Probing cosmic geometry suggests the universe is flat

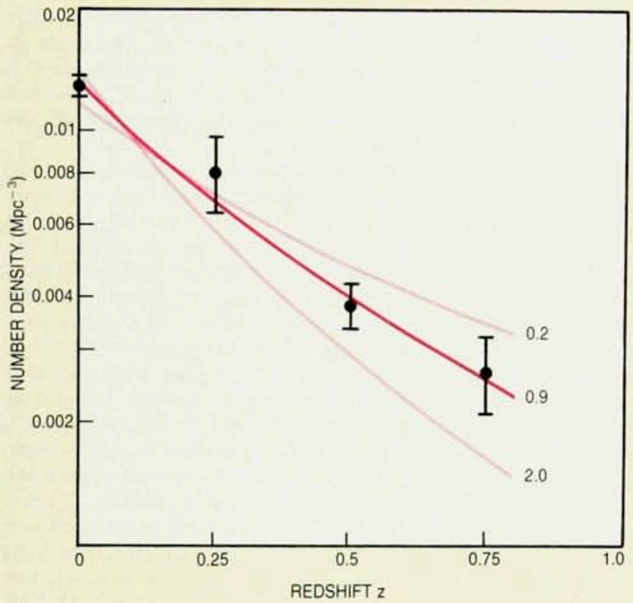
The tug of gravity is gradually slowing down the general Hubble expansion of the cosmos. We don't, however, have a good observational fix on the rate of this universal deceleration. In particular, we don't know whether we're slowing to an ultimate halt, to be followed by a recontraction to a "Big Crunch," or whether the Hubble expansion will go on forever.

Assuming that the gravitational deceleration is not hindered by a general repulsion implicit in a nonvanishing "cosmological constant"—a general-relativistic free parameter that went more or less out of fashion with steady-state cosmology—one can readily calculate a critical mass density, $\rho_c(t_0)$, for the universe at time t_0 (the present) such that the Hubble expansion will ultimately be reversed if and only if $\rho(t_0)$, the actual average density, exceeds $\rho_c(t_0)$. If H_0 is the present Hubble constant,

$$\rho_c(t_0) = \frac{3}{8\pi}\frac{H_0^2}{G}$$

where G is the gravitational constant.

This critical "closure" density corresponds to about 10^{-29} gm/cm^3, a few hydrogen atoms per cubic meter throughout the universe. It would appear that the total mass trapped in galaxies falls an order of magnitude short of ρ_c. But because there is nowadays a strong theoretical prejudice favoring the *precise* equality of ρ and ρ_c, astronomers and particle physicists are fervently searching for the "missing mass" required by this limiting case of a universe just dense enough to be closed.



Number density of galaxies per unit volume of redshift space, as a function of redshift, and normalized so the density at $z = 0$ is just the ordinary spatial density Φ^* observed in our vicinity. Data points are from the survey by Edwin Loh and Earl Spillar.[2] Loh and Spillar fit these data (labeled curves) for different values of Ω (the mean mass density of the universe divided by the critical density), assuming that the "comoving density" of galaxies remains unchanged at least as far back as $z = 1$. The best fit (bright red) is obtained for $\Omega = 0.9$, very close to the exact criticality favored by theorists, but much larger than the density accounted for by all the luminous matter we see.

Edwin Loh and Earl Spillar, physicists at Princeton, have taken a different tack. The direct search for missing mass having thus far failed to bring us close to the sought-for critical density, they have undertaken instead to look directly at the geometry of the cosmos, using the redshifts of "galaxies solely as benchmarks in space to measure the change in expansion with distance and the curvature of space," as Loh described it recently in *Physical Review Letters*.[1] Counting the galaxies in a patch of sky as a function of redshift is equivalent to measuring the volume of a cosmic cone. Having measured the redshifts of 1000 galaxies in five small patches of sky, Loh and Spillar reported[2] last summer that the cosmic geometry thus revealed implies that ρ/ρ_c is 0.9 (their 95% confidence limits put the range between 0.4 and 1.6). Both ρ and ρ_c decrease as the universe expands, but the ratio ρ/ρ_c, designated by Ω, is less time dependent, and rigorously constant if Ω is 1.

