THE UNIFICATION OF ELECTROMAGNETISM WITH THE WEAK FORCE

Maxwell unified electricity, magnetism and optics. The theory that now unifies quantum electrodynamics with the weak nuclear force is a comparable triumph.

Paul Langacker and Alfred K. Mann

The year 1983 marked the end of a particularly noteworthy decade in the development of elementary-particle physics. Between 1973 and 1983 there were three accomplishments of special, perhaps historic, importance:

▷ The theoretical and experimental unification of the forces that govern all of the phenomena of both electromagnetism and the weak interactions.

▷ The recognition that the strongly interacting particles (nucleons, mesons and other hadrons) are in fact made of still smaller entities, now known as quarks, and the development of a tentative theory of the force between the quarks.

▷ The identification of three almost identical (except for mass) families of elementary "particles," each family consisting of two quarks, one charged lepton (the electron, the muon or the tau) and a neutrino.

The unified theory of the electromagnetic and weak forces, combined with the tentative theory of the strong force, referred to as the "standard model," has been remarkably successful in providing an accurate description of the fundamental particles and their interactions. It is a mathematically consistent theory that, when combined with general-relativistic gravitation, either predicts or at least incorporates all known properties of matter under ordinary terrestrial and astrophysical conditions down to 10^{-16} cm, the smallest distance that has been probed at the most powerful particle accelerators.

Despite the astonishing successes of the standard model, few physicists regard it as a serious candidate for the ultimate theory of matter. The theory is just too complicated, and it leaves too many things unexplained. For example, it involves a large number (21 in the simplest version) of fundamental constants or parameters, including the masses of the elementary particles and the strengths of the various forces. The numerical values of these parameters can be taken from experiment, but they are not predicted by the model or understood on any fundamental level. Similarly, the standard model incorporates, but does not explain, the fact that the electric charges of the proton and electron are opposite in sign but precisely equal in magnitude.

The family classification of the elementary particles is particularly bewildering. Matter under ordinary conditions is made almost entirely of particles from the first, and lightest, of the three families. Although the heavier families are routinely produced in the laboratory and in cosmic-ray interactions, we still have no understanding of their ultimate meaning or role. Furthermore, the quantitative experimental testing of "quantum chromodynamics," the mathematical theory of the strong interactions of quarks and hadrons, is far from complete. Finally, the dream of a grand theoretical unification of the strong force with the electromagnetic and weak forces, not to mention the even more distant goal of their unification with gravity, is yet to be fulfilled.

Electroweak unification

Which then, if any, of the accomplishments of 1973–83 remains as an essentially complete, self-consistent theory, well tested by experiment and ready to be canonized in physics textbooks? The answer is the "electroweak theory," the relativistic quantum field theory from which flow, in a direct and natural way, exquisitely quantitative descriptions of all known electromagnetic and weak-interaction phenomena over a range of energies that spans many orders of magnitude. The electroweak theory represents the fusion of two bodies of knowledge—quantum electrodynamics and quantum weak dynamics—just as Maxwell's electrodynamics in the 19th century represented the fusion of classical electricity and magnetism.

The electroweak theory is a gauge theory. This means that the forms of the equations describing a physical

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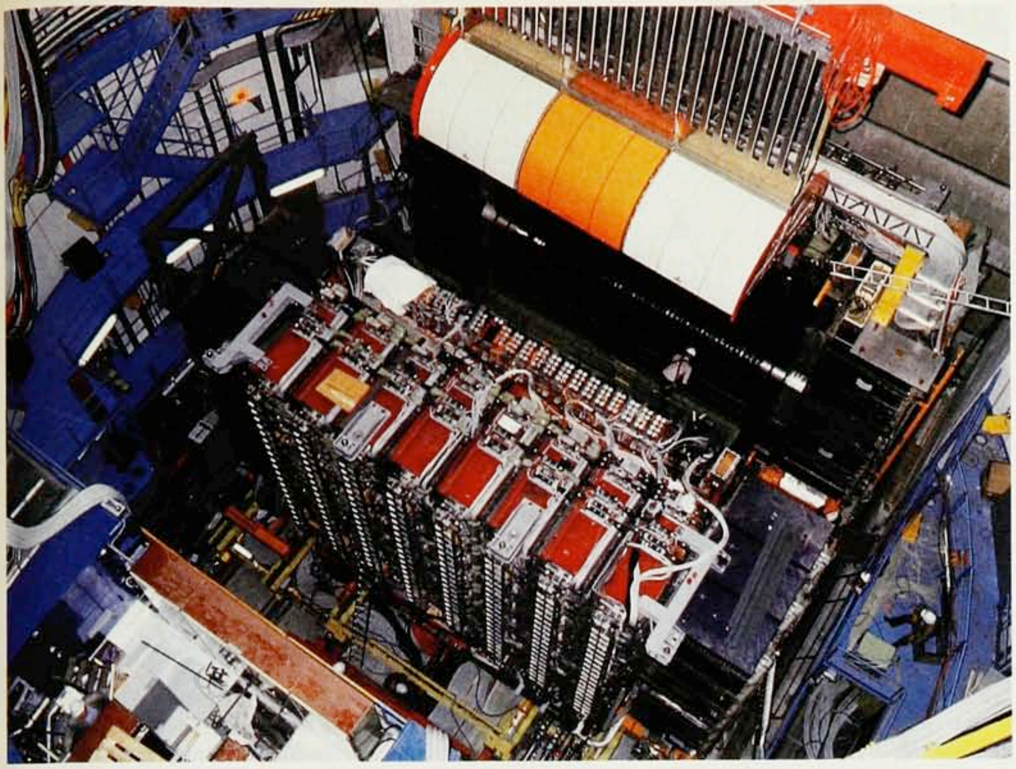


CERN

Large UA1 detector at CERN was built to record $\bar{p}p$ collision products when the CERN Super Proton Synchrotron was converted into a 270×270-GeV proton–antiproton storage-ring collider. In 1983 the UA1 group was the first to report[17] the discovery of the $W^{\pm}$ and Z^0 intermediate vector bosons predicted by the electroweak theory. **Figure 1**

system are unchanged by certain (gauge-symmetry) transformations of the quantities appearing in them, and that the transformations can be performed independently at different points in space and time. Surprisingly, the equations describing the motion of free (noninteracting) particles are not gauge invariant: their kinetic energies are changed by gauge transformations. A gauge-invariant theory *requires* additional terms in the equations to compensate. These extra terms correspond to new physical particles that can mediate interactions between the original particles and in some cases with each other. Because these mediating particles must have unit spin, they are referred to as vector bosons. The requirement of gauge invariance dictates not only the existence of these vector bosons but also the detailed form of the interactions they mediate.

