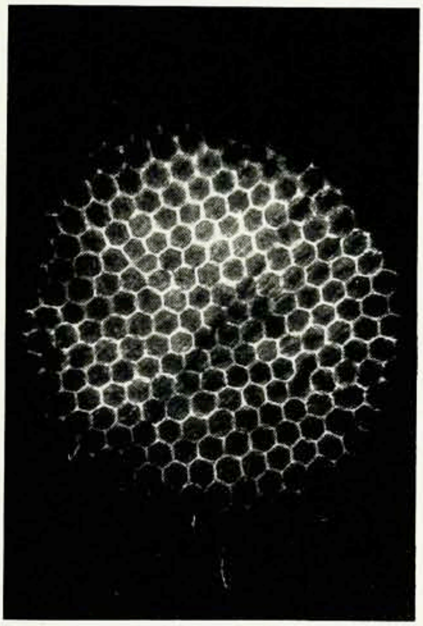
Electrons and ions at the helium surface

Two-dimensional arrays of charged particles exhibit plasma oscillations and phase transitions such as crystallization and melting in the low-density classical limit, but their high-density quantum regime remains to be explored.

Arnold J. Dahm and W. F. Vinen

In the last two decades, two-dimensional systems have been found to exhibit phases and phase transitions unlike any seen in their three-dimensional counterparts. Quantization of the Hall conductance of an electron gas in a strong magnetic field, localization of electronic wavefunctions in the presence of infinitesimally small amounts of impurities, and infinite-order phase transitions in magnets and crystals are some examples of unusual phenomena in two dimensions. However, it was not merely a pursuit of the novel and the unexpected that got physicists interested in two-dimensional systems. In fact, studies of two-dimensional systems are useful for understanding surfaces of three-dimensional solids, interfaces between two three-dimensional phases and anisotropic solids in which the interactions in a plane of symmetry are much stronger than interplane couplings. Moreover, our understanding of why and how behavior of physical systems depends on spatial dimensionality has been enhanced considerably by studies of two-dimensional systems.

Fabricating genuine two-dimensional systems in our three-dimensional world is a major experimental challenge. As an example of the difficulties encountered, consider a layer of atoms adsorbed on a graphite surface. Obviously the graphite surface must be atomically smooth if the adsorbed layer is to constitute a good two-dimensional system. It is also desirable that the surface interact weakly with the adsorbed atoms so that their motion on the surface is determined only by their interaction with one another. Such a system of adsorbed atoms has been successfully used for studying phase transitions in two dimensions. The graphite surface, however, although it has been widely used as a substrate, is atomically smooth over a range no greater than 10^4 Å. Furthermore, the ordered surface potential due to the graphite crystal structure often influences the properties of the adsorbed layers.



Periodic array of charged dimples on a liquid helium surface. Each dimple is about a millimeter wide, a tenth of a millimeter deep and contains about 10^7 electrons. (Photograph taken by Paul Leiderer at University of Mainz.) Figure 1

Superfluid helium provides an ideal surface for forming and studying two-dimensional arrays of electrons and charged particles. The helium surface is free from impurities and anisotropies, and it is atomically smooth on a macroscopic scale. Electrons above the surface are bound very weakly to it by their interaction with image charges in the bulk—the lowest energy state is at a mean separation of 114 Å from the surface. At low temperatures, the vapor above the liquid causes no significant scattering as it is quite rare, but waves on the liquid surface are an important perturbation on the electrons. The coupling of electrons to surface waves gives rise to many interesting phenomena and also limits the maximum density of electrons that may be trapped above the surface by applying an electric field. However, this coupling is quite small for most experimentally accessible densities; the electrons trapped above the helium surface, therefore, form a clean two-dimensional system whose microscopic dynamics are dominated by the two-body Coulomb interaction.

Our purpose in this article is to review what we have learned about a two-dimensional system of charged particles from experiments done on electrons and ions trapped at the helium surface. A large part of our discussion will be based on results obtained from experiments done on the two-dimensional electron gas formed *above* the helium surface, but we will mention where appropriate the experiments on the two-dimensional system of ions trapped *below* the surface. For densities on the order of $10^8/\mathrm{cm}^2$, the two-dimensional electron gas behaves as a classical gas at temperatures on the order of a few hundred millikelvins. We will discuss plasma oscillations and electron mobility in this classical two-dimensional gas and indicate how changes in these properties are interpreted as evidence for crystallization.