The result of Loh and Spillar is both pleasing and surprising. Pleasing because, as Joseph Silk (University of California, Berkeley) tells it, "cosmologists are obsessed with the idea that Ω is precisely equal to unity. Were it not so . . . the initial conditions of the Big Bang must have been highly contrived, to within a part in 10^{60}, to allow the universe to expand to its present dimension." More explicitly, the very attractive inflationary Big Bang cosmology requires that Ω equal 1, assuming that the cosmological constant vanishes. If Ω is 1, space, on the grand scale, is essentially Euclidean; there is no general cosmic curvature. In the inflationary scenario, all curvature would have been exponentially inflated away in the first 10^{-35} seconds after the Big Bang. (See Silk's article in last month's PHYSICS TODAY, page 28.)

On the other hand, the Loh–Spillar

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result is something of a surprise, not easily reconciled with the only two serious estimates we have of the universal mass density. From the theory of primordial nucleosynthesis and the observed abundance of the light elements one concludes that nuclear matter cannot account for an Ω in excess of 0.2. The other serious estimate comes from dynamical measurements of galactic interactions. From the observed relative velocities of galaxies near one another, one gets a very conservative upper limit of 0.5 for Ω due to all matter—whether nuclear or nonbaryonic—associated with galaxies. Thus if Loh and Spillar are correct in telling us that Ω is close to unity, it would appear that the bulk of the mass that accounts for this high cosmic density is neither nuclear nor galactic.

"This result is attractive to those of us who believe that the universe is dominated by 'cold dark matter,' for which the galaxies are a severely biased tracer," comments Marc Davis (Berkeley). Davis argues that luminous galaxies are generated only in regions of extraordinarily high primordial density, so that most of the cosmic mass distribution remains unobserved and unassociated with the galaxies we can see.

In general-relativistic cosmology, the universe can have an overall spatial Riemannian curvature, in addition to the local wrinkles in the neighborhoods of massive objects. But the "cosmological principle"—the postulate that the large-scale universe in any given epoch looks the same in all directions to all observers "comoving" with the Hubble expansion—requires that any such grand-scale curvature must be uniform throughout the cosmos. If this curvature is nonvanishing, it will evolve as the universe expands or contracts.

To the extent that one can ignore the other components of the energy–momentum tensor, the Einstein equations tell us that this universal curvature depends only on the average mass density. If Ω is 1, there is no curvature, and space is Euclidean. The non-Euclidean curvature of 3-space is easiest to visualize for Ω greater than 1, the case in which the average density exceeds the critical density. In that case the radius of curvature $R(t)$ can literally be thought of as the radius of the universe, regarded as the three-dimensional hypersurface of a hypersphere expanding (and ultimately recontracting) in a four-dimensional Euclidean space (not to be confused with four-dimensional Minkowski space–time). This is referred to as the case of positive curvature, with the proper distance element exceeding its Pythagorean value, just as it does in ordinary spherical geometry.

Unlike the positive-curvature case, where the entire universe is of finite volume—"closed"—the negative-curvature case, with Ω less than 1, corresponds to an infinite—open—universe. In this low-density case, the universe never stops expanding, but its ever increasing curvature parameter $R(t)$ can no longer be thought of as its literal radius. For negative curvature, the proper distance element is smaller than its Pythagorean value, as if one were at a saddle point.

For the limiting Euclidean case, with precisely critical density, the universal expansion is never quite reversed, but the expansion rate goes asymptotically to zero. Space being flat, $R(t)$ loses its role as a radius of curvature, but it retains the role of "cosmic scale factor," describing the expansion of the universe.

In the expanding universe, one can define a "comoving" coordinate system, such that a "fixed" point (x,y,z) moves with the Hubble expansion and the proper distance between any two such points grows proportionately as $R(t)$ grows. The conventional number density of galaxies decreases like $R^{-3}(t)$ as the universe expands. But per unit volume of this comoving coordinate grid, which is itself expanding like $R(t)$, the number of galaxies is, on average, fixed, to the extent one can ignore the creation or merging of galaxies. This is the essential working hypothesis of Loh and Spillar. Normalizing the comoving coordinate system to the present, one can determine this unchanging "comoving number density" Φ^* from low-redshift surveys of galaxies relatively close by. The number density Φ^* turns out to be roughly one galaxy per hundred cubic megaparsecs.