Quantum electrodynamics is the classic example of a gauge theory. In QED the photon is the vector boson associated with changes in the phases of the fields associated with electrically charged particles. The electroweak theory exhibits an even larger class of gauge symmetries. The variables describing particles of different electric charge (such as the electron and neutrino) can be interchanged in the electroweak equations without changing the form of these equations. This larger symmetry requires (in addition to the massless photon) three massive vector bosons: W^+, W^- and Z^0. The existence of all three was first verified at the CERN Super Proton Synchrotron in 1983, by means of the enormous detector shown in figure 1.

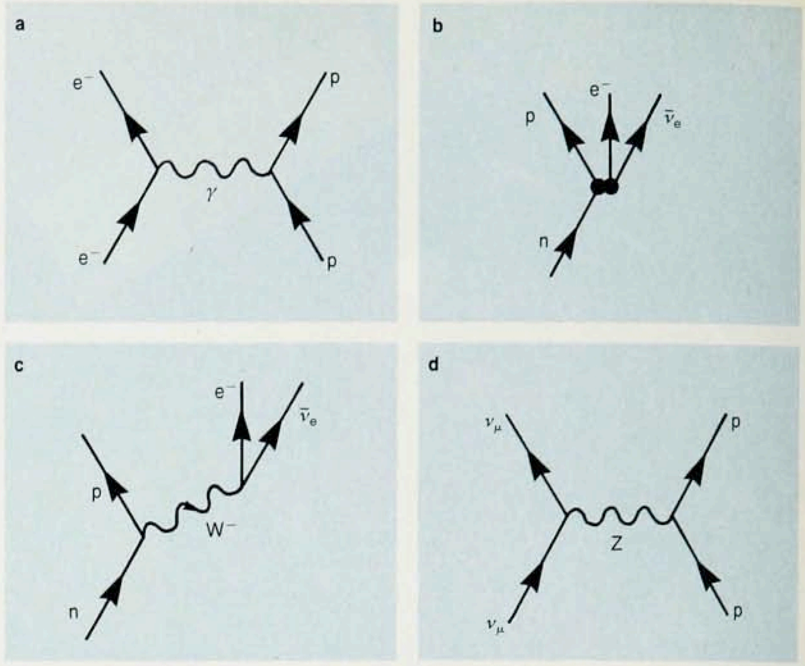
The electroweak theory and the phenomena it describes have been extensively reported and reviewed in the specialized literature. But only recently has it become possible to display the remarkable quantitative agreement and consistency of experiment and theory in such a wide variety of physical processes and over such a wide range of energies as we will describe in this article. We will document the extraordinary success of this unifying theory with accounts of representative experiments ranging from low-energy atomic physics to high-energy neutrino beams, to electron–positron colliders and finally to ultrahigh-energy proton–antiproton colliders.

History

Quantum electrodynamics is unquestionably the most successful theory in the history of science. It represents the merging of three great ideas of modern physics: classical electrodynamics, quantum mechanics and special relativity. The basic formulation of QED was completed by 1930. It combined Dirac's theory of the electron (with its correct predictions of the existence of the positron and the first-order magnetic moment of the electron) with the quantization of the electromagnetic field into individual photons (as indicated in figure 2a). But the calculation of higher-order processes was plagued by divergent integrals over the energies of intermediate states.

The correct interpretation (or, more accurately, evasion) of these infinities was completed in the late 1940s by Hans Bethe, Richard Feynman, Sin-Itiro Tomonaga, Julian Schwinger and Freeman Dyson. It was found that one could rescale the fields so that the divergences all occurred in the renormalization of the mass and charge of the electron from their lowest-order values in the absence of interactions. All physical quantities became finite when they were expressed in terms of the renormalized (that is to say, experimentally measured) values of the mass and charge.

QED thus became a mathematically consistent and well-defined theory. It has been tested to incredible precision (sometimes to a part in 10^7) by measurements of the Lamb shift, the hyperfine splitting in hydrogen, the positronium spectrum and the magnetic moments of the electron and the muon. At much higher energies—for example, in electron–positron elastic scattering at tens of GeV—quantum electrodynamics has in recent years been tested successfully down to the few-percent level.



Feynman diagrams for photon exchange in quantum electrodynamics (**a**) and neutron beta decay in the old Fermi theory, with no finite-range mediator (**b**). In the modern, unified electroweak theory, the weak interactions are mediated by massive vector bosons: the charged $W^\pm$ for beta decay (**c**), or the neutral Z^0 for elastic neutrino scattering (**d**). **Figure 2**

The weak force

Our acquaintance with the weak interactions dates back to the discovery of radioactive decays by Henri Becquerel in 1896. In one form of nuclear disintegration, namely beta decay, the emitted beta rays (electrons) exhibited a continuous distribution of energies. This appeared to be a two-body decay violating the conservation of energy, momentum and angular momentum.

In 1931 Wolfgang Pauli suggested that the conservation laws could be saved if an unobserved massless (or very light) particle was also emitted in beta decay. This supposed particle, named the neutrino (in this case it's actually the antineutrino) by Enrico Fermi, could carry off the missing energy and momentum in the beta-decay process $n \to pe^- \bar{\nu}_e$. The presumption was that the electrically neutral neutrino interacted so weakly with matter that no experiment had yet detected its presence.

The neutrino hypothesis was finally verified in 1953 when Frederick Reines and Clyde Cowan detected the antineutrinos from beta decay by observing their rescattering ($\bar{\nu}_e p \to e^+ n$) in a massive detector they had put up alongside a reactor at the Savannah River nuclear plant. The subscript "e" denotes the later recognition that these are "electron neutrinos," as distinguished from the "muon neutrinos" ν_μ first recognized in 1962 by the Columbia–Brookhaven experiment for which Leon Lederman, Melvin Schwartz and Jack Steinberger won last year's Nobel prize (see PHYSICS TODAY, January 1989, page 17).

In 1933 Fermi incorporated Pauli's hypothesis into a mathematical theory of beta decay patterned loosely after QED. Fermi postulated the existence of an interaction Hamiltonian

$$\mathsf{H} = \frac{G_F}{\sqrt{2}} \sum_{\lambda=1}^{4} J_\lambda^\dagger J^\lambda \qquad (1)$$

in which the vector "current" $J_\lambda^\dagger$ described the transition $n \to p$, and J_λ described $\nu_e \to e^-$ or the corresponding creation of an $e^- \bar{\nu}_e$ pair. The Fermi constant, $G_F \simeq 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}(\hbar c)^3$, is a measure of the underlying strength of the weak transition. The current operators $J_\lambda^\dagger$ and J_λ are known as "charged currents" because they describe transitions in which one particle changes into another particle differing in electric charge by one unit. In QED, by contrast, particles do not change identity when emitting or absorbing photons. In addition to the charge-changing nature of the currents, equation 1 differs from its QED analog in that it describes a pointlike "four-fermion" interaction, as shown in figure 2b. That is to say, the weak force, according to the Fermi theory, acts only between overlapping particle wavefunctions at *zero* spatial separation. The QED interaction, by contrast, is mediated over large distances by the massless photon.

