other experimental trial unscathed.

With LEP's present complement of 128 copper rf accelerating cavities, its energy cannot exceed 55 GeV per beam. (See the cover of this issue.) Over the next several years the copper cavities will gradually be replaced by superconducting niobium rf cavities being developed at CERN. By the end of 1992 the schedule calls for 196 superconducting rf cavities to be in place, yielding beams of 96-GeV electrons and positrons.

—Bertram Schwarzschild

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DOES QUANTUM MECHANICS HAVE NONLINEAR TERMS?

One hears about testing quantum electrodynamics or relativity theory, but the theoretical basis of quantum mechanics is rarely questioned. Nevertheless, Steven Weinberg (University of Texas) has recently called for high-precision tests of quantum mechanics that are independent of any particular quantum mechanical theory.[1] To pave the way for an examination of quantum mechanics, he has suggested one possible way of generalizing quantum mechanics to make it nonlinear. As Eugene Wigner pointed out in a 1939 paper, the linearity of quantum mechanics is an important assumption and one that may not necessarily always prove true. Weinberg stressed to us that he does not really feel that quantum mechanics is in any immediate danger, but he does believe that we can learn by questioning it: If we find that quantum mechanics cannot be generalized any further, we may come to understand better why it works so well. If we find that the theory can be generalized in a plausible way, then we must ask why ordinary quantum mechanics is so nearly valid—and we may discover some hidden physics in the process.

Weinberg has suggested one particular example in which a nonlinearity might manifest itself, and several experimental groups have taken up the search for it. The first results are now in: John Bollinger, Daniel Heinzen, Wayne Itano, Sarah Gilbert and David Wineland of the National Institute of Standards and Technology (Boulder, Colorado) have set a very low upper limit on the size of a possible nonlinear term in the hyperfine splitting of a beryllium atom. Bollinger presented these results at the annual meeting of the APS division of atomic, molecular and optical physics, held in Windsor, Ontario, last May.[2]

Quantum mechanics has been tested in several ways before. One set of experiments aimed to distinguish it from local hidden-variable theories. (See the article by David Mermin in PHYSICS TODAY, April 1985, page 38.) These experiments succeeded in ruling out hidden variables, but Weinberg points out that they did not test quantum mechanics to better than about one percent. Another approach to testing quantum mechanics dealt with the incorporation of nonlinear terms into the Schrödinger equation. In 1980, at the suggestion of Abner Shimony[3] (Boston University), a team led by Clifford Shull[4] (MIT) used neutron interferometry to search for possible nonlinear terms of a form suggested by Iwo Bialynicki-Birula and Jan Mycielski[5] (University of Warsaw). Roland Gahler (Technical University of Munich), Anthony G. Klein (University of Melbourne, Australia) and Anton Zeilinger[6] (Technical University of Vienna), working at the Laue–Langevin Institute in Grenoble, followed with a similar experiment that set more stringent limits on the size of the nonlinear term.

Nonlinear formulation

Weinberg set out to make the most general formulation possible that is still consistent with essential properties of quantum mechanics. A key requirement was to preserve homogeneity—to ensure that if one wavefunction is a solution of the Schrödinger equation, then so is another wavefunction that is just a constant multiple of the first. The nonlinear term introduced by the Polish theorists did not satisfy this condition. Weinberg assumes that the time dependence of the wavefunction is given by:

$$i\hbar\frac{d\Psi_k}{dt} = \frac{\partial h(\Psi,\Psi^*)}{\partial \Psi_k{}^*}$$

This equation reduces to the standard form if the Hamiltonian function h has the bilinear form $\Psi_k{}^* H_{kl} \Psi_l$, but allows for treatment of possible terms in the Hamiltonian that are not bilinear. In the example of a generalized two-component system, Weinberg shows that the Hamiltonian function h can be put in the form $n\,\bar{h}(a)$, where n is the norm $|\Psi_1|^2 + |\Psi_2|^2$ and $\bar{h}$ is an arbitrary function of the variable $a = |\Psi_2|^2/n$.

