

QUANTUM HALL EFFECT SHOWS SURPRISING EVEN-DENOMINATOR PLATEAU

Yet another surprise from the quantum Hall effect. Since its first observation in 1980 by Klaus von Klitzing and coworkers, this remarkable quantum behavior of two-dimensional electron systems at low temperature and high magnetic field has not ceased to challenge theorists with the totally unexpected. The first great surprise was the observation of broad plateaus of constant Hall conductivity at quantized values that agreed with a manifestly simplistic theory to a quite incredible accuracy of a few parts in 10^8 . Von Klitzing won the 1985 Nobel Prize in Physics for his discovery.

In the spring of 1982, just when the theorists thought they finally understood why the Hall conductivity should be quantized in integral multiples of e^2/h to such extraordinary accuracy, Dan Tsui, Horst Störmer and Arthur Gossard at Bell Labs discovered that there were also quantized states at *fractional* multiples of e^2/h . "It knocked our socks off," recalls theorist Robert Laughlin (now at Stanford), who eventually provided the canonical explanation for this new "fractional quantum Hall effect," explaining, among other things, why the effect was seen only at odd-denominator rational fractions of e^2/h . The first and most prominent of these fractional states has a Hall conductivity of $\frac{1}{3} e^2/h$.

An even-denominator state

"After that, we thought we had no socks left to knock off," says Serge Luryi (Bell Labs). "But we were wrong." Three months ago Störmer, Tsui, Gossard and their colleagues James Eisenstein, John English and Robert Willett reported¹ the discovery of an *even-denominator* fractional quantum Hall state—with a conductivity of $\frac{5}{2} e^2/h$. A whole array of quantized fractional multiples of e^2/h had been found in the past five years— $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{9}$,...—but this was the first with an even denominator. Though no one had produced a rigorous argument prohibiting even-denominator states, a collocation of experimental findings and

strong theoretical suggestion had led most everyone, including the Bell Labs group, to assume that they simply did not exist.

A year earlier, Robert Clark and his colleagues at Oxford University and Philips Research Laboratories in England had in fact reported² the first suggestion of this new state. Theory and experience tell us that every quantized Hall conductivity plateau is accompanied by a deep dip in the component of resistivity along the direction of current flow; the current in the two-dimensional electron system becomes essentially lossless. The English group had seen a weak resistivity minimum at a magnetic field level appropriate to $\frac{5}{2}$, but not yet the Hall conductivity plateau essential to clinching the case. There being a widespread prejudice against such even-denominator states, their inconclusive data met with skepticism.

"The experimenters have alerted us to the inadequacies of the present theory," commented Richard Prange (University of Maryland) when the $\frac{5}{2}$ plateau was reported in October. But things are moving rapidly on the theoretical and experimental fronts. In November, Duncan Haldane (University of California, San Diego) and his colleague Edward Rezayi (California State University, Los Angeles) submitted for publication³ their theory that appears to explain why a fractional quantum Hall state should have been seen at $\frac{5}{2}$ —but not at $\frac{1}{2}$ or $\frac{3}{2}$. Whereas all the previously seen integral and odd-denominator plateau states are presumed to be states of fully polarized electron spin, Haldane and Rezayi attribute the new $\frac{5}{2}$ plateau to a spin-singlet state of mixed polarization. Preliminary new data from the Bell Labs group strengthen this sort of conjecture. The Haldane-Rezayi argument requires that the Zeeman splitting between energy levels of opposite spin be appropriately small. The Bell Labs group concludes tentatively that the $\frac{5}{2}$ plateau does indeed go away, as theories of this kind would predict, when they increase the Zeeman split-

ting in a way that leaves the system otherwise essentially unaltered.