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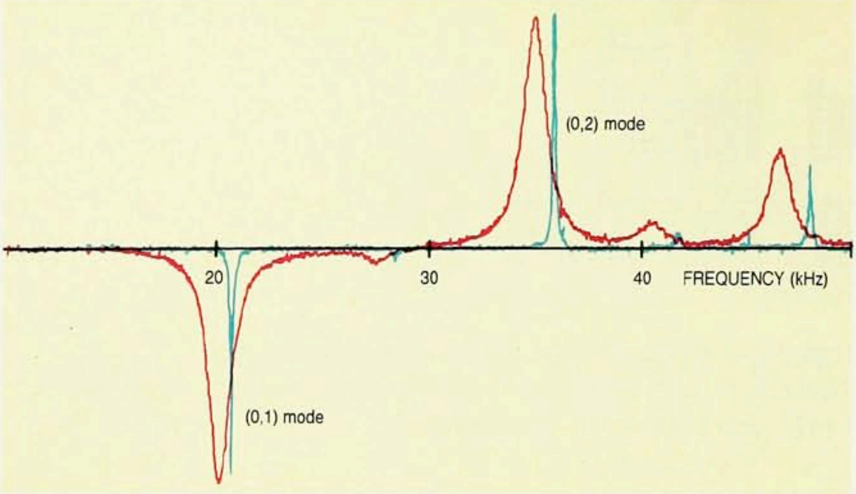


Temperature dependence of plasma-resonance linewidth in a two-dimensional array of negative ions. The graph shows the response of the array to an rf signal at 54 mK (red) and 81 mK (blue). At temperatures above about 30 mK the ionic mobility is limited by phonons in the bulk of the helium; the mobility increases and the resonance linewidth decreases as the number of phonons decreases with decreasing temperature. The peaks marked (0,1) and (0,2) correspond to the two lowest-frequency axisymmetric plasma modes of a circular array. The position of the resonance line is used to determine the effective mass.[4] Figure 2

We will also discuss how the detailed studies of the melting of this electron crystal compare with the predictions of a remarkable theory, according to which a two-dimensional crystal melts when dislocations can move freely through the sample. Next we will discuss the self-trapping of electrons due to their interactions with the surface waves in the superfluid, and the formation of a "dimple" lattice (figure 1). The helium surface becomes unstable and does not hold electrons when the areal density is increased beyond $2\times10^9/\mathrm{cm}^2$. We will describe recent attempts to overcome this difficulty by using only a thin film of helium and increasing the electron density to values at which quantum mechanical many-body effects become important. We will conclude with a brief discussion of the phase diagram of this two-dimensional quantum mechanical system. The article[1] by Charles Grimes is a good review of the work done up to 1978.

Trapping of charged particles

The prediction that electrons may be trapped above the surface of liquid helium was made by Warren Sommer in 1964 in his PhD thesis at Stanford University. Sommer's thesis, however, was not widely known and Milton Cole and Morrel Cohen in 1969 and Valery Shikin in 1970 independently considered this possibility. To understand the mechanism of forming a two-dimensional electron gas above the helium surface, consider first a single electron. By symmetry, the electron is free to move in a plane parallel to the surface and we need only consider the forces acting on it normal to the surface. If it were a classical particle, the only force would be that due to the dielectric image potential