The detailed forms of the currents in the weak Hamiltonian H have been modified several times over the years, mainly to incorporate the weak decays of the newly discovered particles of the heavier families. New terms were added to describe transitions between heavier charged leptons—the muon and the tau—and their associated neutrinos, ν_μ and ν_τ, and also to accommodate decays involving the related heavier quarks—strange, charmed and bottom. Another modification incorporated the violation of parity conservation, as suggested by T. D. Lee and C. N. Yang in 1956 and observed by C. S. Wu and her collaborators a few months later. The modified Fermi interaction is now known to give approximately correct descriptions of a great variety of weak decays, as well as various neutrino scattering processes, such as $\nu_\mu e^- \to \mu^- \nu_e$ and $\nu_\mu n \to \mu^- p$.

Fermi theory fails

On the other hand, it was clear that the Fermi theory was only an approximation. Its high-energy limit violated unitarity, the requirement that probabilities add up to unity. For example, the cross section for elastic $\nu_e e^-$ scattering computed from equation 1 is given at high energies by $G_F{}^2 s/\pi$, where s is the square of the total center-of-mass energy. However, the unitarity limit on the cross section for S-wave scattering (which is all you can get from a point interaction) is $16\pi/s$. It follows that the four-fermion description must break down at $s = 4\pi/G_F$, corresponding to a center-of-mass energy of about 500 GeV per particle. Normally, one expects to preserve unitarity by including higher-order terms in perturbation theory. But this does not work for the Fermi theory, because higher-order diagrams computed from equation 1 lead to severe divergences that cannot be absorbed into a finite number of measured parameters. That is to say, the theory is not renormalizable.

The intermediate-vector-boson theory was first proposed by Hideki Yukawa in his 1935 paper on meson theory and revived in a more realistic form by Schwinger in 1957. It was a modification of the four-fermion theory in which the exchange of massive spin-1 charged bosons, W^+ and W^-, were presumed to mediate the weak force, as illustrated in figure 2c. In beta decay, for example, the neutron emits a W^- and turns into a proton, and the virtual W^- then decays into $e^-\bar{\nu}_e$. This scheme is analogous to QED and to the theories of the strong interactions in that the force is no longer of zero range. But if the W's are heavy enough, they cannot propagate very far, and the model approximates the predictions of the Fermi theory at low energies, with $G_F/\sqrt{2} = g^2/8M_W{}^2$.

The coupling constant g is the analogue of the electron charge in QED; it is proportional to the amplitude for emitting a W, and it measures the strength of the interaction. Despite its apparent success, this early intermediate-vector-boson theory did not really solve the unitarity problem. The high-energy behavior of $\nu_e e^- \to \nu_e e^-$ was no longer such a serious problem because the interaction now had a finite range and was therefore no longer purely S wave. However, the high-energy amplitude for $e^+e^- \to W^+W^-$ again violated unitarity for energies on the order of $G_F{}^{-1/2}$, because of the Feynman diagram in figure 3a. Higher-order diagrams were severely divergent and nonrenormalizable.

Neutral vector bosons

One can solve the unitarity problem by introducing a new, electrically neutral vector boson, W^0. For properly chosen couplings of the W^0 to fermions and to the $W^\pm$, the high-energy divergences of diagrams like those in figure 3 cancel each other out, leading to a unitary, renormalizable theory.

Such cancellations would seem contrived if they did not emerge naturally from some underlying principle. Fortunately, the necessary relations between the $W^\pm$ and W^0 couplings emerge automatically in a gauge theory. In such theories the equations of motion are left unchanged in form when the fermions are rotated or otherwise transformed into each other by a group of symmetry operations. For these symmetries to be independent at different space–time points, it is necessary to introduce one vector (gauge) boson for each generator of the symmetry group. The interactions of these gauge bosons with the fermions (the quarks and leptons) are determined by the group-representation matrices, and their interactions with one another are dictated by the commutation rules of the group generators.

For example, the $W^\pm$ and W^0 emerge from the special unitary group SU(2), which is closely related to the rotation group in three dimensions. The proton and neutron (or, more precisely, their up- and down-quark constituents u and d) transform as a doublet under SU(2), in exact analogy with the transformations of the up and down states of a spin-$\frac{1}{2}$ particle under rotations. The e^- and ν_e also transform as a doublet. The $W^\pm$ bosons are associated with the symmetry between the u and d quarks and between the e^- and ν_e. Therefore they can be emitted or absorbed in transitions between these particles. The W^0 is associated with group transformations that change the phase, but not the identity, of the fermion states. It is therefore emitted or absorbed, in this theory, in "diagonal," neutral-current transitions such as $e^- \to e^-$.

It is possible to have different symmetries for the positive (right-handed) and negative (left-handed) helicity fermions, that is to say, particles whose spin projections on the direction of motion are respectively $+\frac{1}{2}$ or $-\frac{1}{2}$. This implies different couplings to gauge bosons for different helicity states. The observed parity violation in nature can be incorporated into the theory if only the fermions of left-handed helicity (and antifermions of right-handed helicity) transform nontrivially.

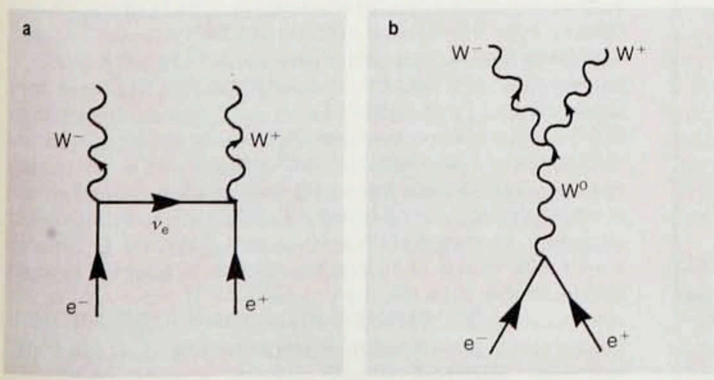
This SU(2) model, however, is not realistic. In 1961 Sheldon Glashow constructed a semirealistic model[1] based on the larger group SU(2)×U(1), which possesses an extra phase symmetry and, in fact, naturally unifies the weak and electromagnetic interactions. In addition to the gauge bosons $W^\pm$ and W^0 of the SU(2) theory, a second neutral boson, B, is associated with the U(1) symmetry. The W^0 and B are linear superpositions of the two *physical* neutral bosons. In particular the superposition

$$A = \cos\theta_W B + \sin\theta_W W^0 \qquad (2)$$

which couples to the electromagnetic current, can be identified as the photon. The weak mixing angle parameter θ_W in equation 2 is defined by $\tan\theta_W \equiv g'/g$, where g and g' are the coupling constants that determine the strengths of the interactions of the SU(2) and U(1) gauge bosons, respectively. The elementary electric charge e is related to g by $e = g\sin\theta_W$.