Quantum Hall effect

The classical Hall effect is simply the development of a transverse voltage across a current flowing in the presence of a perpendicular magnetic field. This "Hall voltage" is normal to both the current flow and the magnetic field. To see the *quantum* Hall effect one first creates a two-dimensional electron gas by confining electrons in the narrow potential well at a semiconductor-heterostructure or MOSFET interface, at a temperature so low that the electrons are all in the ground state of that potential well. The electrons have no freedom of motion normal to the interface, but they can respond freely to an electric field in the plane. One then imposes a strong magnetic field perpendicular to the plane. The electrons execute tiny cyclotron orbits around the flux lines. In response to an electric field in the plane these orbits will tend to drift in the direction $\mathbf{E} \times \mathbf{B}$ —the Hall effect. The energy associated with the cyclotron motion of each electron takes on quantized values $(n + \frac{1}{2}) \hbar \omega_c$, where ω_c is the cyclotron frequency at the imposed magnetic field and n is the quantum number corresponding to the "Landau level."

At low temperature all the electrons seek to crowd into the lowest Landau level—up to the limit imposed by the Pauli exclusion principle. It turns out that the maximum (two-dimensional) density of electrons allowed in any one Landau level at a given magnetic field is $2Be/h$ —that is to say, one electron per elementary flux quantum ($\Phi_0 = h/e$) for each of the two electron spin orientations normal to the plane. One can visualize this as the tightest possible packing of cyclotron orbits in the plane. As the field B gets stronger, the orbits get tinier, and there's room for more electrons. The "filling factor" ν is the electron density normalized to Be/h . When ν reaches 2, for example, both spin states of the first Landau level ($n = 0$) are fully occupied. If the

magnetic field is strong enough, there is considerable Zeeman splitting between the two spin sublevels of each Landau level. The lower-energy spin-up state must then be filled before electrons move into the spin-down state. One can vary the filling factor experimentally by varying B for a fixed carrier density at a heterostructure interface.

When B corresponds to an integral number of completely filled Landau levels, an electron orbiting and drifting in crossed electric and magnetic fields cannot lose energy by scattering. All its neighboring states are occupied and the energy gap to the next unoccupied Landau level ($\hbar\omega_c$) is too great to jump at low temperature and high B . The same is true of the Zeeman energy gap between spin sublevels. Thus the electron drift at integral values of the filling factor will be precisely orthogonal to any electric field in the plane, and current flow will be lossless. In this special case the Hall conductivity—the current density divided by the transverse field—takes on the simple, precisely quantized value ve^2/h . Why this quantized value should persist (with great accuracy) for a finite range of ν values around the integers, forming broad plateaus on the curve of Hall conductivity versus B , is a more subtle question that spawned a considerable theoretical literature on the role of localized defect states.

If this “integral quantum Hall effect” can be understood simply in terms of the exclusion principle in an independent-particle model, without invoking the repulsion between electrons, that is certainly not the case for filling factors like $1/3$ or $1/5$ —the fractional quantum Hall effect. The independent-particle picture provides no energy gaps for fractionally filled Landau levels. It must be that the Coulomb repulsion produces incompressible states of highly correlated carrier motion in high magnetic fields at specific fractional filling levels.

In 1983 Laughlin shed the essential light on the fractional quantum Hall effect by writing down a variational multiparticle wavefunction that exhibits all the essential properties of the states found at $\nu = 1/3, 1/5, \dots$ (see *PHYSICS TODAY*, July 1983, page 19). For every odd integer m it describes a ground state at filling factor $1/m$ that behaves like an incompressible liquid. This stiffness is a consequence of the strong mutual repulsion of the electrons, which takes on highly correlated quantum-liquid character at these special filling levels.

