Quantized vortices in superfluid helium-4

Singular whirlpools give rise to such diverse phenomena as superfluid turbulence, two-dimensional phase transitions and quantum nucleation.

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When liquid helium-4 is cooled to 2.17 K it changes from an ordinary fluid to a superfluid somewhat similar in nature to superfluid helium-3 and to the electron fluid in superconductors. Extensive investigation over the last five decades has given us a highly refined phenomenological description of how superfluid He^4 behaves, even though the underlying quantum theory is only partially understood. Much of the current research on superfluid He^4 is part of a continuing effort to work out the implications of this phenomenological description, in the same way as classical fluid dynamicists are still working out the implications of the Navier–Stokes equations.

Although the microscopic quantum theory of superfluid He^4 appears to be even more complicated than that of its cousin superfluids, its actual behavior as a fluid is often comparatively simple. Unlike superfluid He^3, superfluid He^4 is isotropic (see the article on superfluid He^3 by Pertti Hakonen and Olli V. Lounasmaa, page 70), and unlike the superconducting electron fluid, superfluid He^4 is not affected by electromagnetic fields or by a host lattice. Most importantly, in superfluid He^4 the quantum coherence length is on the order of only 1Å, so that a classical continuum description remains useful down to truly microscopic dimensions. These characteristics give the study of superfluid He^4 a unique flavor that combines traditional condensed matter physics and fluid dynamics (figure 1).

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The flow of superfluid He^4 has a number of special features.

▶ The spectrum of allowed excitations in the fluid is so restricted that at moderate velocities it is simply impossible to convert kinetic flow energy into random internal excitations. Consequently superfluid He^4 behaves as a frictionless, ideal fluid.

▶ Because the velocity field $\mathbf{v}_s$ is interpreted as the gradient of a quantum mechanically defined phase field, the flow must be strictly irrotational:

$$\nabla \times \mathbf{v}_s = 0$$

▶ For the same reason, the circulation κ must obey a Bohr–Sommerfeld rule,

$$\kappa \equiv \oint \mathbf{v}_s \cdot d\mathbf{l} = \frac{nh}{m_4}$$

Here h is Planck's constant, m_4 is the mass of the helium atom and n is 0, 1, 2, ... This rule says that the circulation of the superfluid around any closed path in the fluid is quantized.

Save for the peculiarly modern condition of quantized circulation, one is left with the appealing notion that superfluid He^4 is in fact the first real example of that simplest possible construct of 19th-century fluid mechanics, the ideal fluid undergoing potential flow. This provides a useful starting picture to which we can add the complications that arise from thermal fluctuations and the breakdown of quantum coherence.

We begin this article with a discussion of the purely fluid dynamical issues presented by rotating helium and by superfluid turbulence. We then move on to topics such as thin-film phenomena and flow through microscopic orifices, where thermal fluctuations play an important role. We finish up with a brief discussion of how quantized vortices can arise through direct quantum nucleation.

Quantized vortices

The dynamical behavior of an ideal fluid becomes interesting when one adds the notion of a vortex filament. To represent such a filament, draw a line that either forms a closed loop in the fluid or terminates on the boundaries of the fluid, as figure 2 shows. Then require that the circulation of the velocity field $\mathbf{v}_s$ around this line have a given value. The problem of determining the velocity field that arises from this added boundary condition is mathematically identical to the problem of determining the magnetic field generated by a current-carrying wire, and can hence be solved by writing down the Biot–Savart integral. Thus a vortex filament consists of a circulating flow around some central, singular curve, the velocity dropping off away from the filament in such a way that the circulation κ stays constant.

One interesting aspect of vortex filaments is that by introducing them one can construct arbitrarily complicated velocity fields. Another is that they lead to complicated, nonlinear dynamical behavior. Simple conservation of momentum requires that each point on such a singularity move in response to