The second physical neutral boson is given by

$$Z^0 = -\sin\theta_W B + \cos\theta_W W^0 \qquad (3)$$



The first attempts to solve the unitarity problem in Fermi's theory of the weak interactions involved introducing charged intermediate vector bosons $W^\pm$. But then the amplitude for W^+W^- pair production (**a**) blew up at high energies. Invoking SU(3) symmetry in this early theory generated a third neutral boson W^0, whose role in pair production (**b**) precisely canceled the bad behavior of diagram **a**. **Figure 3**

The Z^0 mediates a new neutral-current interaction. Figure 2d shows a typical process of this kind—elastic neutrino–proton scattering. The strength of the diagonal couplings of the Z to the various fermions is predicted in terms of g and $\sin^2\theta_W$. Both left- and right-helicity fermions couple to the Z, but with different strengths, so that this new weak interaction is predicted to violate parity.

Spontaneous symmetry breaking

At this point the $SU(2)\times U(1)$ model was still not realistic, because the gauge invariance needed for a renormalizable theory did not allow mass terms for the gauge bosons in the equations of motion. That is to say, all the gauge bosons had to be massless. That's fine for the photon, but not for the $W^\pm$. The very short range of the weak force had to correspond to heavy mediators. The problem was finally solved in 1967 by Steven Weinberg[2] and, independently, by Abdus Salam,[3] who applied an idea of Peter Higgs[4] to the $SU(2)\times U(1)$ model. The "Higgs mechanism" is a gauge-theoretic realization of a more general phenomenon known as spontaneous symmetry breaking. The idea is that the ground-state solutions of a theory may have less symmetry than the equations of motion. A familiar example is a ferromagnet: The equations of motion are rotationally invariant, but in a given ferromagnet the spins are aligned in some specific direction. Choosing that direction spontaneously breaks the symmetry.

Weinberg and Salam introduced a pair of scalar fields (called Higgs fields) that transform as a doublet under SU(2). Depending on the parameters associated with this doublet of Higgs fields, the lowest-energy state of the theory might not be empty space, but instead a Bose condensate (that is to say, a nonzero classical value) of the neutral Higgs field. The $W^\pm$ and Z bosons and the fermions couple to this field that permeates all space. In this way they acquire their effective masses, much as a photon acquires an effective nonzero mass when it propagates through a plasma. Because this background Higgs field is neutral, the photon remains massless in the absence of ordinary matter. In the resulting unified electroweak theory one can calculate the masses of the W and Z bosons in terms of the fine-structure constant $\alpha \simeq 1/137$, the Fermi constant and $\sin^2\theta_W$:

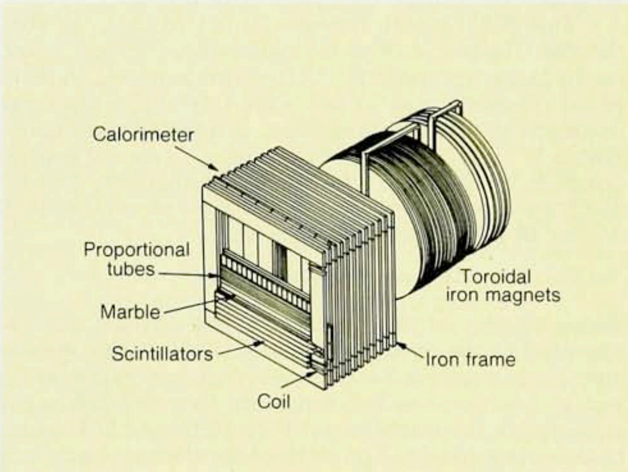
$$M_W = \frac{A_0}{\sin\theta_W (1-\Delta r)^{1/2}}$$
$$M_Z = \frac{M_W}{\cos\theta_W} \quad (4)$$

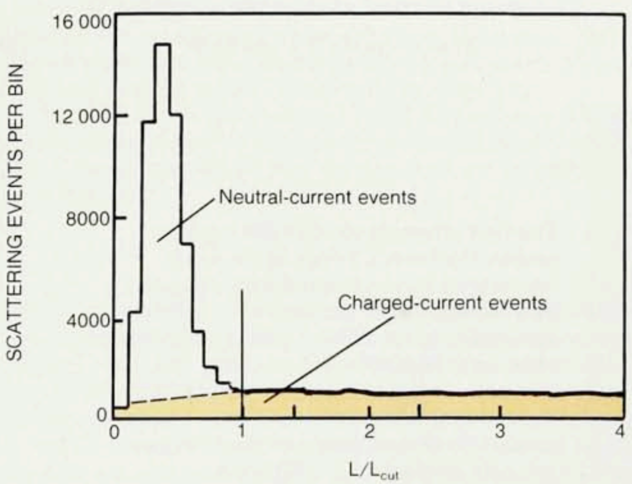
where $A_0 = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.281$ GeV and Δr is a small higher-order correction, due mainly to vacuum-polarization effects.

This unified electroweak theory was largely ignored for about five years, until Gerard 't Hooft and Martinus Veltman[5] proved in 1972 that the spontaneous symmetry breaking did not spoil the renormalizability of the theory. This crucial proof and the observation of neutral-current interactions at CERN and Fermilab the following year[6] attracted enormous attention to the electroweak theory. In the scattering of ν_μ beams off nuclear targets, a neutral-current event manifests itself by the absence of a muon in the final state, which shows that the neutrino has been scattered without changing its identity.