The liquid can of course ultimately be compressed, but only at the cost of

bridging an energy gap that involves the generation of exotic pseudoparticle excitations of fractional electric charge. Laughlin's wavefunction is restricted to odd integer values of m by the requirement that it be antisymmetric under the exchange of any pair of electrons, as demanded by Fermi statistics. The freedom that might be afforded by two electron spin orientations was not then considered, it being assumed that any partially filled Landau level would be fully spin polarized at high magnetic field. It was soon pointed out by Haldane, Bertrand Halperin (Harvard), Laughlin and others that these $1/m$ Laughlin states generate a host of derivative states with fractional fillings like $2/3, 2/5, 3/5$ and so on—but always with odd denominators. Halperin did point out that there could be even-denominator states *if* electrons were somehow bound into bosonic pairs. But he did not suggest a mechanism for such pairing.

Finding the new state

The discovery of the new even-denominator fractional quantum Hall state last summer was the achievement of a Bell Labs-Princeton-MIT collaboration; Tsui is now at Princeton, and Willett was an MIT graduate student jointly supervised by Peter Wolff, director of the MIT Francis Bitter Magnet Lab, and Störmer at Bell Labs. For the examination of lower-filling-factor states, the group exploited the uniquely high magnetic fields available at the Bitter Magnet Lab. The $5/2$ state was discovered in a single-interface GaAs/AlGaAs heterostructure Gossard and English had grown by molecular beam epitaxy. The potential well at the interface was formed by modulation doping, that is to say, varying the doping as a function of distance from the interface. The group measured the transport parameters in the two-dimensional electron gas trapped in this well at temperatures down to 25 millikelvin and magnetic fields up to 30 tesla at Bell Labs and MIT.

Observing the fractional quantum Hall effect has always required much cleaner interfaces than one needs for seeing the integral effect. “Our gallium arsenide heterostructures are the result of two decades of intense materials research in molecular beam epitaxy,” says Störmer. “They give us exceedingly smooth interfaces of unprecedentedly low disorder. We can get elastic scattering mean free paths as long as 20 microns, a truly macroscopic distance.” It also becomes increasingly difficult to see the quantum effect at higher filling fac-

tors, as one goes to lower magnetic field. The use of a heterostructure of extraordinarily high mobility (10^6 cm²/V sec) was therefore essential to the group's success in finding the $\nu = 5/2$ plateau. “With our 1982 materials, we wouldn't have seen it.”

At $5/2$ filling the experimenters found a clearly discernible Hall conductivity plateau developing, with its conductivity centered at $5/2 e^2/h$ to within half a percent. The plateau is accompanied by a well-defined minimum in the resistivity component along the current direction. “Though they are not yet fully developed,” Eisenstein told us, “these features emerge in much the same way as do the conventional odd-denominator states. [See figure on page 19.] They provide striking evidence for the first even-denominator state.”

A filling factor of $5/2$ implies that both spin states of the first Landau level are full, and that the effective filling factor of the second Landau level is $1/2$. Because a completely filled Landau level is much like a closed atomic shell, exerting essentially no influence on the partially filled level above it, one might well expect that a quantum state at $\nu = 5/2$ would imply the existence of a similar state at $\nu = 1/2$. But in fact no such state has been found. The $5/2$ state is the only even-denominator state anyone has seen. That is one of the key issues the theorists must address.

For a fixed interface carrier density, as one has in semiconductor heterostructure experiments of this kind, the filling factor is inversely proportional to the imposed magnetic field. In this experiment the $5/2$ plateau is seen at the relatively modest magnetic field of 5 tesla. At this field intensity the Zeeman splitting between up and down electron spins is considerably smaller than the Coulomb repulsion energy between neighboring electrons. The smallness of the Zeeman splitting is essential to the argument of Haldane and Rezayi. To test theories of this kind, Eisenstein and company have in recent weeks undertaken a modification of the experiment: They have increased the magnetic field, but tipped the heterostructure sample to such an angle that the field component normal to the interface, which determines the cyclotron frequency and the splitting between Landau levels, remains unchanged. Stressing that the results so far are tentative, Störmer told us that they find that the $5/2$ plateau vanishes promptly, while neighboring odd-denominator states are unaffected by the increased Zeeman splitting. This result strong-

ly suggests that the $\frac{5}{2}$ plateau is indeed due to a state of mixed up and down electron spins.

