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On complexity

Leo P. Kadanoff



The physicist is largely concerned with abstracting simple things from a complex world. Newton found simple laws that could predict planetary motion. Results from atomic physics have demonstrated that the Schrödinger equation is a correct description of almost all observed atomic behavior. In fact, the hydrogen atom is used as a metaphor to describe the search for simple and illuminating examples in all areas of the physical sciences.

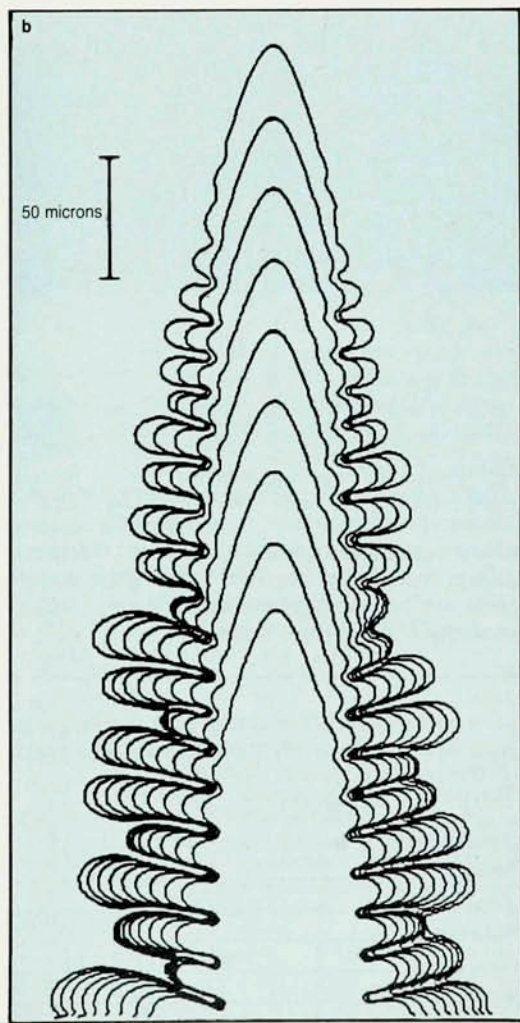
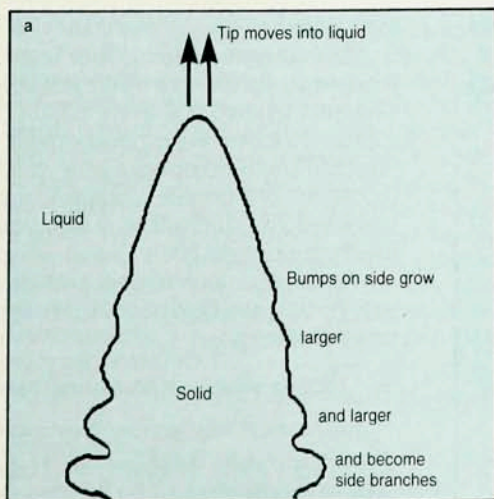
In some ways, this search for the simple makes our description of the world seem like a caricature rather than a portrait. Most elementary physics textbooks describe a world that seems filled with very simple, regular and symmetrical systems. A student might get the impression that atomic physics is the hydrogen atom, that electromagnetic phenomena appear most often in the world in guises like the parallel plate capacitor or dipole radiation, and that regular crystalline solids are "typical" materials. (Hydrodynamics is the one subject in the standard physics curriculum that deals most fully with richly complicated outcomes of physical laws. It is now only

rarely taught.) If our hypothetical students look at the world with an unbiased eye, they will see a richness quite unlike anything described in physics texts. In the real world, simplicity is a rare exception. Waves break upon reaching a beach, producing swirls, foam and spray. Every planet is different from the others, some with moons (again all different), some with beautifully structured rings. Atomic and molecular spectra show many different characteristic energy or frequency differences and a richness of structure not seen in the hydrogen atom. Even "elementary" particle physics includes a complexity that belies its name. Almost every decade of energy from millielectron volts to hundreds of GeV is described by its own individual phenomenology. Although complexity is the most obvious feature of the world about us, the textbook description of physics ignores this diversity or treats