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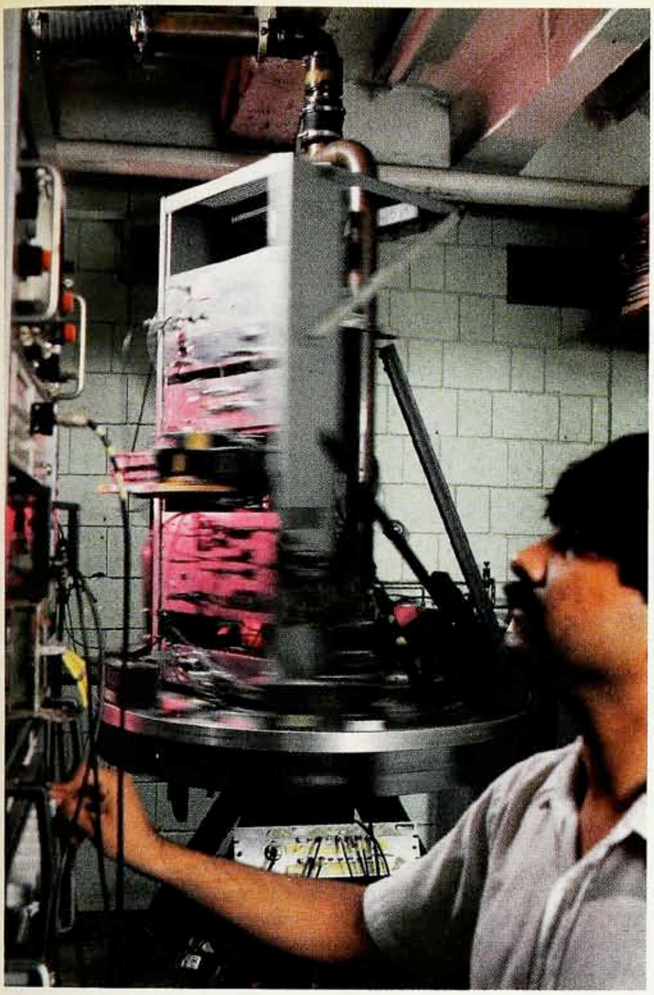


Rotating table for superfluid helium experiments at Rutgers University. Graduate student Rajive Tiwari is at the controls. All of the low-level electronics, as well as a He^3 refrigerator, rotate. A set of concentric slip rings maintains electrical connections, and a rotating vacuum seal connects the pumped helium bath to a remote vacuum pump. The table is capable of rotating at up to 12 radians per second, corresponding to an equilibrium vortex line distribution with an average line spacing of 0.06 mm. (Photograph by Nat Clymer.) Figure 1

the translational velocity of the surrounding fluid. Hence the complicated velocity field generated by a singularity acts back at each instant to cause equally complicated changes in its configuration.

It was first suggested by Lars Onsager, and convincingly argued by Richard Feynman,[1] that vortex singularities of this kind should in fact exist in superfluid He^4. In accord with the Bohr–Sommerfeld equation above, the circulation of this fluid must then be quantized, although in practice only the case where n is 1 has ever been observed. In real life, of course, the filaments must possess a core structure that cuts off the divergence of the circulating velocity field. For superfluid He^4 the quantum coherence length, which marks the distance at which fluid dynamical concepts become meaningless, provides this cutoff. The exact structure of the vortex core is a challenging problem that has attracted considerable theoretical attention. Both theory and experiment indicate a core radius on the order of only an angstrom, sufficiently small that the detailed core physics becomes largely irrelevant to the dynamical behavior of the vortex. It follows that quantized vortices in superfluid He^4 do in fact behave as classical vortex filaments, the only differences being that their circulations and core radii are preselected by quantum mechanical effects.

One can detect quantized vortices indirectly through the dissipation they cause and through the way their presence modifies the dynamical behavior of the superfluid. A complementary detection technique makes use of the fact that a charged ion injected into the superfluid can become trapped on a vortex core like a cork in a whirlpool. By tracking the fate of a pulse of such ions as it travels through the fluid, one can often obtain quite detailed information about the vortices. Thus the existence of quantized vortices in superfluid He^4 is amply borne out by experiment, and is known to account for the remarkable diversity of dynamical phenomena exhibited by this superfluid.

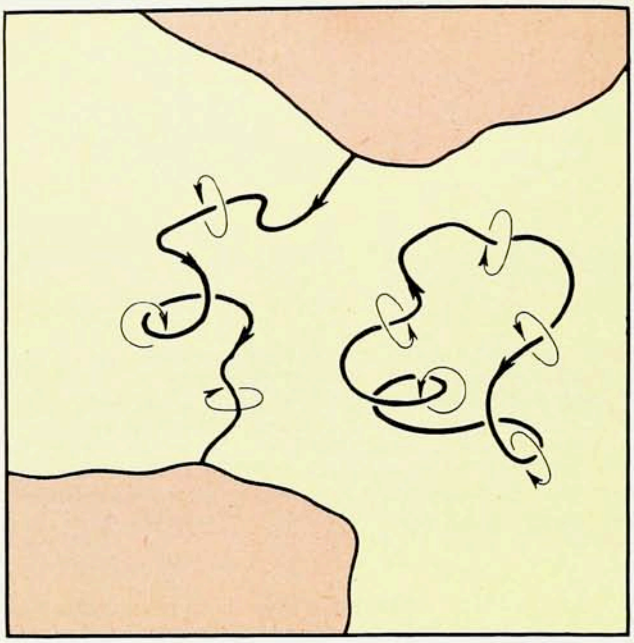
While we have stressed that one should think of a quantized vortex as a classical object, much of the current interest is devoted to phenomena that test the limits of this description. At finite temperatures, for example, the superfluid plays host to a gas of elementary thermal excitations, which acts as an independent normal fluid. These excitations usually propagate unimpeded, but scatter when they encounter the core of a quantized vortex. Each such scattering event causes a microscopic change in the configuration of the vortex, producing an average friction-induced motion of the vortex as well as thermally activated statistical fluctuations in its configuration. The statistical effects become especially important near the superfluid transition temperature and also in connection with certain phenomena that occur on a very small scale. The thermal fluctuations totally dominate the properties of very thin helium films, where the unbinding of thermally activated vortex pairs destroys the superfluid state.