$$V(z) = \frac{(K-1)}{4\pi\epsilon_0(K+1)}\,\frac{e}{4z}$$

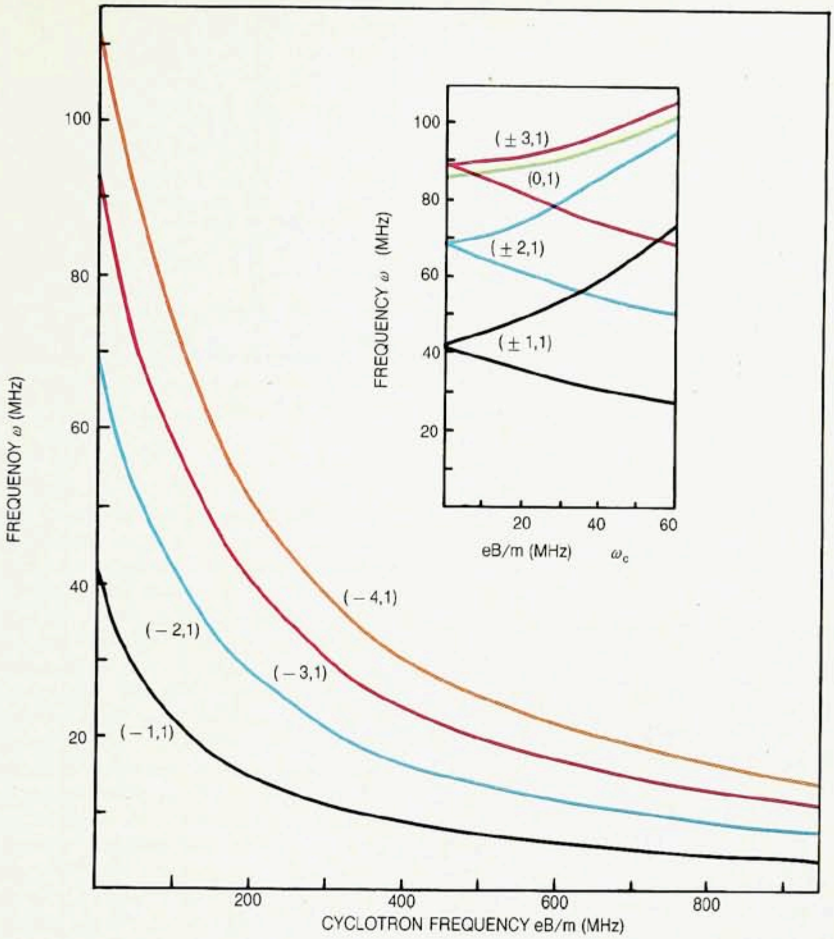
where z is the distance from the surface and K is the dielectric constant of the liquid. In quantum mechanics, however, the Pauli principle requires that the wavefunction of an excess electron in liquid helium be orthogonal to the wavefunctions of the 1s core electrons; consequently, an electron within an interatomic distance of the surface experiences a potential barrier, about 1 eV in magnitude, that repels the electron away from the surface and prevents it from entering into the liquid. Therefore, in its motion normal to the surface, the electron may occupy only the quantized "hydrogenic" states. These states are solutions of the one-dimensional Schrödinger equation for a potential that is the sum of the Coulomb potential due to the image charge $e(K-1)/(K+1)$ and a short-range repulsion. If the repulsion at the helium surface is approximated by an infinite barrier at the surface then the wavefunction must vanish at the surface, and the binding energies and orbit radii of the eigenstates are given by the Bohr formulas for hydrogenlike atoms. The binding is very weak: Because the dielectric constant of liquid helium is only slightly larger than 1, the image charge is very small. Grimes and Truman Brown[1] investigated these "one-dimensional hydrogenic states" normal to the surface by inducing transitions between them in a microwave resonance experiment.

When many electrons are accumulated above the helium surface, each occupying the lowest energy state in the Coulomb potential of its image charge, they form a two-dimensional array because the lowest energy state is at a fixed mean distance of about 114 Å from the surface. The electrons' weak binding to the surface is a very useful property—it ensures that the two-dimensional array of electrons is not perturbed by its substrate, except for its coupling to ripples on the surface (see the box on page 47). But the weak binding also makes it very easy for an electron to pick up enough energy from thermal fluctuations and by scattering off other electrons to ionize out of the bound state. An electric field that presses the electrons toward the surface is therefore necessary to check the array's tendency to auto-ionize. The electric field increases the electrons' coupling to ripples on the helium surface. We will discuss below some of the many interesting consequences of this electron–ripplon coupling.

The mechanism of forming two-dimensional arrays of charged particles *below* the surface of liquid helium is similar to the one discussed above. The image potential due to a charged particle inside liquid helium is repulsive, but the addition of a uniform external field, E_0, forcing the charges toward the surface gives rise to a total potential having a minimum at a distance $[(K-1)e/16\pi\epsilon_0(K+1)E_0]^{1/2}$ below the surface. Two types of charged particles, loosely called ions, have been produced in helium. The positive ion, popularly known as a snowball, is believed to be a $He_2{}^+$ ion embedded in a sphere of *solid* helium. (The helium solidifies around the ion because electrostatic attraction due to the ion enhances the local pressure.) The sphere of solid helium has a radius of about 5.5 Å and an effective mass of 30–40 times the mass of the helium atom. The negative ion is formed from an electron, which forces the helium atoms away from it, so that it becomes self-trapped in an otherwise empty bubble of radius about 17 Å and effective mass about 240 times the mass of the helium atom.

Although many experiments have been carried out on ions in bulk superfluid helium over the past 30 years, Jacqueline Poitrenaud and Francis Williams[2] were the first to study, in 1972, ions trapped near the surface. They deduced effective masses for both types of ions from the observed frequencies of their vertical oscillation in the potential well that binds them to the surface. Recent studies of plasma oscillations (discussed below) in the ionic arrays have shown that the mass of the positive ion has a strong temperature dependence, even at very low temperatures.[3,4] This effect is not understood yet, but it may have its origin in the dependence on superfluid velocity of the energy associated with the interface between the solid core of the ion (the snowball) and the surrounding superfluid.

Two-dimensional arrays of charged particles formed below the helium surface differ in many ways from the electron array above the surface. Because the ions are in the liquid, they interact not only with the ripplons (to an extent that depends on the depth at which they are trapped) but also with excitations—phonons and rotons—in bulk helium. Moreover, because the ions have large effective masses their energy levels are very closely spaced, so even at very low temperatures an ion has a significant probability for making transitions between levels. These transitions change the ions' vertical position with respect to the surface and can spoil the two-dimensional character of the array. However, the extent of the vertical motion is very small compared with the typical interionic spacing, so for most purposes the ionic array can be regarded as a good two-dimensional system.