For an atom undergoing a radiative transition (such as that between two hyperfine levels), Weinberg's equation predicts that the resonant transition frequency will be sensitive to the occupancy of the two levels. This transition frequency will change as these occupancies change. If one drives a certain transition with an applied monochromatic oscillator, detuning will result because the resonant transition frequency will shift as the oscillator continues to populate the upper levels. For a weak nonlinearity, the shift in frequency will be small. To detect it, one must make the coupling between the oscillator and the atom as weak as possible so that the transition takes a long time. The sensitivity of the technique thus varies inversely with the time of perturbation.

Weinberg used data from a 1985 experiment performed at NIST[7] (then called the National Bureau of Standards) to estimate an upper limit on the size of the nonlinear term. In that experiment, Be^9 ions were driven from one state to the other with a single pulse about 1 second long. Because the transition could be driven with such a pulse, Weinberg inferred that the nonlinearity could not lead to a shift of more than the inverse of that pulse length, or 1 Hz, which corresponds to a nonlinear term on the order of 10^{-15} eV.

Norman Ramsey (Harvard University) in 1949 originated a technique for measuring atomic resonant transitions and hence for setting frequency standards. This so-called method of separated oscillatory fields gives a sensitive way to detect changes in transition frequencies. First, an rf pulse is applied to an atom and then turned off. This creates a superposition state, with specific populations of the two atomic levels, that depends on the amplitude, frequency and length of the pulse. After a certain time interval, a second rf pulse of the same length, coherent with the first, is applied. If the two rf pulses are at the resonant transition frequency, the second pulse will be in phase with the superposition state and it will continue to transfer the ions to the upper state. The rf frequency is changed until this resonant condition is met.

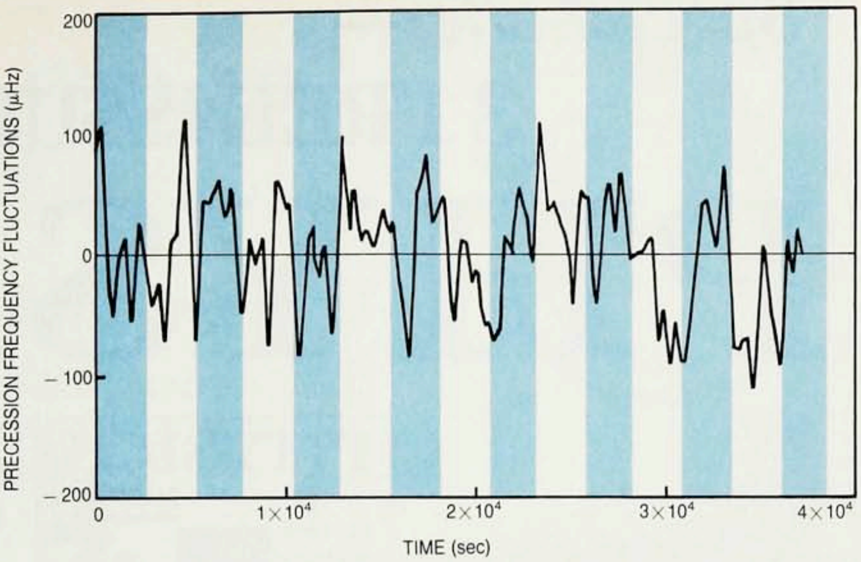
If a nonlinearity is present, the phase of the superposition state will not be determined simply by the difference between the energy eigenvalues of the two levels but will also depend on the state amplitudes created by the first pulse. Thus the resonant frequency measured by the Ramsey method will depend on the length of the first pulse. In this method for detecting a nonlinearity, the accuracy is inversely related to the time between pulses.