A different kind of limit is reached under a very special circumstance: when tiny ions are pulled through the superfluid at very high velocities. Here we may enter a rather eerie regime where neither fluid dynamics nor statistical mechanics applies, but instead quantized vortices appear as the result of a direct quantum mechanical transition.

Thus, depending on the situation, quantized vortices in superfluid He^4 may be governed by fluid dynamics, by statistical effects or by quantum mechanics. Our aim in the rest of this article is to give an impression of this diversity, concentrating in each instance on topics of greatest current interest.

An elegant special case

Consider a cylinder containing superfluid He^4 set into rotation about its



Flow patterns associated with arbitrary vortex filaments in superfluid He^4. At any instant, the velocity field everywhere is uniquely determined by the configuration of the filaments. The way any filament develops with time depends on the filament's effect on itself, the influence of other filaments, the influence of boundaries, and the effect of normal and superfluid driving fields. Figure 2

axis. At sufficiently low temperatures, where one can ignore the thermal contribution to the free energy, the equilibrium state is that with the smallest kinetic energy as measured in the rotating reference frame. For a classical fluid, of course, the result is solid-body rotation. For the superfluid, however, solid-body rotation is forbidden by the restriction $\nabla \times \mathbf{v}_s = 0$. Instead, quantized vortex lines parallel to the axis of rotation establish themselves in a pattern that minimizes the kinetic energy. For small enough angular velocities, the equilibrium state contains no vortices at all. As the rotation speed increases, increasing numbers of vortex lines appear in characteristic configurations. At sufficiently high rotation speeds, the equilibrium configuration approximates a regular triangular lattice, at least near the center of the cylinder. Because a rotation rate of 1 radian per second already produces an average areal density of 2000 vortex lines per square centimeter, the vortex array can under many circumstances be treated as a continuum.

Because of its elegant simplicity and the clarity of the theoretical issues it raises, the vortex array in rotating superfluid He^4 has been a most important laboratory for testing ideas about quantized vortices.[2] We can trace much of our current understanding of how quantized vortices move and how they interact with the normal fluid to Henry Hall and William Vinen's pioneering investigations at Cambridge University.[3] Close to two decades later, Richard Packard and his collaborators at Berkeley[4] obtained actual images of quantized vortices as they appear in the slowly rotating superfluid. Observed patterns, such as the one shown in figure 3, match those predicted theoretically.

Have all interesting questions about rotating He^4 been answered? Despite the venerable nature of the subject, the answer is certainly no. Physicists have long appreciated, for example, that the existence of vortex arrays in the superfluid implies that its macroscopically averaged behavior depends on additional internal stiffness parameters. Gordon Baym and Eileen Chandler of the University of Illinois have reexamined this problem and have found that in effect the macroscopic equations become those for a "three fluid" system: the superfluid, the normal fluid and the vortex deformation field.[5] Among the solutions of these equations are the so-called Tkachenko waves, propagating shear waves in the vortex lattice in which each vortex executes an elliptical motion about its equilibrium position. Experiments have produced evidence for such modes.[6]

On a more detailed level, the competition between the triangular symmetry of the vortex lattice and the circular symmetry imposed by the rotation field—here the long-range, logarithmic interaction between the vortices is important—leads to very interesting defect structures that remain to be investigated.