To determine the density of galaxies at earlier times, when the universe was smaller, Loh and Spillar counted galaxies at larger redshifts. Seeing a galaxy whose optical wavelengths are redshifted by a fractional amount $\Delta\lambda/\lambda$, designated by z, one is looking at light emitted at a time t_e, when the cosmic scale factor $R(t_e)$ was smaller than its present value by a factor $z+1$. (A different normalizing convention simply equates $R(t)$ to $1/(z+1)$.) We have no reliable direct measurement of the distance of the emitting galaxy. Counting the number of galaxies ΔN to be seen in a small redshift interval Δz and a solid angle of sky $\Delta\omega$, one determines the corresponding element of comoving volume ΔV from the relation $\Delta N = \Phi^* \Delta V$. That is to say, one assumes that the comoving number density of galaxies was the same then as it is now. Galaxies may enter or leave a comoving volume element as it expands with time, Loh and Spillar assume, but on average its population does not change—at least not for redshifts less than 1, the range of their survey. If one were to look much further back in time, to significantly larger redshifts, one would face the problem of significant galaxy formation in the early universe.

All this is of course complicated by the fact that galaxies appear fainter with increasing distance, and there is a limit to how faint a galaxy one can detect. But more of that later. The redshift volume element $z^2 dz d\omega$ is related to the comoving volume element dV by a complicated but calculable expression that depends on Ω. The mass density ratio Ω plays this geometrical role because it equals twice the "deceleration parameter" $-\ddot{R}/(RH^2)$, which describes the rate at which the Hubble expansion is slowing down at any particular time.

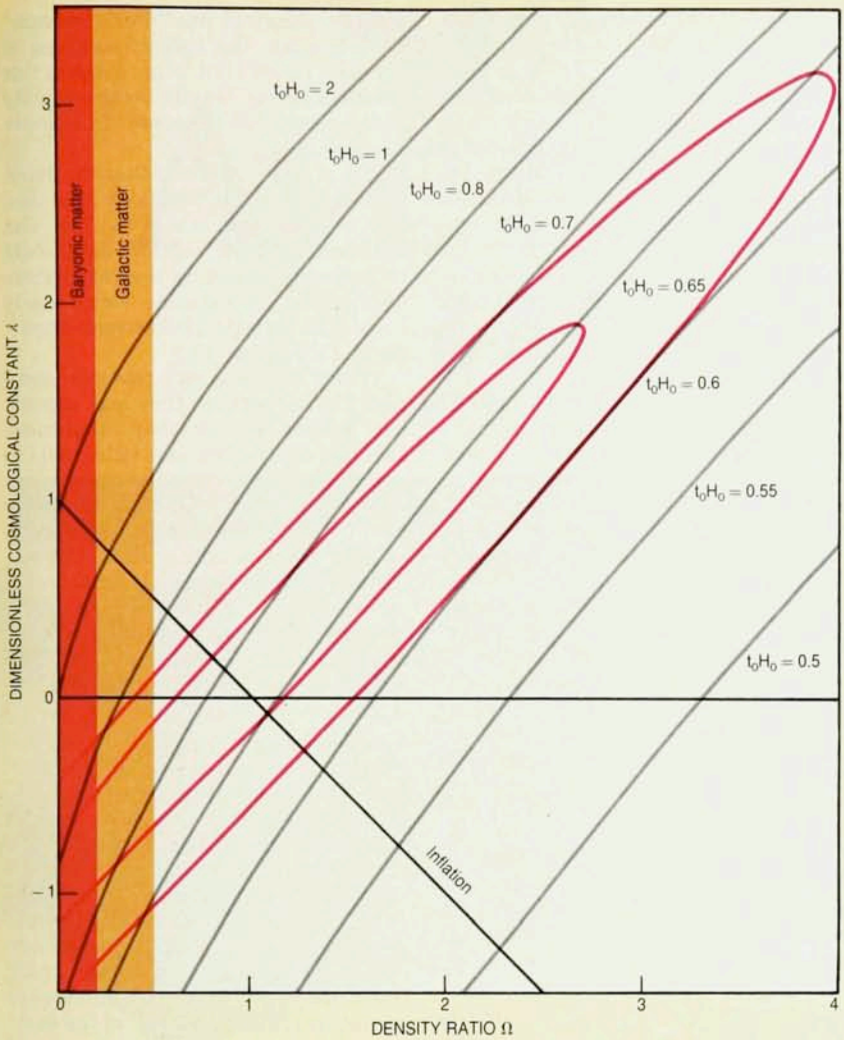
If r is the (dimensionless) radial coordinate of a z-shifted galaxy in a comoving spherical polar coordinate system with us at its origin, the comoving volume element is given by

