



ON THE CALCULATION OF THE FINE-STRUCTURE CONSTANT

David J. Gross

One of the best of the many Pauli jokes tells of Pauli's arriving in Heaven and being given, as befits a theoretical physicist, an appointment with God. When granted the customary free wish, he requests that God explain to him why the value of the fine-structure constant, $\alpha = e^2/\hbar c$, which measures the strength of the electric force, is 0.00729735.... God goes to the blackboards and starts to write formulas furiously. Pauli watches with pleasure but soon starts shaking his head violently, his characteristic mode of disapproval. Even God could not live up to Pauli's standards!

Like Pauli, every physicist has a wish list of questions that he or she would like to see answered. The progress of physics can be tracked by the changing nature of these questions. Until recently, a large percentage of physicists would have put explaining the value of α near the top of the list. This was natural in the days when the only satisfactory theory of elementary interactions available was electrodynamics and the fundamental significance attached to this dimensionless number was obvious. In addition, it had acquired a special mystique since $1/\alpha$ was initially thought to equal precisely the unlikely integer 137. In this column I shall explore how our view of this constant has changed.

The apparent integer value of $1/\alpha$ stimulated much numerical mumbo jumbo. The most famous case involved Arthur Eddington, a distinguished astronomer and early proponent of general relativity who fell into a numerological abyss in his later career. His theory of α was based on the notion that the strength of the electric force was inversely proportional to the number of varieties of the elementary charges. Edding-

ton claimed that every electron could be assigned one of 16 labels (the number arose first from the square of the dimension of space-time and later from the number of elements in Dirac's matrices). For two electrons there were 136 different pairs of labels (combinations such as 1,2 and 2,1 were counted only once owing to the indistinguishability of electrons). Therefore α was equal to $1/136$. The fact that this bizarre theory was slightly off did not faze Eddington, who wrote, "I think if we can account for $136/137$ of the quantum, the remaining $1/137$ will not be long in turning up."

This absurd theory provoked one of the best practical jokes in physics, a spoof on Eddington by G. Beck, Hans Bethe and W. Riezler. They proposed that the value of α was related to the value of the absolute zero of temperature, T_0 , in centigrade! Since, according to Eddington, an electron had $1/\alpha$ degrees of freedom, and since all degrees of freedom are frozen at absolute zero, they equated $-T_0$ to $2/\alpha$. The factor of 2 was there because every electron is accompanied

by a proton, and 1 was subtracted because the orbital motion of the electron is not frozen! This yields $T_0 = -273$, which, as they noted, agrees well with the experimental value. They submitted a paper to *Naturwissenschaften*, the preeminent scientific journal of the day. The paper slipped through the refereeing process and was duly published. (Such a slip could, of course, never happen today.)

A severe blow to simple numerology was provided by increased experimental precision, which demonstrated that $1/\alpha$ was not an integer. This did not completely stop further speculation. The most recent example that attracted widespread attention was the work of the mathematician Armand Wyler, who related α to the ratio of some (strangely normalized) group volumes. His formula,

$$\alpha = \frac{9}{8\pi^4} \left(\frac{\pi^5}{2^4 5!} \right)$$

is accurate to 1 ppm (see PHYSICS TODAY, August 1971, page 18). Its appearance set off a flurry of semi-serious searches for similar relations



David Gross, a particle theorist, is Eugene Higgins Professor of Physics at Princeton University.