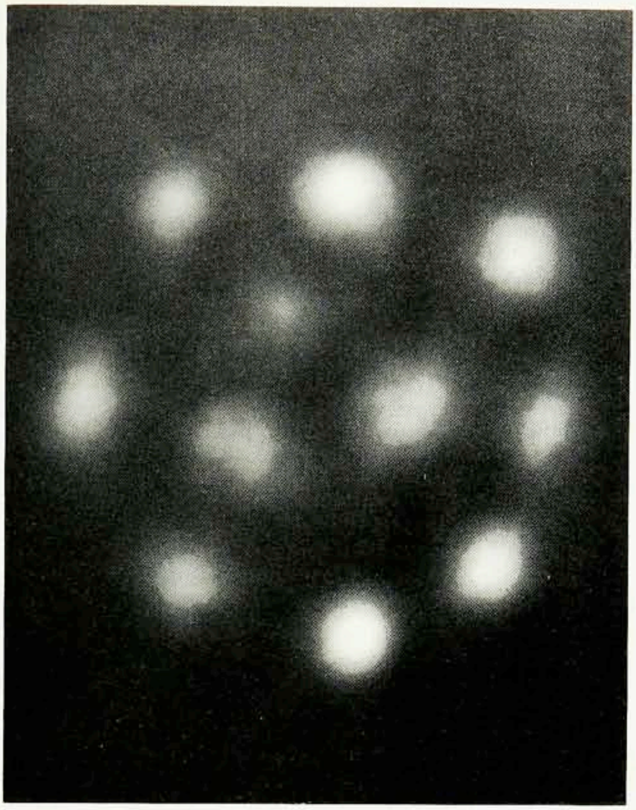
Perhaps of greatest interest, however, is a question that until recently has hardly ever been raised: Where does the vortex array come from? The answer to this question is likely to involve concepts now developing in the theory of superfluid turbulence and critical velocities.

Vortices on the computer

The vortex array appears as the equilibrium state of the rotating superfluid, just as solid-body rotation is the equilibrium state of a classical fluid in a rotating container. Quantized vortices, however, can also arise when the superfluid is driven far from equilibrium, generating the kind of spectacularly nonlinear behavior one associates with chaos and turbulence. Thus when the superfluid is made to flow more and more rapidly, it eventually reaches a critical velocity at which laminar flow gives way to a new state of motion in which the fluid suddenly fills up with a random, constantly changing tangle of quantized vortex lines, as figure 4 illustrates.[7,8] In contrast to subcritical flow, this state of motion is dissipative, the vortex tangle providing a nonlinear pathway through which the superflow can couple to its environment. The similarities between the appearance of the vortex tangle and the onset of turbulence in a classical fluid are striking, yet the underlying fluid dynamical mechanisms are substantially different.

How can one deal with the situation represented by figure 4? Perhaps the most significant recent development in the study of superfluid turbulence and critical velocities is the realization that numerical simulation provides a potent methodology for exploring this kind of problem.[9] As pointed out above, the vortex tangle shapes its own evolution: The instantaneous motion of each vortex filament is determined by the total configuration of all vortices. Although in general this gives rise to an impossibly complicated nonlocal problem, for the situation in which long-range fields are unimportant one can reduce the rules of the game to a few, relatively simple elements. These basic rules, some of which were themselves identified as a result of numerical work, can be implemented quantitatively on the computer to explore the nonlinear dynamics of the vortex tangle.

The first, and most successful, application of such numerical simulation was to the problem of fully developed, homogeneous turbulence in a superfluid. This kind of turbulence occurs when the driving velocity is high and the temperature is above 1 K. Under these conditions a very dense vortex tangle is maintained entirely by its interaction with the normal fluid. Remarkably, when an initial set of vortices is allowed to evolve on the computer according to the appropriate deterministic, nonlinear set of rules, it quickly turns into a random tangle as shown in figure 4. A crucial assumption in producing this self-sustaining chaotic behavior is that vortex lines that come close to each other will break and reconnect. Both the velocity dependence and the temperature dependence of the average vortex line length density (the total length of vortex lines per unit volume) calculated in this way agree well with experiment. Inspection of what the vortex tangle is doing reveals that the frictional interaction tends to drive the lines across the fluid toward the walls of the channel in which the fluid is flowing. At the same time, the reconnection of lines keeps the tangle randomly kinked and generates enough new vortex loops to replace the singularities that are annihilating at the walls, thus maintaining the system in a chaotic steady state.



