Blumenthal, S. M. Faber, J. R. Primack, M. J. Rees, Nature **311**, 517 (1984). S. D. M. White, C. S. Frenk, M. Davis, G. Efstathiou, Astrophys. J. **313**, 505 (1987).
14. S. Lilly, Astrophys. J. **333**, 161 (1988).
15. See A. Vilenkin, Phys. Rep. **121**, 263 (1985), for a review.
16. R. J. Scherrer, A. L. Melott, E. Bertschinger, Phys. Rev. Lett. **62**, 379 (1989).
17. R. Brandenberger, N. Kaiser, D. N. Schramm, N. Turok, Phys. Rev. Lett. **59**, 2371 (1987). E. Bertschinger, P. N. Watts, Astrophys. J. **316**, 489 (1988).
18. N. Turok, Phys. Rev. Lett. **55**, 1801 (1985).
19. A. L. Melott, R. J. Scherrer, Nature **328**, 691 (1987). E. P. S. Shellard, R. H. Brandenberger, Phys. Rev. D **38**, 3610 (1988).
20. A. Vilenkin, Phys. Rev. D **23**, 852 (1981). W. A. Hiscock, Phys. Rev. D **31**, 3288 (1985).
21. J. R. Gott III, Astrophys. J. **288**, 422 (1985).
22. L. L. Cowie, E. M. Hu, Astrophys. J. **318**, L33 (1987).
23. N. Kaiser, A. Stebbins, Nature **310**, 391 (1984).
24. F. R. Bouchet, D. Bennett, A. J. Stebbins, Nature **335**, 410 (1988).
25. C. J. Hogan, M. J. Rees, Nature **311**, 109 (1984).
26. L. A. Rawley, J. H. Taylor, M. M. Davis, D. W. Allan, Science **238**, 761 (1987).
27. E. Witten, Nucl. Phys. B **249**, 557 (1985).
28. J. P. Ostriker, C. Thompson, E. Witten, Phys. Lett. B **180**, 231 (1986).
29. D. N. Spergel, W. H. Press, R. H. Scherrer, Phys. Rev. D **39**, 379 (1989).
30. E. M. Chudnofsky, G. B. Field, D. N. Spergel, A. Vilenkin, Phys. Rev. D **34**, 944 (1986). ■