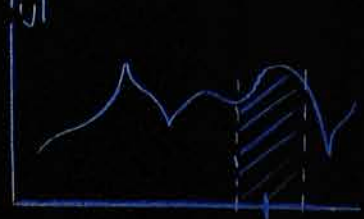
$$y \sim \frac{36}{3} \frac{1}{3} \cos(t) + \frac{E}{72} (-\cos(t) + 3\cos(3t)) + \dots + PV' \left(\frac{2V'}{r} + V^2 \cot(\Theta) \right)$$

$$+ V' \left(V' \frac{\partial P}{\partial r} + V^2 \frac{\partial P}{\partial \theta} + V^3 \frac{\partial P}{\partial \varphi} \right) + \frac{VISC}{TER}$$

control pitch thru $\vec{u}: \vec{y}' = A\vec{y} + B\vec{u}$

$$TFM_{1,1} = \frac{\alpha}{s^3 - 2s^2 + s - 2\alpha}$$

$$\left[\text{Matrix} \cdot t \right]_{1,2} = \frac{6e^{5/2 t}}{\sqrt{33}} \sinh\left(\frac{\sqrt{33}}{2} t\right) \text{ from Macsyma}$$

$$|TFM_{ij}|$$


$$\text{Fourier}|\sin(t)| \rightarrow \frac{2}{\pi} \left(1 - \sum_{n=1}^{\infty} \frac{(1+(-1)^n) \cos(n)}{n^2 - 1} \right)$$

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(see PHYSICS TODAY, November 1971, page 9). It proved surprisingly simple to find formulas of equal simplicity, accuracy and vacuity.

With the emergence of the standard model of the unified electromagnetic and weak interactions (see the article by Paul Langacker and Alfred K. Mann on page 22) and the deepening understanding of quantum field theory, our view of the significance and calculability of α has changed considerably. First, we have realized that because of the effects of renormalization, coupling constants must be regarded as *running* parameters whose values change as we explore physics at different scales. In quantum electrodynamics, the renormalization of the electric charge arises from the polarizability of the physical vacuum, which is full of virtual electron-positron pairs. This phenomenon causes the effective fine-structure constant to change with distance or, equivalently, with energy. The fine-structure constant itself is simply the zero-energy limit of the electric coupling. But who is to say that the low-energy value is fundamental?

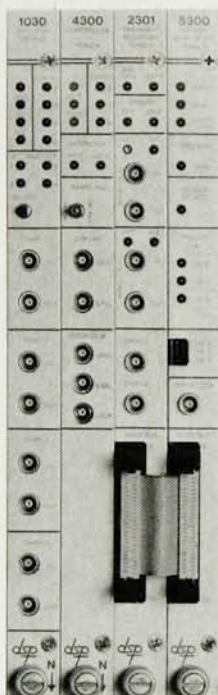
Unification sheds further light on this issue. Electromagnetism and the weak interactions have been unified into a combined electroweak theory that possesses two independent coupling constants. In this framework, nothing appears to fix the value of either coupling. They are equally fundamental and equally noncalculable.

The more ambitious "grand unification" schemes combine the electroweak and the strong interactions in theories with a unique coupling constant. In the most ambitious of these, say, string theory, one can imagine deducing the value of this coupling. However, even if one were to achieve this goal one would end up calculating the value of the unified coupling constant at the unification scale, which lies anywhere from 10^{15} to 10^{19} GeV. The value of α itself would be related to this coupling constant by a long succession of renormalization, symmetry breaking and other contortions that the theory undergoes as it makes its way down in energy by 17 orders of magnitude.

Today's physicist, given a similar opportunity to ask one question of the Supreme Physicist, would probably not waste it on α , but would rather ask, "Why are there three generations of quarks and leptons?" or "Why does the cosmological constant vanish?" or "Why is space-time four dimensional?" In answering these questions, the value of α might emerge as an incidental by-product. ■



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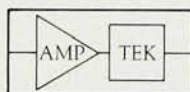
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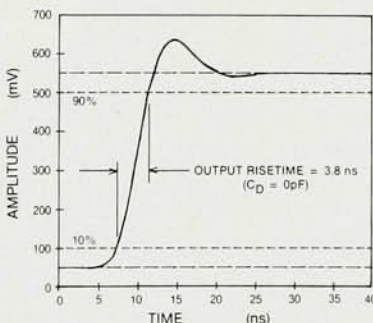
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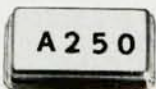


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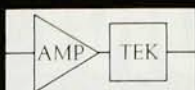
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