Array of vortices, photographed in superfluid He^4. Excess electrons are injected into the superfluid and become trapped on the vortex lines. An applied electric field forces the trapped electrons through the free surface of the liquid and accelerates them toward a fluorescent screen. The spots of light produced on the screen correspond to the locations of the vortices in the liquid, or at least to the points where the vortices intersect the free surface. Edward Yarmchuk, now at IBM, photographed this pattern in a 2-mm-diameter cylinder rotating at 0.59 rad/sec. Figure 3

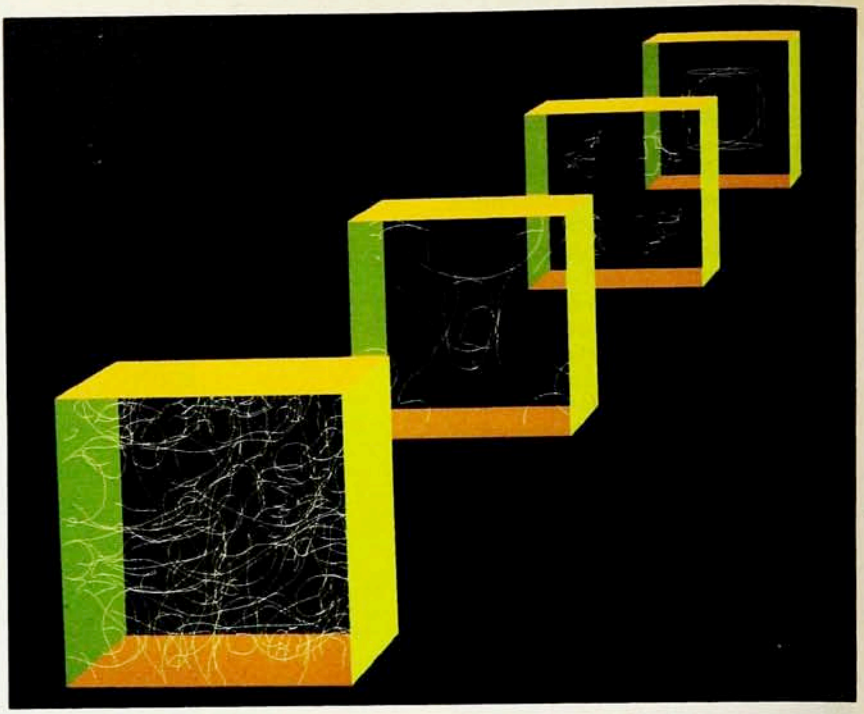
Attention is now turning to the problem of the critical velocity at which laminar flow gives way to the dynamic tangle of quantized vortex lines. It is already obvious from the above description that when the line length density becomes very small and not enough new vortex loops are being created by the reconnection of vortices, the vortex tangle ceases to be self-sustaining. Calculations assuming an ideally smooth channel in fact give critical velocities of the right order of magnitude. However, recent studies by Hugo van Beelen and his coworkers at the Kamerlingh Onnes Laboratory in Leiden[10] and by James Tough and his collaborators at Ohio State University[11] have shown that, unlike the homogeneous behavior in the limit of high velocities, the regime for the onset of the tangle is extremely complicated. New numerical studies that include the possibility of rough walls indicate that this may occur because the vortex tangle can be driven by two distinct mechanisms—not only do the vortices interact with the normal fluid, but they also couple to the microscopic surface roughness on the channel walls. Prospects appear good that we will soon have substantial further insight into this fundamental problem of superfluid dynamics.

Discrete dissipation

Although the flow properties of superfluid He^4 have been studied for decades, even simple experiments continue to produce unexpected results. The most recent of these surprises comes from an experiment by Olivier Avenel of Saclay and Eric Varoquaux of Orsay, in which a vibrating diaphragm drives an oscillating superflow through a tiny orifice, as shown[12] in figure 5a. As the amplitude of oscillation builds up, there is no dissipation until the velocity in the orifice reaches a critical velocity of order $(\kappa/2\pi D)\ln(D/a_0)$, where D is the size of the hole and a_0 is the radius of the vortex core. At the critical velocity the diaphragm amplitude decreases discontinuously, corresponding to a sudden dissipative event. Surprisingly, every time the flow reaches the critical velocity, the same discrete amount of energy dissipates, leading to the unusual pattern shown in figure 5b.

The historical motivation for studying flow through microscopic orifices has been the idea that such a constriction in the superflow might act as a quantum mechanical weak link leading to a superfluid He^4 analog of the Josephson effect. However, all orifices, including that of Avenel and Varoquaux, appear to have been of a size that is still several orders of magnitude too large for quantum tunneling effects to be important. Indeed, at first sight it seems that we can interpret both the critical velocity and the dissipation of energy in purely fluid dynamic terms, the former as the typical velocity at which a vortex pinned in the orifice by surface imperfections breaks loose, and the latter as the amount of energy dissipated when such a vortex crosses the entire orifice. This has led to the suggestion[9] that the discrete dissipative events observed by Avenel and Varoquaux correspond to a vortex shuttling process in which individual pinned vortices break loose from near a corner of an orifice, cross the orifice and repin in the opposite corner. A second remarkable result obtained by the same group, however, shows that their experiment operates on the borderline between statistical mechanics and fluid dynamics. They observe that the critical velocity is in fact a linearly decreasing function of temperature, a dependence that they have followed down to a few millikelvin. This implies that thermal vortex fluctuations must play an important role in the initiation of discrete dissipative events even near absolute zero.