Frequencies of magneto-plasma modes, labeled by azimuthal and radial mode numbers, as a function of the magnetic field, here measured by the cyclotron frequency. The inset shows the splitting of the degeneracy of the lowest three azimuthal modes by a magnetic field; the lowest unsplit axisymmetric mode (0,1) obeys equation 3 of the text. The main figure shows only the lowest branch of the azimuthal modes. Figure 3

Two-dimensional plasma waves

When a plasma of charged particles in a stationary charge-neutralizing background is perturbed, the displacement of charged particles results in an electric field that tends to restore local charge neutrality; if the charged particles are free, the entire plasma oscillates collectively. The frequency of this plasma oscillation is given by

$$\omega^2 = ne^2/\epsilon_0 m \qquad (1)$$

in three dimensions, and

$$\omega^2 = (ne^2 k/2\epsilon_0 m)F(k) \qquad (2)$$

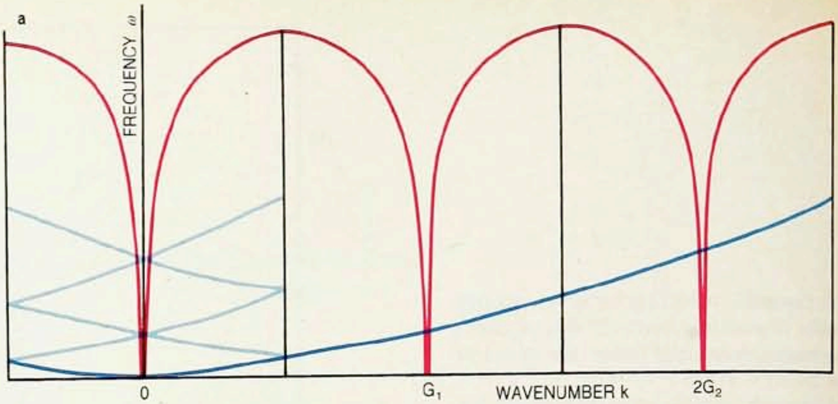
in two dimensions. Here m is the mass of the charged particle, n is the particle density for the respective dimensionality, and k is the wavenumber of the displacement. The factor $F(k)$ in equation 2 is usually of order unity and depends on the spacing of the surrounding electrodes (the background "neutralizing" charge for the electron or ion layer is in these electrodes).

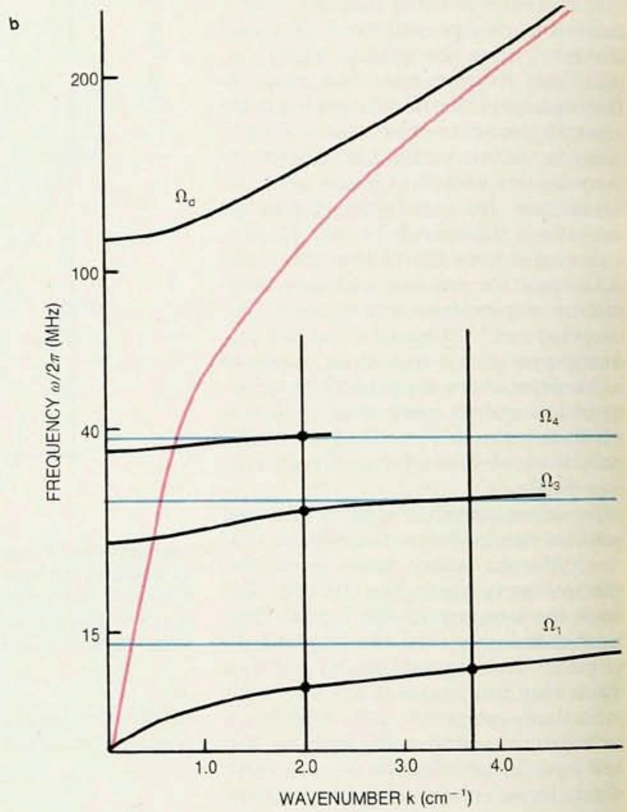
The factor k in the two-dimensional dispersion relation, equation 2, arises because the average restoring electric field in two dimensions is due to an infinitely long line of charges, so it is inversely proportional to the wavelength. In three dimensions the average restoring field is independent of the wavelength because it is due to a large charged plane. Grimes and Gregory Adams[1] provided the first experimental verification of the unusual dispersion relation for two-dimensional electron arrays.

Gary Williams[3] and his colleagues at UCLA first observed plasma waves in two-dimensional systems of positive ions. Extensive studies of plasma waves in the electron system and in both the ionic systems have now been carried out. As we have already mentioned, observed plasma frequencies have yielded much useful information about ionic masses. (See the box on page 49 for experimental details.)