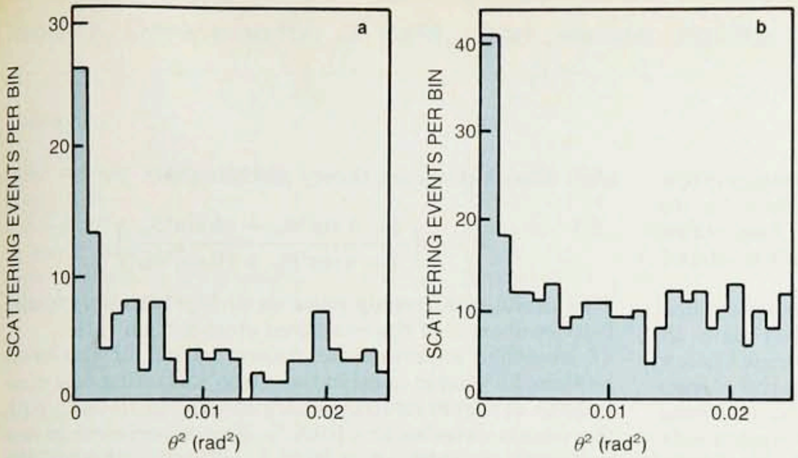
Charm, the fourth quark

One serious remaining difficulty in 1972 was that there were only three known quarks: the up quark, u, with charge $+\frac{2}{3}e$, and the down and strange quarks, d and s, with charge $-\frac{1}{3}e$. The u and d were related by SU(2), but the s quark had no partner. Strangeness-changing, charged-current decays such as $K^+\to\mu^+\nu_\mu$ could, it was





Neutrino scattering experiments require massive detectors of highly modular construction. The CHARM detector at CERN (top) has a fine-grained calorimeter 3×3 m^2 in cross section, followed by a magnetic muon spectrometer. From another CERN detector,[11] the histogram (bottom) plots the distribution of L, the length of the longest, most penetrating track from each scattering event, scaled to a cutoff length. The flat background (orange) of charged-current events with muons of all lengths is clearly distinguishable from the sharp peak of short events, where the absence of a penetrating muon is the signal of neutral-current neutrino scattering. **Figure 4**



Scattering-angle distribution in a Brookhaven experiment[13] looking for elastic scattering of medium-energy neutrinos (**a**) and antineutrinos (**b**) off electrons. Kinematics restricts the square of θ, the angle of the struck electron relative to the neutrino beam, to less than 10^{-3} rad^2. These θ^2 histograms both show clear elastic-scattering peaks at small angles over an isotropic background. **Figure 5**

thought, be accommodated by assuming a little bit of mixing. That is to say, it was suggested that the SU(2) partner of the u contains a small admixture of s quark.

But this explanation for the familiar decay $K^+ \to \mu^+ \nu_\mu$ produced horrendous overestimates when applied to certain very rare decay modes. The mixing of the down and strange quarks necessarily implied that electrically neutral but strangeness-changing transitions between the s and the d could be mediated by the Z boson, just as the Z mediates the diagonal neutral-current processes u→u and d→d. With such "flavor-changing neutral currents" allowed by mixing, it was difficult to understand why the decay $K_L \to \mu^+\mu^-$ is seen only once in a hundred million decays of the long-lived neutral K meson.

This nagging problem could be eliminated by invoking an idea put forward by Glashow, John Iliopoulos and Luciano Maiani[7] at Harvard in 1970. They had introduced a fourth type of quark, the charmed quark, c, with charge $+\frac{2}{3}e$, which would be the doublet partner of the strange quark. With the d and s quarks now treated symmetrically, the flavor-changing neutral currents would cancel, relegating the decay $K_L \to \mu^+\mu^-$ to very rare, higher-order mechanisms.

In 1972 there was no direct evidence for the existence of such a fourth quark, and high-energy physicists were reluctant to introduce new particles. But in November 1974 the charmed quark was discovered with great fanfare in the production of the J/ψ meson, a bound state of c and its antiquark.[8] Evidence for charm had, in fact, been seen in charged-current neutrino reactions.[9]

The proof of renormalizability, the existence of neutral currents and the discovery of the charmed quark gave substance to the $SU(2)\times U(1)$ electroweak theory. In the subsequent decade this unifying theory was quantitatively tested in an impressive array of experiments, which clearly established it as correct, at least to first approximation.

Since the earliest observations of weak-neutral-current reactions in 1973, the experiments that bear on the electroweak theory have concentrated primarily on measuring the value of $\sin^2\theta_W$, the fundamental unspecified parameter of the theory. We will now describe several of the measurement methods and their results, to illustrate the diversity of the methods and the extensive range of the dynamical variables. Reference 10 offers more comprehensive surveys of these results.

Inelastic neutrino–nucleus scattering

The first weak-neutral-current interaction to be observed, after more than a decade of vain searches, was the inelastic scattering of energetic neutrinos by stationary target nuclei.[6] These neutrino beams, generated by the decay of mesons produced at high-energy accelerators, consist overwhelmingly of muon neutrinos. The neutral-current signature was the absence of a prompt muon in the final state of the reaction $\nu_\mu N \to \nu_\mu X$, where X represents an arbitrary hadronic state. By contrast, the usual charged-current reaction, $\nu_\mu N \to \mu^- X$, produces a muon, which is distinguished from pions by its ability to penetrate a considerable thickness of iron. The difficulty lay in demonstrating that the neutral-current signature was not the spurious result of a failure to record all outgoing muons or to distinguish very low-energy muons from hadrons.

After these first experiments, the method of delineating the muonless events from the more abundant charged-current events was refined, and the statistical precision of the experiments substantially improved. A typical detector for the study of neutrino–nucleus reactions is shown in figure 4, which illustrates the large target mass needed to compensate for the low neutrino-interaction cross section, and the modular structure required to distinguish the muons from the shorter-range hadrons and electromagnetic showers. The histogram in figure 4 gives the length distribution of the longest charged track emerging from each collision in a recent neutrino scattering experiment. One can clearly distinguish with high efficiency the short, muonless events from the long charged-current events. Furthermore, the range spectrum yields the cross-section ratio $R_\nu \equiv \sigma(\nu_\mu N \to \nu_\mu X)/\sigma(\nu_\mu N \to \mu^- X)$. This ratio gives the weak mixing angle by the relation

$$R_\nu = \tfrac{1}{2} - \sin^2\theta_W + \tfrac{5}{9}\sin^4\theta_W(1+r) \qquad (5)$$

where r (roughly 0.4 in these experiments) is the ratio of antineutrino to neutrino charged-current cross sections integrated over the same neutrino spectrum with the same experimental cutoffs. (In practice one must also apply a number of small corrections for such effects as quark mixing and the "sea" of $s\bar{s}$ strange-quark pairs in the nucleon.) Two of the highest-precision experiments in recent years, both done at the CERN Super Proton Synchrotron, have yielded R_ν values of 0.3072 ± 0.0032 (from the CDHS collaboration[11]) and 0.3093 ± 0.0031 (from the CHARM collaboration[12]).

Elastic neutrino scattering

Neutrino scattering experiments have advanced to the extent that one can now measure the cross sections for elastic scattering of neutrinos by protons *and* electrons. The detector employed for this purpose at Brookhaven was designed to accept a ν_μ or $\bar{\nu}_\mu$ beam of modest energy (a few GeV) from the Alternating Gradient Synchrotron, to discriminate against inelastic (multiparticle) final states and to identify unambiguously either the single scattered proton or electron.