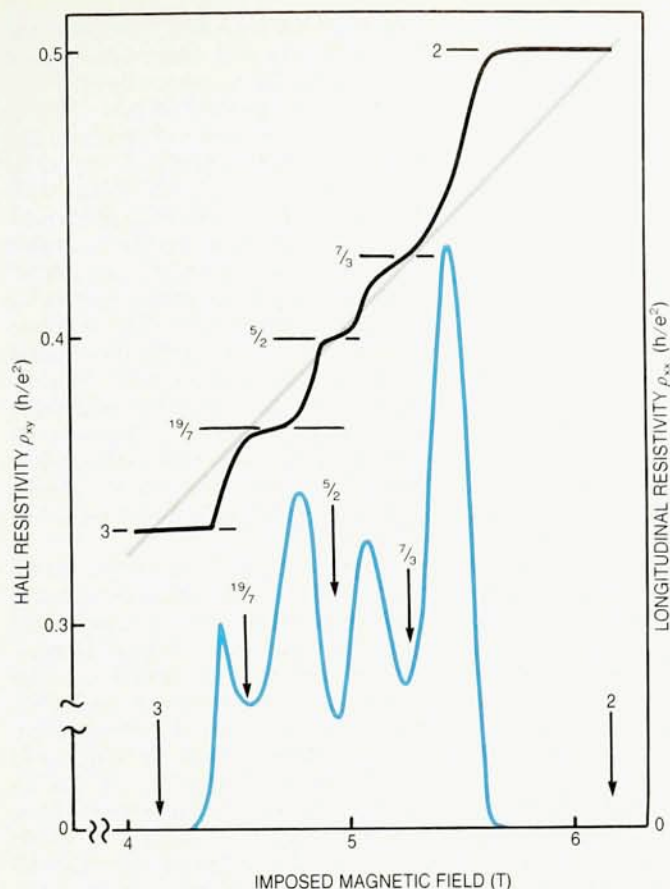
A new wavefunction

"Right now we're kicking ourselves because the experimenters found the $\frac{5}{2}$ state before we did," Haldane told us. "Last summer we were looking at mixed-spin states in the first Landau level, and at the second Landau level, but only in a fully spin-polarized state. We didn't put the two together until after we saw the data." The answer, when it eventually came, required mixed spins in the second Landau level.

At 5 tesla the Zeeman splitting is so small that it is no longer safe to assume that a fractionally filled Landau level has all its electron spins lined up. Can the mixing of spins, providing, as it does, an extra degree of freedom for satisfying the exchange-antisymmetry requirement for fermions, give us even-denominator quantum Hall states?

Haldane and Rezayi used analytic and numerical methods in tandem to study the fractional quantum Hall effect. The analytic approach involves replacing the real Hamiltonian, with its Coulomb and magnetic interactions over all distances, by a simple, solvable "model potential." They express these model potentials in terms of the energy cost for any pair of electrons to be in a particular state of relative orbital angular momentum l . In 1983 Haldane pointed out that Laughlin's variational wavefunction was in fact the *exact* solution to a model potential in which there is no energy cost for two electrons being in any orbital state except $l=1$. Smaller l , of course, means smaller separation and thus greater Coulomb repulsion. In the Laughlin wavefunction, which assumes complete spin polarization, Fermi statistics permit only odd values of l . Haldane noted that the Laughlin ground states are characterized by *zero* probability for, any two electrons being in an $l=1$ orbital state. The nodes of the wavefunction keep the electrons well apart. Subsequent numerical studies diagonalizing a more realistic Hamiltonian verified that the Laughlin wavefunction remains a very good approximation even after one turns the long-range interactions back on.