The damping of plasma oscillations, measured by the width of the plasma resonance, tells us about the scattering of charged particles in these two-dimensional arrays. In the case of electrons above the helium surface, plasma resonance studies show that, as the theory leads us to expect, electron–

Frequency of coupled phonon–ripplon modes as a function of wavenumber k for the electron crystal. **a:** Schematic diagram of the uncoupled dispersion relations for longitudinal phonons (red), for which the frequency ω is proportional to $k^{1/2}$, and for ripplons (dark blue), for which the frequency ω is proportional to $k^{3/2}$, in the extended zone scheme. Light blue lines show the ripplon dispersion in the reduced zone scheme. The vertical lines denote the Brillouin zone boundaries and G_1 is the smallest reciprocal lattice vector along k. **b:** Schematic diagram of the dispersion relation of the longitudinal coupled phonon–ripplon modes (black) for small wavenumbers. Uncoupled modes (light blue and red) are also shown for comparison. The vertical lines represent the wavevectors excited in the experiment of Grimes and Adams. The observed resonances are labeled with spots. The coupled mode near the origin is the "acoustic" mode; Ω_d is the "optical" mode. (See reference 7.) Figure 4





ripplon scattering becomes very small and independent of temperature at low temperatures. Figure 2 shows an example of the temperature dependence of the plasma-resonance linewidth in a negative-ion system. Clearly the damping decreases in a striking way with decreasing temperature as the density of bulk phonons decreases. But at very low temperatures—below about 30 mK—the damping in the ionic system is also dominated by interaction with ripplons, and if the ions are not too close to the surface so that this interaction is weak, the plasma resonances become so sharp that the linewidth becomes very hard to measure. In the ionic cases,[3,4] many interesting nonlinear effects have also been observed at large plasma-wave amplitudes. Some of these nonlinear effects may be associated with the nucleation of quantized vorticity in the superfluid (see the article by William Glaberson and Klaus Schwarz on page 54).

Magnetoplasmons are plasma waves in a magnetic field normal to the two-dimensional plane. These were studied in the electron fluid[5] by David Mast, Alexander Fetter and Dahm, in the electron crystal phase by Francis Williams and colleagues, and very recently in the ionic system[4] by Vinen's group. In an unbounded two-dimensional plasma the frequency of a magnetoplasma wave of fixed wavenumber k increases with increasing magnetic field according to the relation

$$\omega^2 = \omega_p(k)^2 + \omega_c(k)^2 \qquad (3)$$

where $\omega_p(k)$ is the zero-field frequency of the plasma wave, and ω_c is the cyclotron frequency eB/m. However, in the bounded plasma the situation is complicated by the need to satisfy boundary conditions at the edge of the plasma. Consider the simple case of a circular pool of charge. Modes of oscillation that are axisymmetric have wavevectors (determined by the boundary conditions) that are unchanged by the magnetic field. Equation 3 continues to hold for these modes, with $\omega_p(k)$ independent of the field. But the modes that are not axisymmetric (having angular dependence $e^{im\phi}$) behave in a more complicated way. In the absence of the magnetic field such modes tend to be localized near the edge of the disc, especially for large values of m, and modes with equal and opposite values of m are degenerate. The degeneracy is removed by the magnetic field: The frequency of one mode increases while that of the other decreases, as shown in results obtained by Mokyang Kim and Dahm (figure 3). The lower-energy mode becomes more and more localized near the edge of the disc, the radial wavevector becoming imaginary. This means that the mode becomes a "perimeter" wave, and may be regarded as the two-dimensional analog of a surface wave in three dimensions.

Two-dimensional crystals

The two-dimensional plasmas with which we are dealing are very cold, in the sense that the particle interaction energies are large in comparison with

Electron–ripplon coupling

Consider an electron in an eigenstate of the image potential and the short-range repulsion it feels at the helium surface. When an electric field pointing away from the surface is applied, the electron moves toward the surface to lower its electrostatic energy. This downward displacement of the electron causes a depression in the helium surface below it so that quantum conditions—which require that the electron wave function vanish at the surface and determine the distance from the surface of the eigenstate in the absence of the electric field—are satisfied (see main text). Such a local depression in the helium surface is popularly known as a "dimple."

Because of surface tension, every fluid surface when disturbed supports short-wavelength waves called capillary waves or ripples. In superfluid helium, these surface ripples are quantized; their quanta are called ripplons. Just as a downward displacement of the electron forms a dimple on the helium surface, so do ripples on the surface perturb the electron. In quantum mechanical parlance, this situation is an electron–ripplon coupling.

the thermal kinetic energy. At not too low a temperature such plasmas are fluids, but Richard Crandall and Richard Williams pointed out that they may crystallize[1] at a sufficiently low temperature, when a parameter

$$\Gamma = \frac{n^{1/2}e^2}{4\pi^{1/2}\epsilon_0 k_B T}$$

describing the ratio of potential to kinetic energy, exceeds a critical value. A similar result for crystallization of a low-density electron gas in three dimensions was obtained by Eugene Wigner in 1934. That such a phase transition does indeed occur in two dimensions was confirmed in 1979 in experiments on plasma oscillations in the electron plasma by Grimes and Adams[6] at Bell Labs.