$$dV = \frac{R^3(t_0) r^2 dr d\omega}{\sqrt{1 - kr^2}}$$

where $R(t_0)$ arbitrarily normalizes the (unchanging) comoving coordinates to the present, and k is ± 1 or 0, depending on the sign of the cosmic curvature. The geometrical role of Ω in the conversion from redshift space to comoving space is most easily seen in low-z approximation, where r is given, to first order, by $(t_0 - t_e)c/R(t_0)$. Because r between us and the z-shifted galaxy we're looking at is approximately proportional to the light transit time, we see that r for a given redshift z decreases as Ω increases: Larger Ω implies faster deceleration of the Hubble expansion. The faster the deceleration, the more recently was $R(t_e)$, the cosmic scale factor, $1/(z+1)$ of its present size. The more recent the emission for a given z, the smaller is the comoving radial coordinate of that redshifted galaxy.

Thus for a given redshift volume element, the corresponding comoving volume element is smaller for larger Ω. By assumption, the number of galaxies in a given redshift volume is simply its comoving volume times the constant, comoving number density Φ^*. Therefore the bigger Ω is, the fewer galaxies we expect to find in a particular redshift interval. What Loh and Spillar have done is fit the observed redshift distribution of galaxies found in several patches of sky to the cosmologically expected distribution, with Ω as a free parameter to be determined.

This 'redshift–volume test' of cosmic curvature is by no means a new idea.



Fits for the cosmological constant as well as Ω from the data of Loh and Spillar.[1] The elliptical red curves enclose the 95% and 67% confidence regions of the Ω,λ plane. The inflation-theoretic constraint $\Omega + \lambda = 1$ is indicated by the diagonal line. With this constraint, the best fit gives $\Omega = 0.9$ and $\lambda = 0.1$, with relatively small uncertainties. It is widely supposed that λ is precisely 0. The shaded regions at left indicate that even the most generous upper limits on the matter we know about cannot account for values of Ω near 1. The gray curves are lines of contant t_0H_0, the age of the universe times the Hubble constant. An Einstein–de Sitter universe requires that t_0H_0 equal $\frac{2}{3}$.

H. P. Robertson and Thomas Noonan at Caltech pointed out 25 years ago that a redshift–volume measurement out to $z = 0.5$ ought to suffice to determine the geometry of the cosmos. But measuring redshifts for distant galaxies has until recently been a laborious process, and adequate redshift surveys out to sufficient distance have not been available. Half a century ago, Edwin Hubble and Richard C. Tolman carried out essentially the same test in the absence of adequate redshift data by invoking a statistical relationship between z and apparent luminosity. They concluded that Ω lay between 2 and 3, implying a universe so implausibly small that Hubble felt this result cast doubt on the entire connection between redshifts and the cosmic expansion.

Loh and Spillar have produced the first data sample adequate for the redshift–volume test by availing themselves of a number of new techniques.[3] At the University of Wyoming's 2.3-meter telescope in 1983, they photometrically determined the redshifts of 1000 galaxies in five $7' \times 9'$ fields, out to $z = 1$. "Using the new techniques, we were able to complete a field [about 200 galaxies] in just three hours," Loh told us. "With standard spectroscopic methods it would take 20 times as long with a 4-meter telescope and lower-z [nearer] galaxies."

Loh and Spillar determine redshifts by looking at broad features of the galactic spectrum rather than individual spectral lines. The virtue of this idea, which goes back to William Baum at Lowell Observatory in the 1950s, is that one can make do with much less dispersion of the meager galactic light if one doesn't have to look for individual lines. "When you don't split the data into so many wavelength bins, you don't need as high a signal-to-noise," explains Loh. Baum carried out a precursor of the Loh–Spillar test in the 1960s. Measuring galactic angular diameters against redshift, he, like Loh and Spillar, obtained an Ω much larger than that implied by the abundance of luminous matter.