If one admits both spin orientations into the same partially filled Landau level, one can have even as well as odd l . Under these circumstances of small Zeeman splitting, Haldane had looked to see if anything interesting happens at a filling factor of $\frac{1}{2}$ in the first Landau level. He saw nothing,



First even-denominator fractional quantum Hall conductivity plateau has been found, with a conductivity (reciprocal of Hall resistivity) of $\frac{5}{2} e^2/h$, at a filling factor of $\frac{5}{2}$. Black curve shows Hall resistivity versus imposed magnetic field, observed¹ at 25 mK. Other developing plateaus are seen here at odd-denominator filling factors of $\frac{7}{3}$ and $\frac{19}{7}$. Filling factors, inversely proportional to the magnetic field, are indicated by arrows. Corresponding multiples of e^2/h are indicated by horizontal lines. Fully developed plateaus are seen at integral multiples: 2 and 3. Blue curve shows ordinary (longitudinal) resistivity, manifesting deep minima at filling factors corresponding to the plateaus.

nor was there any evidence of even-denominator states in the second Landau level, as long as they looked only at fully polarized states.

"As soon as we heard the $\frac{5}{2}$ state had been found, we extended our numerical search to mixed-spin states in the second Landau level," Haldane recalls. "We had faith that the Laughlin picture was substantially complete—that no really new physics was needed." The crucial difference between the first and second Landau levels, it turns out, is the wavefunction, or "cyclotron form factor," describing the cyclotron orbit of a single electron. In the lowest Landau level this form factor has a Gaussian distribution peaked at the center of the tiny orbit. The form factor in the second Landau level, by contrast, has a node in its middle. Thus two electrons sitting on top of one another experience less mutual repulsion in the second Landau level. The energy cost of having an $l=0$ pair is substantially

cheaper than it is in the lowest Landau level.

Haldane and Rezayi began their new numerical study with a potential appropriate to the first Landau level, with most of the energy cost coming from $l=0$ and 1 pairs. Then they gradually lowered the energy cost for $l=0$. What they found was a sharp transition when the $l=0$ energy term fell to a level appropriate to the second Landau level. The two-electron correlation function at $\nu = \frac{5}{2}$, which had maintained a deep hole at the smallest interelectron distances, suddenly manifested a large pairing peak at small separation. "It was very dramatic," Haldane recalls. "We were sure we had explained the $\frac{5}{2}$ state."

At the end of October they topped all this by finding an analytic wavefunction that yields the $\frac{5}{2}$ ground state—essentially a generalization of the Laughlin wavefunction to mixed spins. Haldane and Rezayi had noted

that continuing to lower the $l = 0$ energy cost in their numerical calculations below anything physically reasonable changes very little. So they undertook to solve a model potential with the $l = 0$ energy—the cost of two electrons of opposite spin sitting on top of one another—reduced to zero. In this simple “hollow core” model there is no Coulomb energy cost for any pairing other than $l = 1$. This is the model for which the new wavefunction is an exact solution.

As they raise the filling factor in the hollow-core approximation to the second Landau level, Haldane and Rezayi find that the last of the spin-polarized ground states occurs at a filling factor of $2 + \frac{1}{3}$. After that, electron spins begin to reverse, until finally one gets to the spin-singlet $2 + \frac{1}{2}$ ground state—“the same state we were finding numerically with more realistic potentials, and the one the experimenters found,” Haldane asserts.

Not everyone agrees that the hollow-core-model wavefunction will turn out to be a good approximation to the real $\frac{5}{2}$ ground state. Haldane and Rezayi conclude that there is no significant change when one goes from the model potential to the true Coulomb potential. But at the Uni-

versity of Indiana, Daijiro Yoshioka, Steven Girvin and Allan MacDonald are also looking numerically for even-denominator ground states. “With the hollow-core potential, our simulations reproduce Haldane’s $\frac{5}{2}$ spin-singlet ground state,” Yoshioka told us. “But with a more realistic potential, our six-electron simulation yields a partially polarized ground state.” MacDonald does suggest a particular analytic spin-singlet wavefunction that comes close to being the ground state in these six-electron simulations. This state, however, is not the same singlet state put forward by Haldane and Rezayi. In particular, it does not exhibit their strong pairing correlation between electrons of opposite spin.