In the box on page 47 we show that the holding electric field causes a slight depression in the helium surface under an electron. In the fluid phase of electrons, however, no well-defined depressions or dimples are formed on the helium surface because the electron motion is too rapid and irregular for the helium surface to keep up with. The electron–ripplon coupling is therefore of no consequence for the plasma dispersion relation—it is the same as that given in equation 2 for an isolated two-dimensional array of charged particles. In the crystal phase, on the other hand, electrons are confined to the sites of a lattice, and a dimple roughly 10^{-2} Å deep does form in the surface under each electron. If there were no electron–ripplon coupling in the crystal phase, the plasma modes of the fluid would become the longitudinal phonons of the crystal and one might expect to see no change in resonance frequencies through the crystallization temperature. But Grimes and Adams discovered that the dispersion relation changed dramatically when the temperature was reduced below a value close to that at which crystallization was theoretically expected. Daniel Fisher, Bertrand Halperin and Philip Platzman attributed[7] this change to an enhancement of the coupling of phonon and ripplon modes in the crystal phase.

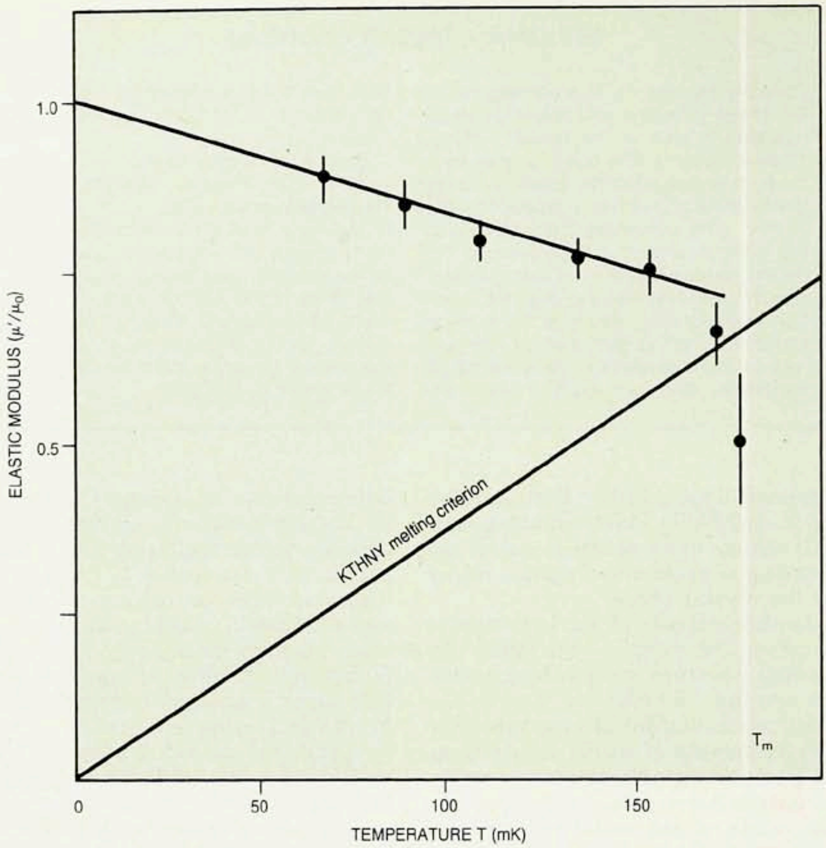
In the presence of electron–ripplon coupling the dimples can follow the moving electrons at low frequencies. As a result, the effective mass of the electrons is substantially increased and the frequencies of low-frequency longitudinal and transverse phonon modes in the electron crystal are reduced. We can think of the resulting modes as "acoustic" modes, because the electrons and the dimples are moving in phase. At high frequencies there are "optical" modes, in which the electron oscillates out of phase with the heavy (almost stationary) dimple. In between, there are modes (figure 4b) that lie near the frequencies at which uncoupled phonon and ripplon dispersion curves cross (see figure 4a). The resonances observed by Grimes and Adams are also shown in figure 4b; they are consistent with this spectrum for coupled phonon–ripplon modes if, as expected, the electrons form a triangular lattice.