Experimental tests

Bollinger and his colleagues decided that, by using unequal pulses, they could improve considerably on the estimate Weinberg had extracted from their earlier data. They undertook a new experiment on $(Be^9)^+$ ions, using a collection of 5000–10 000 ions stored in a Penning trap and cooled below 250 mK. The design of their experiment is best viewed in terms of the precession of the $(Be^9)^+$ nucleus in an applied magnetic field. The precession rate does not vary with the angle between the spin and the magnetic field vector. In a nonlinear quantum theory, however, the precession rate does vary with angle, because the angle is a measure of the particular mixture of states.

To measure the precession rate the researchers applied the Ramsey method with unequal rf pulses. First they prepared the ions in the $(-\frac{1}{2}, +\frac{1}{2})$ state by a combination of optical pumping and rf resonance techniques. (The state is specified by quantum numbers denoting the nuclear and electronic spins, respectively.) The first Ramsey pulse was then applied to create a superposition with the $(-\frac{3}{2}, +\frac{1}{2})$ state. The length of this pulse determined the amplitude of each eigenstate in the superposition, and hence the tipping angle of the Be^9 nuclear spin. After a period of time, typically 100 sec, the second Ramsey pulse was applied. The ion population in the $(-\frac{1}{2}, +\frac{1}{2})$ state was detected by laser-induced fluorescence. The more ions remaining in the $(-\frac{3}{2}, +\frac{1}{2})$ state, the lower the fluorescence signal. The fluorescence thus peaks as a function of the frequency of the Ramsey pulses. The resonance (precession) frequency was determined from the average of the frequencies at half of the maximum intensity. The NIST team measured the precession rate at two angles (see the figure on this page). The precession frequency did not change significantly with angle. Averaging over 25 runs, the researchers found that the frequency difference was 2.7 ± 6.0 μHz, corresponding to an upper limit of 2.4×10^{-20} eV on the nonlinear term.

To judge how stringent this upper limit is, one needs to compare it with some energy scale, but what scale is appropriate? Weinberg maintains that the proper energy for comparison in this case is the nuclear energy. When compared with the binding energy per nucleon for the beryllium atom, the fractional upper limit on the nonlinearity becomes 4×10^{-27}.



Precession frequency is not expected to vary with tipping angle if quantum mechanics is without nonlinearities. An experiment on Be^9 ions measured the precession frequency during alternating time intervals in which the tipping angle had values of 1.02 rad (blue shading) and 2.12 rad. The frequencies are plotted relative to the time-averaged frequency. The precession frequencies for the two tipping angles, averaged over many runs like that shown above, differed by less than 8.7 μHz. (Adapted from ref. 2.)

Work in progress

At least three other research teams are seeking nonlinearities. One of these—a group led by Norval Fortson at the University of Washington—is looking for the variation with angle of the precession frequency of a spin-$\frac{3}{2}$ nucleus, in this case, Hg^{201}. The investigators sense the precession rate through its modulation of an optical signal. Actually the precession rate is the superposition of three frequencies corresponding to the energy separations between four hyperfine levels. If the frequencies remain constant, independent of the amplitude in each state, linear quantum mechanics holds. At Harvard, Timothy Chupp is conducting a similar experiment but with a different nucleus Ne^{21} and a different experimental method.

Isaac Silvera (Harvard University) and his student Ronald Walsworth, in collaboration with Robert Vessot and Edward Mattison of the Smithsonian Astrophysical Observatory, are working with quite a different tool—the hydrogen maser. However, the nonlinear effect Weinberg identified shows up only in a particle of spin greater than or equal to 1, whereas the spin of the hydrogen nucleus is $\frac{1}{2}$. Walsworth has worked out a treatment for this case and applied it to existing data from hydrogen masers. He hopes to improve the accuracy by at least two orders of magnitude by conducting an experiment directly aimed at sensing the nonlinearities. So far he estimates an upper limit in terms of eV that is a factor of ten higher than that of the NIST team. However, compared with what might be the appropriate energy scale for the hydrogen atom—13.6 eV—this fractional upper limit on the nonlinearity is on the order of 10^{-20}.

—Barbara Goss Levi

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