“This raises the interesting possibility,” MacDonald comments, “that there is more than one incompressible state at the same filling factor.” Which one nature chooses as the ground state may depend on subtle effects thus far ignored in the calculations—Landau level mixing and third-dimension correlations due to the finite width of the interface. Also, one will have to undertake much more extensive calculations than the six-electron simulations that have thus far been carried out.

To explain the fractional quantum Hall effect when it made its unexpected appearance five years ago, Laughlin had to invoke an exotic incompressible quantum liquid state with fractionally charged excitations—almost a new state of matter. When the Bell Labs-Princeton-MIT group announced their new experimental surprise last summer, they wrote, “It remains to be seen whether a common theoretical description can be found [for the odd- and even-denominator states], or whether one is dealing with two distinctly different ‘new states of matter.’” It would appear that the more prosaic alternative has won the day. The Laughlin theory, suitably extended to states of mixed spin, now seems adequate to the task.

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MYSTERY SPOTS, X RAYS, γ RAYS: IS THE DUST SETTLING FROM SN1987a?

Supernova 1987a, which appeared as a naked-eye object 23 February 1987 in the Large Magellanic Cloud (see the article by David Helfand in *PHYSICS TODAY*, August, page 25), continues to challenge observers and theorists.¹

The visible light reached its peak three months after the supernova’s appearance, and since then has entered an exponential decline to fade from naked-eye visibility. Supernova models can explain such a decline by Co^{56} radioactive decay. There is some speculation that the supernova’s slow brightening until 20 May could be due to a rapidly spinning neutron star—a pulsar. The neutron star’s formation was probably announced in a burst of neutrinos detected on 23 February. (See the article by Adam Burrows in *PHYSICS TODAY*, September, page 28.)

As the supernova debris spreads from the site of the explosion, observers and theorists expect to detect the radioactive decay products predicted by explosive nucleosynthesis scenarios. Also, photons and the supernova’s blast wave should interact with circumstellar material from the su-

pernova’s blue supergiant stage and, further away, somewhat denser material from an earlier, red supergiant stage. The rapidly fading radio observations of the supernova in the first few weeks are consistent with the supernova shock wave’s running into the low-density wind expected from the blue supergiant progenitor.

The observational fact that a blue supergiant blew up was startling because models of supernova outbursts have generally relied on red supergiant progenitors. But stellar evolution theory can allow a time scale as short as 10 000 years for the progenitor’s evolution from red to blue supergiant. The difference in radius between these supergiants could easily account for SN1987a’s unexpectedly low luminosity at peak brightness compared with other supernovae.

According to Roger Chevalier (University of Virginia), the best way to observe any surrounding gas shell left over from the red supergiant stage would be in the uv data from the International Ultraviolet Explorer satellite. The gas should have been

photoionized by the supernova flash in the initial few hours as the shock wave broke out from the supernova.

Since the summer, Robert Kirshner (Harvard), Nino Panagia (Space Telescope Science Institute, on leave from University of Catania, Italy) and Angelo Cassatella (IUE station, Madrid) have seen uv emission lines near the line of sight from gas estimated at about 2–3 light years from the supernova in the direction toward the observer. Dense gas from the red supergiant stage at a minimum distance of 1 light year (estimated from the 10 000-year time scale mentioned above) should be much further out than where the shock wave should have reached by now. Kirshner has reported uv observations of nitrogen-rich circumstellar material; the observations support the supernova’s former career as a red supergiant.

Out, damned spot . . .

Shortly after the explosion, a group from the Harvard-Smithsonian Center for Astrophysics—Margarita Karovska, Peter Nisenson, Robert Noyes,