The reactions $\nu_\mu(\bar{\nu}_\mu)e^- \to \nu_\mu(\bar{\nu}_\mu)e^-$ are purely leptonic weak-neutral-current processes. Therefore they allow, in principle, a much cleaner test of the electroweak theory than do the "semileptonic" reactions that require corrections for the involvement of hadrons and their strong interactions. In neutrino–electron scattering experiments the kinematic constraints provide the signature of the elastically scattered electrons, permitting good discrimination against backgrounds. The elastic-scattering kinematics require that the square of the scattering angle of the outgoing electron with respect to the incident neutrino be less than 10^{-3} rad^2. The observed distribution of electron scattering angles in figure 5a shows a clear small-angle peak indicating $\nu_\mu e^- \to \nu_\mu e^-$, riding on an approximately isotropic background.[13] The antineutrino beam delivered a similar distribution with an elastic scattering peak of equal clarity (figure 5b).

The ratio $\mathscr{R} = \sigma(\nu_\mu e \to \nu_\mu e)/\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$, which allows for the cancellation of certain systematic uncertainties in the separate cross-section measurements, was found to be $1.38^{+0.40}_{-0.31} \pm 0.17$, where the second uncertainty is systematic. The electroweak theory predicts that

$$\mathscr{R} = 3\left(\frac{1 - 4\sin^2\theta_W + 16/3\sin^4\theta_W}{1 - 4\sin^2\theta_W + 16\sin^4\theta_W}\right)$$

This useful relationship gives us $\sin^2\theta_W$ with only $1/6$ the fractional error of the measured cross-section ratio.

Another experimental determination of the cross sections for elastic neutrino–electron scattering was done in 1983, at higher neutrino energies (around 15 GeV), with the CHARM detector at CERN.[14] This experiment gave a cross-section ratio of $1.26^{+0.72}_{-0.45}$. Thus, although the Brookhaven and CERN experiments used neutrino beams differing in energy by an order of magnitude, contending with very different backgrounds and resolutions, their results are in good agreement.

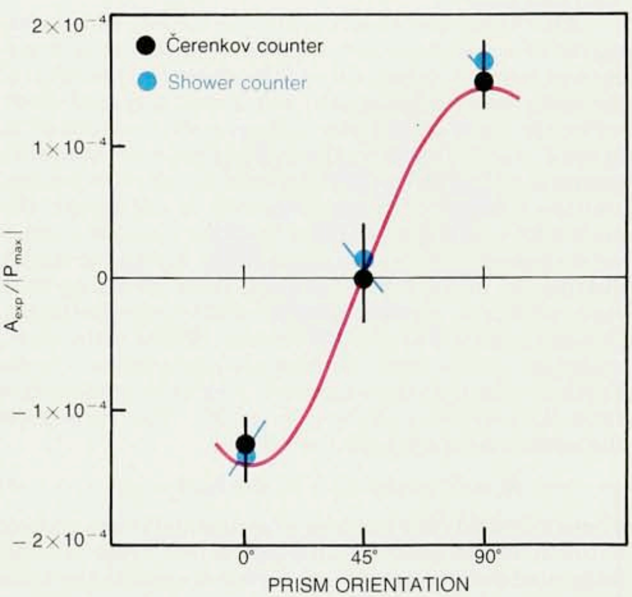
Parity nonconservation

Whenever a virtual photon can be exchanged, the theory tells us, so can a Z. Thus there should be interference effects between photon-exchange and neutral-current amplitudes. Soon after the initial observation of neutral-current neutrino scattering, such electroweak interference effects were eagerly sought in heavy atoms and in the scattering of polarized electrons off deuterons.

The polarized-electron experiment carried out ten years ago at the Stanford Linear Accelerator Center yielded the first positive evidence[15] of a weak-neutral-current contribution to the interaction of electrons with nuclei. Polarized electrons from a GaAs source were scattered inelastically in the deuterium-gas target and then collected by a spectrometer and counted. Because a parity-nonconserving process was interfering with the electromagnetic scattering, the yield, $Y_\pm$, of scattered electrons depended on the helicity (+ or −) of the incident electrons. The observed asymmetry, A_{exp}, defined by $A_{exp} = (\langle Y_+\rangle - \langle Y_-\rangle)/(\langle Y_+\rangle + \langle Y_-\rangle)$, depends on the adjustable orientation of the calcite prism that determines the degree of polarization of the beam electrons emerging from the GaAs source. The basic observation of the experiment (figure 6) shows an unambiguous asymmetry whose magnitude and dependence on the orientation of the prism are in excellent agreement with the electroweak theory.

The prediction depends on the value of $\sin^2\theta_W$ and on a model—the quark–parton model—of the internal structure of the nucleon. Here, as in all reactions involving hadrons, uncertainties concerning the strong interactions introduce uncertainties ranging from a few percent to as much as 10% into the predictions of the electroweak theory and the determination of its parameters.

Entirely different techniques are required to observe parity-violating weak-neutral-current effects in heavy atoms. Such experiments probe the extreme low end of the momentum-transfer range at which neutral currents have been studied. The weak neutral current induces mixing between S and P atomic orbitals. The transition probability for a magnetic-dipole transition induced by circularly polarized photons gains a very small cross term proportional to the electric-dipole amplitude generated by the parity-violating weak interaction. One can observe this cross term because it changes sign with the sense of the circularly polarized incident photons. But because this tiny effect is roughly proportional to the cube of the atomic



Polarized electrons scattered off deuterium atoms in a SLAC experiment[15] exhibit a parity-violating asymmetry due to electroweak interference. The experimental asymmetry A_{exp}, normalized to the maximum polarization P_{max} of the electron beam (about 37%), is plotted against the orientation of the calcite prism that varies the polarization of the electron beam. At each prism setting the scattering is recorded for two helicities of opposite sign. The red curve is the prediction of the electroweak theory. **Figure 6**

Table 1. Determination of $\sin^2\theta_W$ from various reactions

Reaction	$\sin^2\theta_W$ [a]	$\sin^2\theta^0$ [b]
Deep inelastic ν_μN scattering	$0.233 \pm 0.003 \pm [0.005]$	0.242
$\nu_\mu p \to \nu_\mu p$	0.210 ± 0.033	0.208
$\nu_\mu e \to \nu_\mu e$	$0.223 \pm 0.018 \pm [0.002]$	0.221
W and Z masses	$0.229 \pm 0.007 \pm [0.002]$	0.214
Parity violation in heavy atoms	$0.220 \pm 0.007 \pm [0.018]$	0.212
Polarized eD scattering	$0.221 \pm 0.015 \pm [0.013]$	0.226
All data	0.230 ± 0.0048	

Data from ref. 10. See also figure 7.

a Where two uncertainties are shown, the first is experimental, and the second, in square brackets, is theoretical. The latter includes the effect of letting the unknown masses of the top quark and Higgs boson range widely. The central values assume $M_t = 45$ and $M_H = 100$ GeV.

b Values that would be obtained from the data without radiative corrections.

number, this kind of parity violation has been successfully observed only in heavy atoms.