The principal spectral feature exploited by Loh and Spillar to determine redshift is the prominent 4000-Å break due to the start of the Balmer continuum and absorption by heavy elements—mainly calcium—in stellar atmospheres. The redshift measurements were also speeded along by the use of modern charge-coupled-device detectors and the large field of view provided by the unusually short focal length of the 2.3-meter Wyoming telescope. Spillar, who had been Loh's student at Princeton, is now at the University of Wyoming.

The spectral information was obtained by recording CCD images through six different color filters, ranging from 425 nm to 900 nm. Stars, Loh and Spillar report, could be reliably distinguished from galaxies by their different color patterns. Requiring a certain threshold signal in the 800-nm band imposes an apparent-brightness cutoff. Therefore they record a larger fraction of all galaxies at smaller z, where galaxies of a given intrinsic luminosity appear brighter. To take this bias into account, Loh and Spillar do more than just count galaxies (above threshold) in a redshift bin. They also measure the total light flux received from these galaxies. Armed with the flux data and the widely used Schechter luminosity function, which purports to describe the distribution of intrinsic luminosities for galaxies, they can effectively correct the raw counting data for the bias introduced into the distribution of population versus redshift by an apparent-luminosity cutoff.

The empirical luminosity function, introduced by Paul Schechter (Mount Wilson Observatory) in 1976, has a particularly felicitous analytic form. The intrinsic luminosity L appears in the distribution function only as a scaled variable, divided by a characteristic luminosity L^*. The Loh–Spillar analysis requires no external knowledge of L^*. More importantly, the argument goes, the analysis does not suffer if L^* evolves with time. That is to say, the characteristic luminosity of galaxies can have been weaker (or stronger) in the early days we're seeing at z close to 1. "Unless the evolution is of a peculiar kind we can't approximate with an evolving L^* in the Schechter function, we're all right," Loh contends.

An earlier attempt to measure the cosmic curvature had, in fact, come to grief with the evolution problem. The classic redshift–magnitude test, a cousin of the redshift–volume test, was undertaken by Allan Sandage (now at Johns Hopkins) at the 200-inch Mount Palomar telescope in the early 1950s and throughout the 1960s. It exploits the Ω dependence of the relation between apparent luminosity (magnitude) and redshift for a galaxy of given intrinsic luminosity. But unlike the Loh–Spillar work, the redshift–magnitude test required some sort of "standard candle" to compare intrinsic luminosities at different distances.

Sandage invoked the assumption that the brightest galaxy in a cluster could serve as a standard candle. He eventually concluded, however, that the corrections required to take account of the change of galactic luminosity with age introduced uncertainties large enough to render the test ineffective. The principal evolutionary effects for which one could make no useful correction were the death of stars and the consumption of galaxies by their larger neighbors.

The data points in the figure on page 17 show, in effect, the (corrected) number of galaxies per unit redshift-space volume found by Loh and Spillar as a function of redshift. This redshift-space density decreases with increasing z because the redshift volume element corresponds to ever smaller comoving volume elements as z gets larger. The curves show the predicted falloff for different values of Ω. For a universe with positive curvature (Ω greater than 1), the falloff is faster than for the Euclidean critical-density case. For negative curvature (Ω less than 1), the falloff is slower. The best fit, Loh and Spillar report, is obtained with an Ω of 0.9, the bright red curve.

These fits were carried out with Λ, the cosmological constant, held fixed at zero. Thus Loh and Spillar have given the cosmologists what most of them want to hear—the first explicit observational evidence that we live in an Einstein–de Sitter universe (for which Ω is 1 and Λ vanishes), the simplest of all, with no overall curvature, no Hubble reversal and no general repulsion competing against gravity.

In a recent reanalysis of these same data, Loh has relaxed the $\Lambda = 0$ condition.[1] The disappearance of all curvature in the inflationary scenario requires only that $\Omega + \lambda = 1$, where λ is the dimensionless cosmological constant $\Lambda/(3H_0^2)$. Imposing this constraint in place of the requirement that λ vanish, Loh reports that the best fit yields an Ω of 0.9 (the 95% confidence range is 0.7–1.3) and a λ of 0.1 (the 95% confidence range is -0.3 to 0.3), again pointing to an Einstein–de Sitter cosmos. (See the figure on page 19.) With either constraint, Loh writes, his results are compatible with the standard wisdom on primordial nucleosynthesis "only if a large density of nonbaryonic matter exists."