Ravi Mehrotra, Bruce Guenin and Dahm[8] found further evidence for crystallization in their studies of electron mobility. In the fluid phase the mobility is determined by processes in which a single electron absorbs or emits a ripplon. In the crystal phase these are replaced by processes in which the emission or absorption of the ripplon involves the whole crystal and is accompanied by the emission or absorption of a number of phonons in the crystal. Because of the difference in the phonon and ripplon dispersion curves (figure 4a) phonons of small wavenumbers satisfy energy conservation in these processes; momentum is conserved through *Umklapp* processes in which much of the ripplon momentum is transferred to the crystal as a whole. *Umklapp* processes also play an important role in transport processes in three-dimensional crystals. The phonon–ripplon scattering at low temperatures is dominated by low-energy transverse phonons, because the Boltzmann probability factor makes exciting these phonons more likely. The scattering (and therefore the inverse surface electron mobility) increases rapidly with increasing temperature at low temperatures as more phonons and ripplons are available for scattering. But the scattering rate falls as the melting temperature is approached, because the *Umklapp* scattering depends on the Debye–Waller factor. This factor may be thought of as a measure of coherence between electrons and ripplons, and it decreases when melting is approached and fluctations in electron positions increase. A second sharp peak in the scattering occurs very near the melting temperature, possibly as a result of ripplon scattering from dislocations in the lattice. This is followed by a rapid decrease in scattering as the crystal melts and new and less effective scattering processes take over.

Crystallization in a two-dimensional system of charged particles has been observed only in the electron system. Attempts to detect it in the ionic systems have not been successful so far. Owing to the much larger mass of the ions the effects due to electron–ripplon coupling are expected to be much smaller than in the electron case, and hardly observable. A slightly different approach to observing crystallization in the ionic system, based on the coupling between longitudinal and transverse lattice vibrations of the two-dimensional crystal in the presence of a magnetic field, has also yielded negative results.

The melting transition

One of the most active areas of two-dimensional physics in the last decade has been the study of the melting transition. The pioneering work of



Elastic shear modulus μ' of the two-dimensional electron crystal, normalized to its extrapolated zero-temperature value, plotted as a function of the temperature T. The shear modulus drops discontinuously to zero at the melting temperature, in agreement with the KTHNY theory. Figure 5

Michael Kosterlitz and David Thouless, elaborated by Halperin and David Nelson, Peter Young, and others, laid the foundations of a remarkable theory of melting in two dimensions. Melting, according to this theory, is associated with the unbinding of pairs of dislocations in the crystal. A dislocation pair in a two-dimensional crystal consists of two partial rows of atoms entering the lattice from opposite sides and terminating in the same vicinity. The lattice strain produced by one of the dislocations is cancelled at large distances by the opposite strain of the other. This reduction in strain energy provides an attractive interaction between the two dislocations. Dislocation pairs exist as equilibrium defects in a two-dimensional lattice.

In the KTHNY theory the separation between the two partners of a dislocation pair increases as the temperature is raised. As more dislocations enter the lattice, dislocation pairs with small separation, located between the partners of a larger pair, weaken the attraction between the partners of the larger pair, allowing them to separate further. Eventually, above a well-defined critical temperature, some of the dislocation pairs cease to be bound, leaving unpaired dislocations. Such unpaired dislocations move in response to a shear strain in such a way as to relieve the strain, thus destroying the shear rigidity that is characteristic of a crystal. In fact, according to the KTHNY theory the shear modulus starts to decrease just below the transition, and the transition occurs when the ratio of the shear modulus to the temperature falls below a critical value. The observed melting temperature is close to that predicted by this theory.

Gerard Deville, Alliro Valdes, Eva Andrei and Williams[10] carried out one of the most important experiments relating to the melting transition in two dimensions. They measured the frequency of a shear mode in the electron lattice (actually an "optical" shear mode in the sense explained earlier). The observation of the shear mode was itself significant, because it constitutes what is perhaps the most direct evidence for crystallization. Figure 5 shows their results for the shear modulus, as determined from the shear-mode frequency. The linear decrease with temperature at low temperatures is due to phonon–phonon interactions and is unrelated to the KTHNY melting. But the behavior and the magnitude of the shear modulus just below the melting temperature are in agreement with the theory and provide strong evidence in its favor.

Dimple lattice

We mentioned earlier that a holding electric field is necessary to collect a macroscopic density of electrons above the surface. If the holding elecric field is very strong, then the depression in the helium surface under an electron (see the box on page 47) may trap the electron if the temperature is sufficiently low. The resulting state—an electron trapped in a dimple—is similar to the polaronic state of an electron trapped by its interactions with phonons in a three-dimensional crystal. Shirley Jackson, Platzman, Marcos Degani, Oscar Hipolito and others[11] have studied the theory of these polaronic states at the helium surface, and it would be of great interest to study them experimentally. Andrei[11] has reported a sharp drop in the mobility of electrons on a 900 Å film (with capillary length of the order of 30 microns) at a temperature of about 0.5 K and a density of $10^8/\mathrm{cm}^2$. The change in mobility followed the behavior expected for polaron formation, although it occurs at much higher temperatures and smaller electric fields than predicted for a single-electron polaron. Experimental searches for these polaronic states in both electron and ionic arrays are currently under way.