Previous investigators have observed thermally activated flow dissipation near the superfluid transition temperature. Such dissipation is understood at least qualitatively as a kind of thermal nucleation process in which vortex loops have to fluctuate up to some critical size before they can gain energy from the flow field.[13] Analysis of the French experiment shows, however, that the vortex loops present must be very small, at most a few tens of angstroms in radius. A fundamental problem then arises because such loops are much too small to gain energy from a flow field with a velocity of the order of the observed critical velocity. It follows that the activation of isolated vortex loops over a velocity-dependent free energy barrier cannot account for the dissipative events observed.



Evolution of a vortex tangle in superfluid helium. These computer simulations show the development of a tangle in a flow channel, starting with an artifical initial configuration. Calculations are carried out by taking a section of the flow channel and applying periodic boundary conditions along the flow direction. Here an initial configuration of 6 symmetrically arranged vortex rings is subjected to a constant normal-fluid velocity, and the channel walls are assumed to be rough. (Figure by Schwarz and Richard Voss.) Figure 4

One recent suggestion is to combine the two models discussed above. The idea is that very small vortex fluctuations near the boundary of the fluid, while unable to grow on their own, can in fact dislodge pre-existing large-scale vortices pinned on surface imperfections. Consequently, the vortex shuttle model may in fact imply just the kind of thermally activated behavior that is observed. Whether this speculation can survive further investigation remains to be seen, but the very fact that physicists are currently addressing such issues illustrates the exciting new prospects for studying quantized vortex dynamics offered by experiments of the Avenel–Varoquaux type.

Thin films

Superfluidity has been observed in films of liquid helium so thin that the distance between mobile atoms is tens of angstroms. For films up to several atomic layers in thickness, quantized vortices must be strictly two-dimensional structures interacting through a long-range logarithmic potential. Because the energy of each such vortex is small, thermal vortex nucleation becomes very important, to the extent that it destroys the superfluid state above a certain temperature that depends on the thickness of the film. Michael Kosterlitz and David Thouless first elucidated the nature of this phase transition when they were at the University of Birmingham.[14] Although their insights have since been found to apply to other systems, the He^4 film still provides the cleanest prototype of a Kosterlitz–Thouless transition.

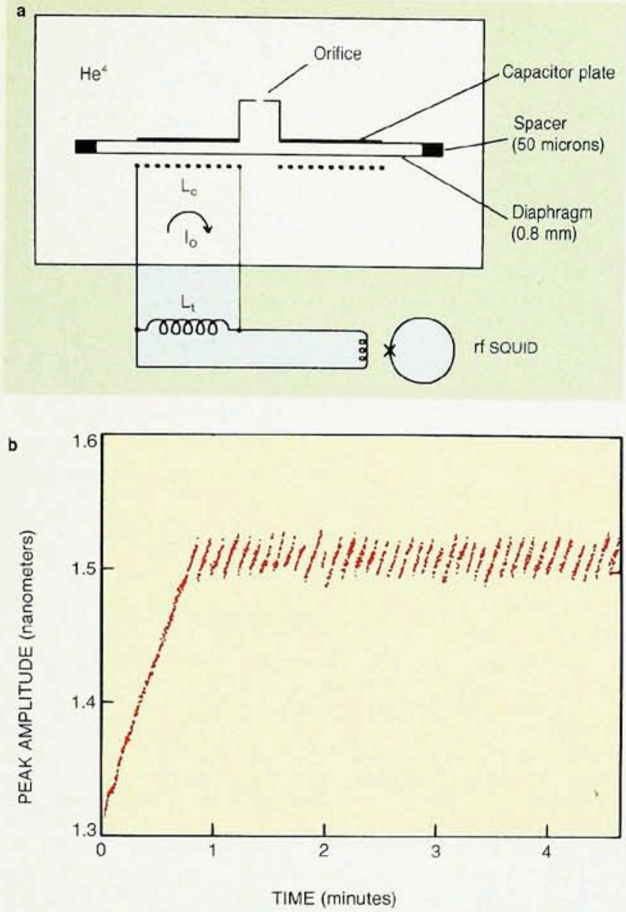
A closely coupled pair of oppositely directed vortices involves less fluid kinetic energy than a single vortex, and such pairs are much more likely to be thermally excited. Kosterlitz and Thouless therefore consider a two-dimensional gas of logarithmically interacting vortex–antivortex pairs. They account for the screening effect of smaller pairs on the interaction between members of larger pairs by introducing a scale-dependent dielectric constant $\tilde{\epsilon}$ that modifies the interaction. A larger test pair polarizes the intervening pair plasma, which in turn screens some of the bare interaction. To obtain a self-consistent determination of the dielectric constant $\tilde{\epsilon}(r)$, one starts with the known behavior of the system at small scales (where $\tilde{\epsilon}$ is 1) and iterates a set of derived recursion relations to infinite scale. The result of this iteration is a peculiar high-order phase transition in which the paired vortices unbind and destroy the superfluid state once a given transition temperature is reached.