The precision of parity-violating experiments on atoms has been steadily increasing. Recent experiments on cesium[16,17] have measured the experimental asymmetry with an uncertainty of only 2%, while the theoretical uncertainty is less than 6% because of the simple, monovalent configuration of the cesium atom.

Electron–positron annihilation

In the last two decades, electron–positron annihilation has been extensively studied at e^+e^- colliders at Cornell, the Stanford Linear Accelerator Center, the DESY laboratory in Hamburg and the new TRISTAN facility in Japan. At all the colliders in operation before the new "Z factories" came on the air this year, the dominant mechanism for $e^+e^- \to f\bar{f}$ (where f is a quark or lepton) is the production and decay of a massive virtual photon ($e^+e^- \to \gamma^* \to f\bar{f}$). However, in the electroweak theory a virtual Z can also mediate the interaction ($e^+e^- \to Z^* \to f\bar{f}$), and the interference of the two contributions to the amplitude leads to forward–backward asymmetries. At collision energies below the Z mass (91 GeV), emerging muons, for example, show a slight preference for the direction of the beam particle of the *opposite* charge. This asymmetry can be larger than 15% at collision energies around 35 GeV. The asymmetry in the theory comes from the axial couplings of the Z to the fermions, which are independent of $\sin^2\theta_W$. Thus the asymmetries are absolute predictions of the theory (except for a small dependence on the mass of the Z) rather than measurements of $\sin^2\theta_W$. Forward–backward asymmetries have been measured for $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$, $c\bar{c}$ and $b\bar{b}$, where b is the "bottom" quark of the third family. All are in excellent agreement with standard-model predictions. The quark–antiquark final states, of course, manifest themselves as jets of hadrons.

This year has seen the completion of two new high-energy e^+e^- colliders—the SLC at Stanford and LEP at CERN—with maximum beam energies exceeding 50 GeV. Therefore these machines make real Z's. At such energies the "weak" amplitude strongly dominates the virtual photon amplitude, so that these machines should provide very precise measurements of $\sin^2\theta_W$. (See PHYSICS TODAY, July, page 17, and October, page 17.)

Intermediate vector bosons

The existence of charged and neutral intermediate vector bosons to mediate the weak interactions is integral to the electroweak theory. The zero range of the earlier four-fermion theory of weak interactions indicated the need for at least one heavy mediator, and many experimental searches were carried out with the limited energies of the accelerators then available. The early neutral-current measurements of $\sin^2\theta_W$ led to predictions of the masses of the intermediate vector bosons, the $W^\pm$ and Z^0, in the range of 80–95 GeV, far beyond the center-of-mass energy available in any then-existing accelerator.

It was recognized, however, that if one could convert the 270-GeV CERN Super Proton Synchrotron from a fixed-target accelerator to a colliding-beam storage ring, one would have a more than 20-fold increase in center-of-mass collision energy—from 23 GeV to 540 GeV, more than enough to make the predicted heavy bosons. In 1982, Carlo Rubbia, Simon Van der meer and their CERN colleagues successfully turned the SPS into a 540-GeV proton–antiproton collider. The following year they found[17] all three intermediate vector bosons, with masses in the range predicted by the electroweak theory.

The lifetimes of the W and Z bosons are thought to be only 10^{-25} sec. They decay directly to all possible lepton–antilepton and quark–antiquark pairs. At the SPS collider they are produced in the reactions $\bar{p}p \to W^\pm X$ and $\bar{p}p \to Z^0X$, frequently with relatively low momentum. The initial detection of the W and Z relied on the observation of the particular decay modes $W \to e\nu_e$ and $Z^0 \to e^+e^-$, because the detection and measurement of relatively isolated electromagnetic showers generated by energetic electrons was a well-established technique. It was necessary that the detector (figure 1) cover as much of the 4π solid angle surrounding the $\bar{p}p$ interaction region as possible, to ensure that no energetic particles from either the initial $\bar{p}p$ collisions or the vector-boson decays (except neutrinos) escaped detection. By now, the Z^0 has also been produced in profusion at the Fermilab Tevatron collider and at the new SLC and LEP Z factories.

Gauge invariance

All of the experimental results thus far are in excellent agreement with the predictions of the standard model, as one can see from the summary in table 1 and figure 7. In particular, $\sin^2\theta_W$ has been obtained from a wide range of probes and for a kinematic range spanning ten orders of magnitude in Q^2, the square of the four-momentum transferred in the interaction. The momentum transfer is inversely proportional to the distance probed, and it equals the (nonphysical) mass of the exchanged virtual particle. The electroweak theory has been successfully tested over the Q^2 range from 10^{-6} GeV^2 to 10^4 GeV^2. The remarkable agreement among the various measurements attests to the internal consistency and broad scope of the theory.

The agreement is especially impressive because it tests not only the lowest-order calculations but also the higher-order (radiative) corrections. From the last column in table 1 it is apparent that without radiative corrections the different experimental determinations of $\sin^2\theta_W$ would not agree with one another. The radiative corrections are required by the data at the level of 3 standard deviations. This is not nearly as precise as the tests of higher-order effects in quantum electrodynamics. However, such corrections are of special importance in the electroweak unification because they are finite and calculable. Were it not a gauge theory, they would be infinite and therefore meaningless.

Remaining uncertainties

There are still several gaps in the impressive accord between this unifying theory and the experiments. The "top quark," which is required to complete the heaviest of the three families, remains to be found. We do not have a good prediction of its mass, but the fact that the top quark has not yet been found in e^+e^- or $\bar{p}p$ reactions places a lower bound approaching 80 GeV on its mass, M_t.

Remarkably, one can obtain an upper bound on M_t from the consistency of the various neutral-current and boson-mass data. The top quark occurs in virtual intermediate states in higher-order calculations (especially in vacuum-polarization diagrams), and its mass affects the radiative corrections. A simultaneous fit to all the data yields an upper limit of 175–200 GeV for M_t at the 90% confidence level. If M_t is less than 100 GeV, the top quark should be detected directly at $\bar{p}p$ colliders in the next few years. If it is in the range of 100–200 GeV, its indirect contribution to higher-order effects in future precise tests of the theory will be enormous.