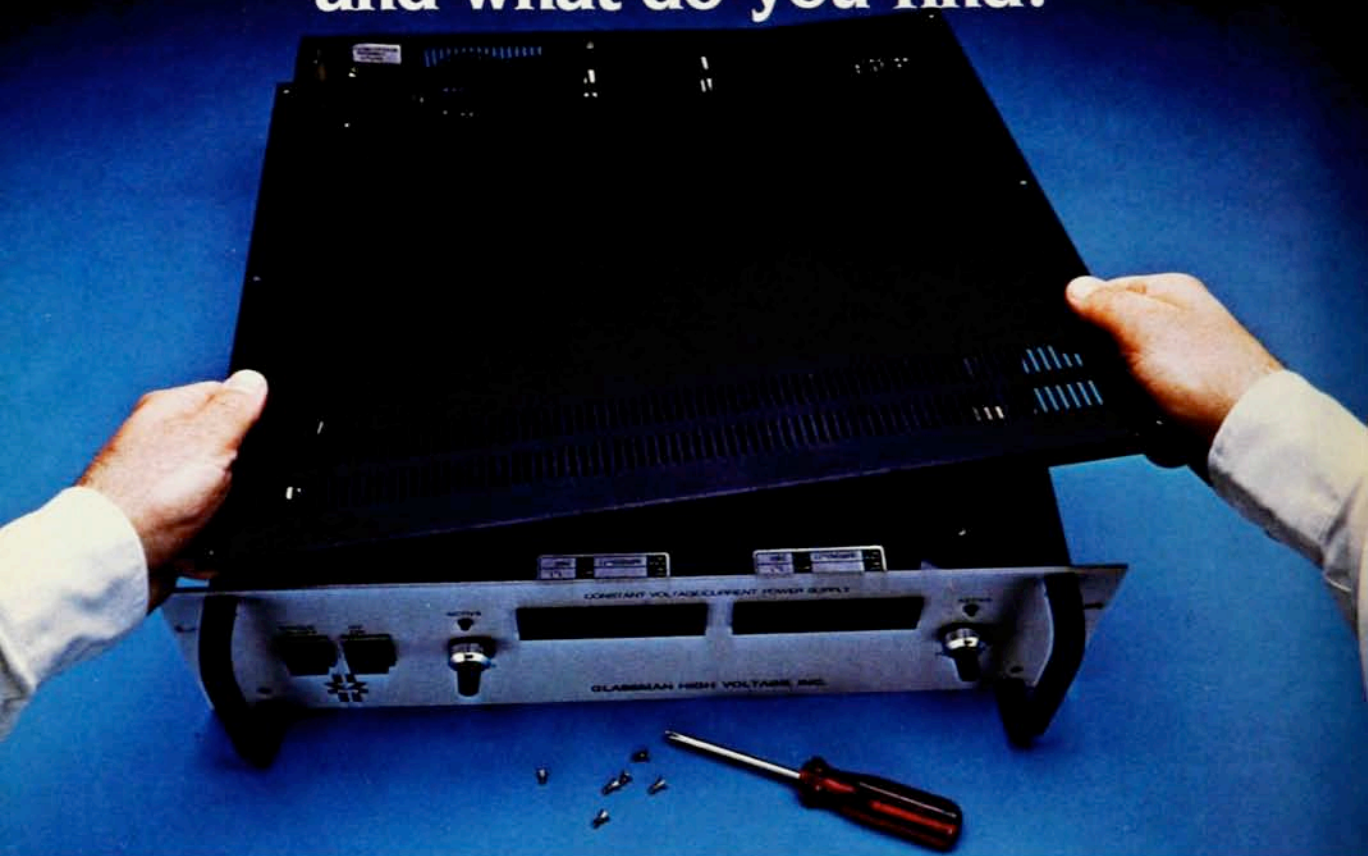
it as an aberration to be circumvented by choosing simple situations that might give us "characteristic" examples.

But I paint too extreme a picture. Physics is concerned with understanding complexity, particularly in those parts of the science aimed at more applied examples. Here I intend "ap-

Dendritic growth, a process in which a needle-shaped crystal grows into a liquid via the forward motion of the needle's point and the continual growth of bumps on its sides. **a:** A schematic view of the region near the point at a given time. **b:** The output of an experiment in which the growth of an ammonium bromide dendrite is observed at 20-second intervals and the resulting contours are superposed. (Reproduced from a preprint of A. Dougherty, P. D. Kaplan and J. P. Gollub.)



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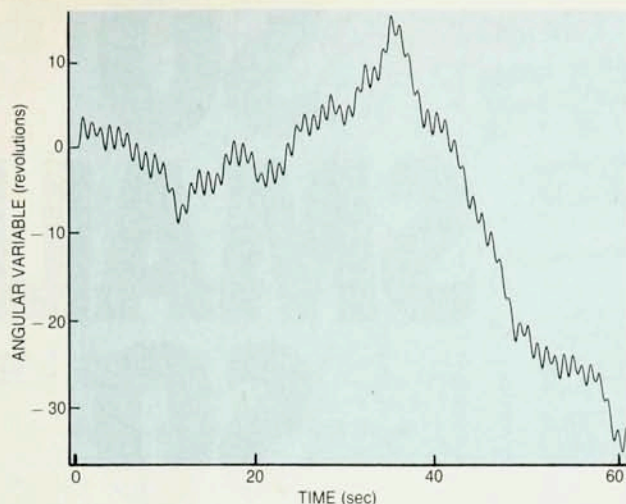
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plied" not to refer to commercial usefulness, but rather to mean focused upon the particular types of systems that really arise in the physical world. The astrophysicist, who must understand the distribution of matter in the universe; the biophysicist, asking perhaps how life arose; the plasma physicist, working with the intertwined structure of flux lines in a swirling ionized gas; the solid-state scientist, looking at the crystallization of a piece of steel—all these scientists must deal with complexity as an everyday issue. Until recently, many physicists have dismissed examples such as these as "dirt physics" or "squalid-state physics"—perhaps intending to suggest that these examples somehow contain less intellectual content than, say, a simple and easily interpreted spectrum. Here, I wish to suggest the possibility of the opposite view: that the observed complexity in the world around us raises questions that are absolutely fundamental to our understanding of the nature of physical law. Three such questions are:

- ▶ How do very simple laws give rise to richly intricate structures?
- ▶ Why are such structures so ubiquitous in the observed world?
- ▶ Why is it that these structures often embody their own kind of simple physical laws?

There is a discipline in and around physics that does study complexity as something interesting in itself, both in general and more especially in the most accessible examples in hydrodynamic flow, astrophysics, solid-state physics, lasers and so on. This subject of study has been given various names, but recently the most popular terms have been "pattern formation" and "chaos." Let me describe one of the central ideas that have emerged from these studies.

A major reason that complexity emerges in the working out of physical laws is that in many situations one has what is called sensitive dependence upon initial conditions. Sensitivity can be seen very directly in a process called "dendritic growth," which occurs when a crystalline solid freezes. Under appropriate conditions the solid moves forward into the liquid in the form of a needle with bumpy sides. (See figure 1.) Each bump is produced by an instability in the solidification process. The bumps are initially formed near the tip of the needle. At that point,



Displacement angle θ for a damped forced pendulum as a function of time. Note that θ is measured in revolutions; the plot shows that the pendulum goes through full circles in a complex and never repeating pattern. Figure 2

they are very, very small. As time goes on, the tip moves forward and the instability magnifies each bump. Thus, eventually, the bumps form a complex pattern of very many side branches pushing out of the needle's sides. The heights and placements of the side branches vary because each branch is produced by the amplification of an initially microscopic perturbation on the interface. Hence the variation of the observed structures is produced by the repeated occurrence of the sensitive dependence of outcomes upon the growth conditions. This repeated sensitivity is a major source of natural complexity.

Dendrites provide an example of chaotic irreversible growth. (The growth of snowflakes is more familiar and probably roughly analogous.) Another example arises when motion occurs in a kind of closed environment so that the variables describing the system cannot ever get out of a bounded domain. In some sense the system must do the same thing again and again. However, if this kind of system exhibits repeated sensitivity to conditions, it too will be chaotic. It will move through its bounded domain, with each go-around being different from all that came before. The system will never precisely retrace the same path.

A prototypical example of this chaotic behavior is one in which a damped pendulum is pushed by an external periodic force. The differential equation involved is terribly simple. If the pendulum's angular position is θ , and it has mass m and length l , the equation is

$$ml^2 \frac{d^2\theta}{dt^2} + \gamma \frac{d\theta}{dt} = -mgl \sin\theta + F \cos\Omega t$$

For a correct choice of damping constant γ , forcing strength F and forcing

frequency Ω , the motion never exactly repeats itself. (See figure 2.) Here, a very intricate motion is produced from the simplest of ingredients.

Some progress has been made in understanding motions such as that plotted in figure 2. We do have a partial phenomenological description in terms of the concepts of fractals and strange attractors. Moreover, there is a good theoretical and experimental description of how, as the parameters in the equation of motion are varied, systems like these go from an orderly to a chaotic motion.

However, our present understanding is limited to systems in which we can describe the motion in terms of a few variables or a few excited modes. Hydrodynamic systems can exhibit much richer behavior in which there are swirls within swirls within swirls and simultaneous sensitive dependences upon conditions at many different length scales. Despite a few tantalizing hints, understanding of this fully turbulent behavior has not yet been achieved.

So what do we say to the student who asks why the laws of physics are so simple, but the world so complicated? Clearly the right answer is "Come into physics and help us find out."

* * *

My series of columns in PHYSICS TODAY has been improved by the helpful criticism of my colleagues Ruth Ditzian Kadanoff, Albert Libchaber, Sidney Nagel, Stephen H. Shenker, Robert Wald and others, including many members of the staff of PHYSICS TODAY. I especially mention this help here because Shenker guided me away from a real bopper in an early draft of this column.

Further reading

- P. Berge, Y. Pomeau, C. Vidal, *Order Within Chaos*, Wiley, New York (1986). □

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