Third-sound[15] and torsional oscillator[16] experiments have confirmed quantitatively the predictions of the Kosterlitz–Thouless theory. The latter measurements, carried out at Cornell University by David Bishop and John Reppy, involve the use of a high-Q (on the order of 10^5) torsional oscillator containing a long, very thin Mylar sheet wound into a roll. Bishop and Reppy condense a thin film of helium onto the surface of the Mylar and, by monitoring the response of the oscillator, determine both the effective superfluid density in the film and any dissipation associated with its motion. However, both these and the third-sound measurements are carried out at nonzero frequencies. To interpret the measurements properly, therefore, one must extend the Kosterlitz–Thouless theory to account for the dynamic response of the vortex plasma to an oscillating field. That is, because of the nonzero frequencies, one iterates the recursion relations only up to the vortex diffusion length, the characteristic distance beyond which pairs can no longer equilibrate. The principal effects of truncating the iteration process are to shift the effective transition to higher temperatures and to induce damping of the superflow.[17]

A recent experiment to determine directly the diffusion length, or more precisely the vortex diffusivity, sets a torsional oscillator containing helium-coated Mylar disks into uniform rotation about an axis perpendicular to the surface of the superfluid film.[18] As discussed above, the overall rotation of the experimental apparatus induces additional vortices, whose population is proportional to the rotation rate. Because the vortex diffusivity is nonzero, the vortices move with respect to the superfluid, causing an excess dissipation. Figure 6 shows how the oscillator's period, which is closely related to the superfluid density, changes with temperature; the figure also shows the dissipation in both a rotating and a nonrotating system. The relatively sharp drop in the period of oscillation reflects the cooperative transition associated with the unbinding of large vortex–antivortex pairs; this drop is unaffected by the vorticity induced by the rotation. The vortex diffusivity obtained from data such as these appears to diverge at the effective superfluid transition temperature.

As yet we have no quantitative understanding of the observed diffusivity, although Vinay Ambegaokar and his coworkers suggest an interesting approach that is at least qualitatively consistent with the data.[17] They suggest that a particular vortex moves in the velocity field produced by the randomly distributed thermally activated pairs. The diffusivity then diverges because the number of pairs of a given size becomes large in the vicinity of the transition. In this picture, the diffusivity is a direct probe of the transition behavior, and it is clear that a complete theory of diffusivity is needed before it can be said that the transition is completely understood.

An intriguing experiment performed at Cornell by Reppy and his coworkers indicates one possible future direction of research in thin superfluid films.[19] The experiment investigates the transition behavior of an extremely thin superfluid film in Vycor glass, a porous medium with a random three-dimensional internal geometry. In this material the distance between mobile atoms, the pore size and the thermal de Broglie wavelength are all comparable—of order 50 Å. Is this system two dimensional, three dimensional or something else entirely? The experiment gives an indication that the mobile atoms in fact behave as a dilute, three-dimensional Bose gas, but other interpretations also have been offered.[20] The problem is attracting considerable theoretical interest,[21] and the Cornell group is pursuing it experimentally.