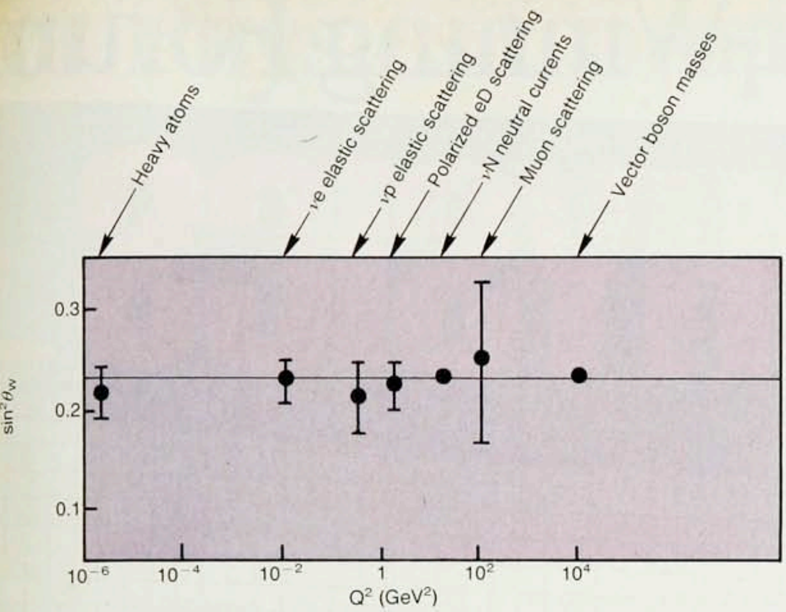
Another uncertain issue is the existence of the Higgs boson, the physical particle presumed to be associated with the Higgs field. The electroweak theory provides no clue to its mass, M_H. Experimental searches have not yet turned up any candidate that might be identifiable as a Higgs boson. But the searches thus far have only been sensitive to masses less than a few GeV. Unfortunately the radiative corrections are rather insensitive to M_H. For example, the modest range of upper limits we've just quoted for M_t result from letting M_H range from 10 to 1000 GeV. Settling the question of the existence of one or more Higgs bosons is an important challenge to particle physics. It should be noted, however, that many physicists suspect that there is no elementary Higgs field. These dissenters conjecture that its assigned role is played by a more complicated bound state or other dynamical effect.

The global fit to all the world's data yields an average value of 0.230 ± 0.0048 for $\sin^2\theta_W$, assuming that M_t is less than 100 GeV. While this average is less interesting than the comparison between different types of experiments, it is still interesting for comparison with the predictions of the *grand* unified theories that seek to unify the electroweak theory with quantum chromodynamics in a gauge group with a single coupling constant. The

Table 2. Planned and proposed high-precision electroweak tests

Measurement	$\Delta\sin^2\theta_W$ [a]	Location	Date
$\Delta M_Z = \pm 20$ MeV	0.0004	LEP	1990
$\Delta M_Z = \pm 45$ MeV	0.0005	SLC	1990
$\Delta M_W = \pm 350$ MeV	0.002	SPS,FNAL	1990
$\Delta M_W = \pm 100$ MeV	0.0006	LEP 200	1995
$e^+e^- \to \mu^+\mu^-$	0.002	LEP	1990
$e^+e^- \to \mu^+\mu^-$ (polarized e^-)	0.0004	SLC, LEP(?)	1990s
$\nu_\mu e \to \nu_\mu e$	0.005	CHARM II	1991
$\nu e \to \nu e$	0.002	LCD (LANL)	1990s
Atomic parity violation	0.003	ILA Boulder, ENS Paris	1990s
$ep \to eX$	0.005	HERA	1990s
$ep \to ep$ (polarized e)	0.002	CEBAF(?)	1990s
$\dfrac{\nu_\mu N \to \nu_\mu X}{\bar{\nu}_\mu N \to \bar{\nu}_\mu X}$	0.004	FNAL(?)	1990s

[a] These expected errors do not include the uncertainty due to ignorance of the top-quark and Higgs-boson masses.



Various determinations of $\sin^2\theta_W$, the fundamental unspecified parameter of the electroweak theory, plotted against Q^2, the square of the four-momentum transfer typical of each class of experiments. Each determination is labeled by experiment type (see table 1). Radiative corrections depend on the assumed mass of the top quark; this plot assumes $M_t = 45$ GeV. The world average of $\sin^2\theta_W$ (horizontal line, not including the recent "Z factory" data) is 0.230 ± 0.0048. **Figure 7**

simplest such models predict $\sin^2\theta_W = 0.216^{+0.003}_{-0.004}$, which is about 2.5 standard deviations below the experimental average value. In any case, these models have been ruled out for some time now by our failure to find proton decay at the rate they predict. The observed average value of $\sin^2\theta_W$ hints at the possibility of a modified grand unified theory with new thresholds below 10^{14} GeV. For example, supersymmetric grand unified theories, which suppose a symmetry between fermions and bosons, predict a value for $\sin^2\theta_W$ in better agreement with the world experimental average.

The neutral-current and boson mass data can also be used to search for or set limits on possible deviations from the standard model. For example, many theoretical extensions of the model predict the existence of additional heavy Z^0 bosons. In most cases the best limits on such new weak interactions are indirect conclusions from the neutral-current data. Depending on the model, one finds lower limits varying from 120 to 300 GeV on the masses of such new neutral bosons. We also have reasonably stringent limits on many other types of possible new physics, such as Higgs bosons with nonstandard $SU(2) \times U(1)$ transformations, heavy fermions with exotic weak interactions, and "leptoquark" bosons that couple to quark–lepton pairs.

From figure 7 it is apparent that the standard electroweak theory is the correct description of electromagnetic and weak-interaction phenomena, at least down to a distance scale of order 10^{-16} cm. Confirmation comes from the charged-current part of the theory, which in recent years has been extensively tested in refined measurements and in theoretical calculations of muon decay, beta decay, strange-particle decays, heavy-particle oscillations and decays, and *CP* violation.

Most particle physicists are convinced that there must be new physics beyond the standard model that will only manifest itself at shorter distance scales. One way to look for such new physics is to carry out high-precision electroweak tests. A new generation of accelerators and experiments is expected to improve the precision with which we know $\sin^2\theta_W$ and other electroweak data by at least an order of magnitude, as indicated in table 2. These experiments include high-precision measurements of the Z and W masses, the asymmetries in e^+e^- annihilations, neutrino–electron scattering, and weak–electromagnetic interference in atoms and deep-inelastic ep scattering. Such experiments will probe the theory and its possible extensions down to at least 10^{-17} cm. They will be an exciting and vital complement to direct searches for heavy particles at the new colliders.

Whatever the future brings, the standard-model unification of the weak and electromagnetic interactions will almost certainly survive as the quantitatively correct theoretical description down to a distance scale one-thousandth the size of the atomic nucleus.

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