A value of λ on the order of 0.1 makes very little sense, Loh points out. "The natural scale of the cosmological constant, the square of the Planck mass, is 10^{121} times larger than [our] limits," he writes. Therefore, he concludes, "some principle must make λ identically zero." Loh also extracts new limits on t_0H_0 from his fits, without any constraint on λ. The Hubble constant having the dimension of reciprocal time, $1/H_0$ is generally taken to be a first approximation to t_0, the present age of the universe. But the theoretical value of t_0H_0 differs from one model to the next. For the Einstein–de Sitter universe, for example, one gets a value $\frac{2}{3}$ for t_0H_0.

Loh concludes that t_0H_0 lies between 0.6 and 0.88. These limits, he told us, are tighter than those we have on t_0 or H_0 separately. Estimates of $1/H_0$ range from 9 billion to 25 billion years. Sandage, for one, is convinced that $1/H_0$ is "precisely" 23 billion years. Until recently, therefore, he could not believe in $\Omega = 1$, he told us, because the globular clusters in our own galaxy were thought to be older than $\frac{2}{3}$ of 23 billion years. "Now that the globular cluster age determinations have been reduced from 17 billion to 14 billion years, I am convinced for the first time that Ω does equal 1."

The physicists are happier with our results than the astronomers," Loh told us. "The technique is quite new, and people have to think about it some more." "Astronomers are cautious from bitter experience," says Richard Kron (Yerkes Observatory). They have more faith in redshifts laboriously measured from spectral lines than in those obtained by the faster photometric technique. Kron expresses concern about the calibration of the Loh–Spillar redshift determinations. Their photometric method was calibrated against spectroscopic redshifts only for a cluster of galaxies at $z = 0.4$, a redshift region where one expects photometry to be at its best. A number of astronomers would like to see more extensive calibration at higher z. "If the galaxy count is off by as little as 30% at $z = \frac{1}{2}$," Silk told us, "that's serious." Furthermore, the correct counting of galaxies in redshift bins, given a flux cutoff, depends heavily on the validity of the empirical Schechter luminosity function.

Silk is also concerned that the determination of Φ^*, the present local density of galaxies, taken by Loh and Spillar from other people's data, could be seriously biased by the very bubbly distribution of galaxies we've lately been discovering. (See PHYSICS TODAY, May, 1986, page 17.)

"Their test is a very powerful one," says Davis, "because they look at number density rather than brightness. Loh and Spillar are less vulnerable to galactic evolution than were the older tests. Among other things, they don't have to look as far back in z. Evolution is still their weakest link, but the method will continue to attract wide attention and we can look forward to great improvements as these details are addressed." As Kron puts it, "This technique will certainly flourish."

Note added in proof. William Fowler (Caltech) has called our attention to new primordial-nucleosynthesis results that call into question the conventional wisdom severely limiting the contribution of baryonic matter to closure. Livermore theorists Grant Mathews, George Fuller and Charles Alcock, calculating the density fluctuations that emerge from a first-order phase transition from the quark-dominated to the hadron-dominated era of the early universe, conclude that baryons would initially have been concentrated in dense pockets (*Bull. Am. Phys. Soc.* **32**, 1123, 1987). A related calculation (to be published in *Physical Review D*), by James Applegate (Columbia), Craig Hogan (University of Arizona) and Robert Scherrer (Harvard–Smithsonian), points out that neutrons, being uncharged, would have diffused out of these pockets much faster than protons. Unlike the conventional picture of primordial nucleosynthesis, which assumes a smoother initial distribution of protons and neutrons and rules out a baryonic Ω greater than 0.2, this scenario yields light-element abundances for $\Omega = 1$ in reasonable agreement with what we see. "It would be premature at this stage," Alcock suggests, "to give up on a universe closed by baryons."

—BERTRAM SCHWARZSCHILD

References

1. E. Loh, Phys. Rev. Lett. **57**, 2865 (1986).
2. E. Loh, E. Spillar, Astrophys. J. **307**, L1 (1986).
3. E. Loh, E. Spillar, Astrophys. J. **303**, 154 (1986). □