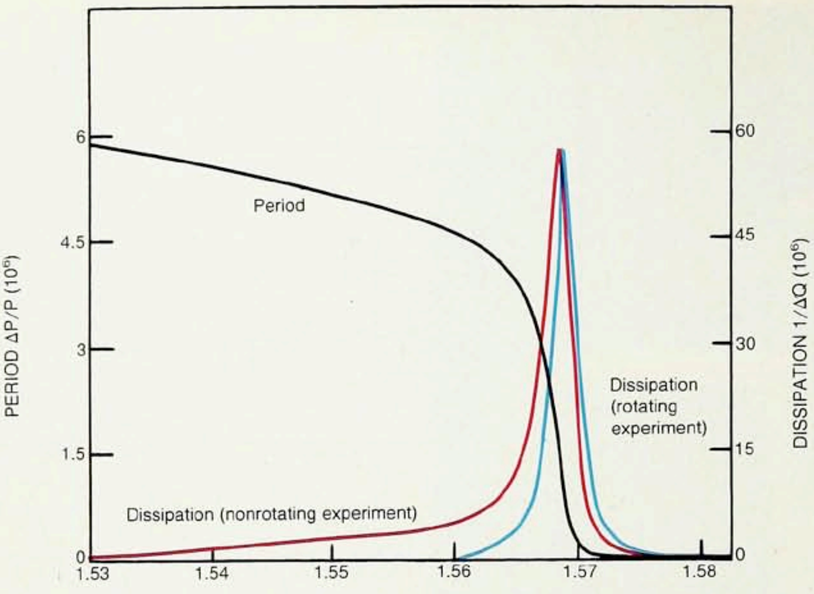
Orifice experiment of Olivier Avenel and Eric Varoquaux. **a:** Schematic diagram showing an oscillating membrane that drives superfluid He^4 through a 0.3×5.0-micron orifice milled into a 0.2-micron-thick nickel film. An rf SQUID capable of detecting displacements of 10^{-13} m monitors the amplitude of the membrane oscillation. **b:** Plot of the peak amplitude of the membrane oscillation as a function of time, for a constant driving power of 2.4×10^{-18} W. Figure 5

Quantum nucleation

As we have seen, quantized vortices can be created either through the reconnection of vortices or through thermal nucleation. Much remains to be learned about these mechanisms and their implications. A different fundamental question that naturally arises, however, is what happens when there is no pre-existing vorticity and the temperature is too low for thermal activation to occur. It appears that vortex creation can actually be observed under such circumstances by injecting ions into the superfluid and subjecting them to an electric field that moves them through the fluid at high velocities.

An ion injected into liquid helium forms a solid structure with a radius of about 7 Å, while an excess electron self-traps in a bubble with a radius of about 12–16 Å, depending on the pressure. The bubble is actually a spherical hole maintained by the quantum pressure of the confined electron. One can investigate the way these relatively large, charged spherical structures move through the superfluid by applying an electric field and looking at their motion in what may be thought of as the superfluid analog of the classical flow-around-a-sphere experiment.

At low applied fields the charged probes exhibit a well-defined drift velocity as frequent collisions with the elementary excitations of the normal fluid limit their progress. However, George Rayfield and Frederick Reif at Berkeley discovered that when a probe reaches a critical velocity on the order of a few tens of meters per second, it again creates quantized vorticity, this time in the form of a quantized vortex ring, or loop, to which the probe re-



Period and damping of a torsional oscillator coated with superfluid helium, plotted as a function of temperature. The oscillator contains a stack of 10 000 Mylar disks, each 2.5 microns thick, 1 cm in radius and coated with a 1.14-nm-thick helium film. The period is roughly linear in the effective superfluid mass. The dissipation peak in the absence of overall rotation of the experiment arises from those thermally activated vortex–antivortex pairs that have relaxation times comparable to the oscillation period. In the rotating experiment, excess dissipation comes from rotation-induced vortices moving relative to both the Mylar substrate and the superfluid. Figure 6

mains attached, so that further application of the electric field causes the vortex ring to grow to macroscopic dimensions.[22] Again, the analogy with the corresponding classical fluid dynamic instability is striking.

In a series of beautiful experiments at the University of Lancaster, Peter McClintock, Roger Bowley and their coworkers have been able to accelerate electron bubbles from very low thermal velocities up to the velocity where the bubbles create a vortex ring, without the bubbles ever experiencing a thermal collision along the way—in essence, the experiment is carried out at absolute zero.[23] Presumably, what we are seeing here is a real quantum instability that cannot be described in fluid dynamical or statistical terms. Supporting this view is the fact that one can make a good estimate for the critical velocity by simply assuming that the creation of vortex rings occurs as a sudden quantum jump that requires the conservation of energy and momentum.[24]

We have seen above that we can treat fluid dynamical issues in some sense completely. We can also deal with statistical fluctuations to some extent, because it is possible to estimate the free energy of a given vortex configuration. The problem of quantum nucleation, on the other hand, appears to be relatively unexplored. In idealized terms, an accelerating ion approximates a spherical boundary moving at an increasing velocity through the zero-temperature superfluid. Perhaps vortices are excited at the point where the macroscopic quantum state representing the superfluid can no longer respond adiabatically to this time-dependent boundary condition. It has also been speculated that the creation of vortex rings is an example of macroscopic quantum tunneling. However, we still lack a convincing interpretation of the fundamental phenomenon of quantum nucleation, a reminder that superfluid He^4 continues to be a fertile source of interesting physical problems.

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