The range of the surface depression due to the electron–ripplon coupling is of the order of the capillary length l associated with the surface. (For a fluid with surface tension σ and density ρ the capillary length is $(\sigma/\rho g)^{1/2}$, where g is the acceleration due to gravity.) If the areal number density of electrons is greater than l^{-2}, each dimple contains more than one electron.

At large areal densities (about $2\times10^9/\mathrm{cm}^{-2}$) the helium surface becomes unstable: A deep trough forms in the surface and the collected charge flows into the electrode. Just before the onset of this instability, as the electric field is increased, a dimple lattice of macroscopic scale forms on the surface. This lattice consists of periodic depressions of millimeter size in the surface, each dimple containing on the order of 10^7 electrons.[12] Figure 1 testifies to the reality of this unique macroscopic phenomenon.

Thin helium films

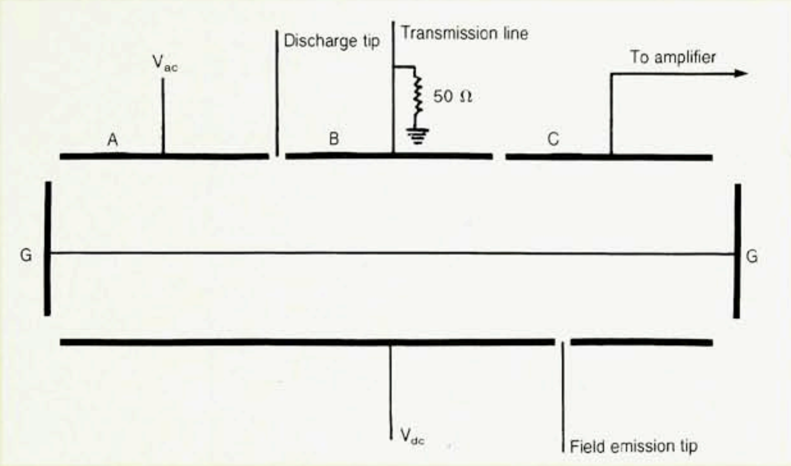
The Fermi energy of a two-dimensional electron gas varies linearly with the density. In all the experiments we have described so far, the electron density was low and the crystallization temperature was higher than the Fermi temperature. In its fluid phase, therefore, the two-dimensional system behaved as a classical gas and did not show any effects that set in below the Fermi temperature due to quantum degeneracy. To understand the consequences of this degeneracy, consider the electron crystal at zero temperature. A quantum mechanical system in its ground state has some kinetic energy, the zero-point energy, even at absolute zero. Therefore, the state of the crystal is now determined by minimizing the sum of the potential and zero-point kinetic energies. At a sufficiently high density the zero-point kinetic energy will dominate the potential energy, in both liquid and crystal phases. The stable phase is then the one in which the zero-point kinetic energy is smaller, and this is the liquid phase. It follows, surprisingly, that at zero temperature the electron crystal ought to melt into a Fermi liquid when the density is increased *above* a critical value. Unfortunately, this critical density (about 10^{12} cm^{-2}) is higher than the value at which the instability of the helium surface mentioned above sets in.

The use of thin helium films allows one to study electron layers of greater density than is possible on bulk helium. In a thin helium film on a suitable substrate, the gravitational force is effectively enhanced by the van der Waals force between the helium and the substrate. This decreases the capillary length of the surface and makes it more stable. Moreover, by using films of different thickness one can study the effect of changing both the ripplon dispersion relation and the strength of the electron–ripplon coupling. Koji Kajita at the University of Tokyo explored in 1982 electron arrays on thin helium films coating a neon or hydrogen substrate.[13]

Helium films on a metallic substrate also allow one to investigate the quantum limit of a very-low-density electron gas at low temperatures. The presence of the metal changes the image potential, which alters the potential of the interaction between one electron and another. The image of an electron in the metal is an equal and opposite charge, so that the interaction between electrons at distances large compared with the film thickness becomes dipolar rather than Coulombic. A two-dimensional Fermi liquid with dipolar interactions solidifies at zero temperature only if the density *exceeds* a critical value, so that at low densities the electrons ought to remain fluid. François Peeters and Platzman[14] used these arguments to propose the phase diagram shown in figure 6a for electrons on a 100 Å helium film on a metallic substrate. In the lower part of the phase diagram in figure 6a, where the electron density is low, the electron spacing is larger than the film thickness, and we get solidification as we *increase* the density; in the upper part, where the electron spacing is less than the film thickness and the interelectron interaction is Coulombic, we get solidification as we *decrease* the density.

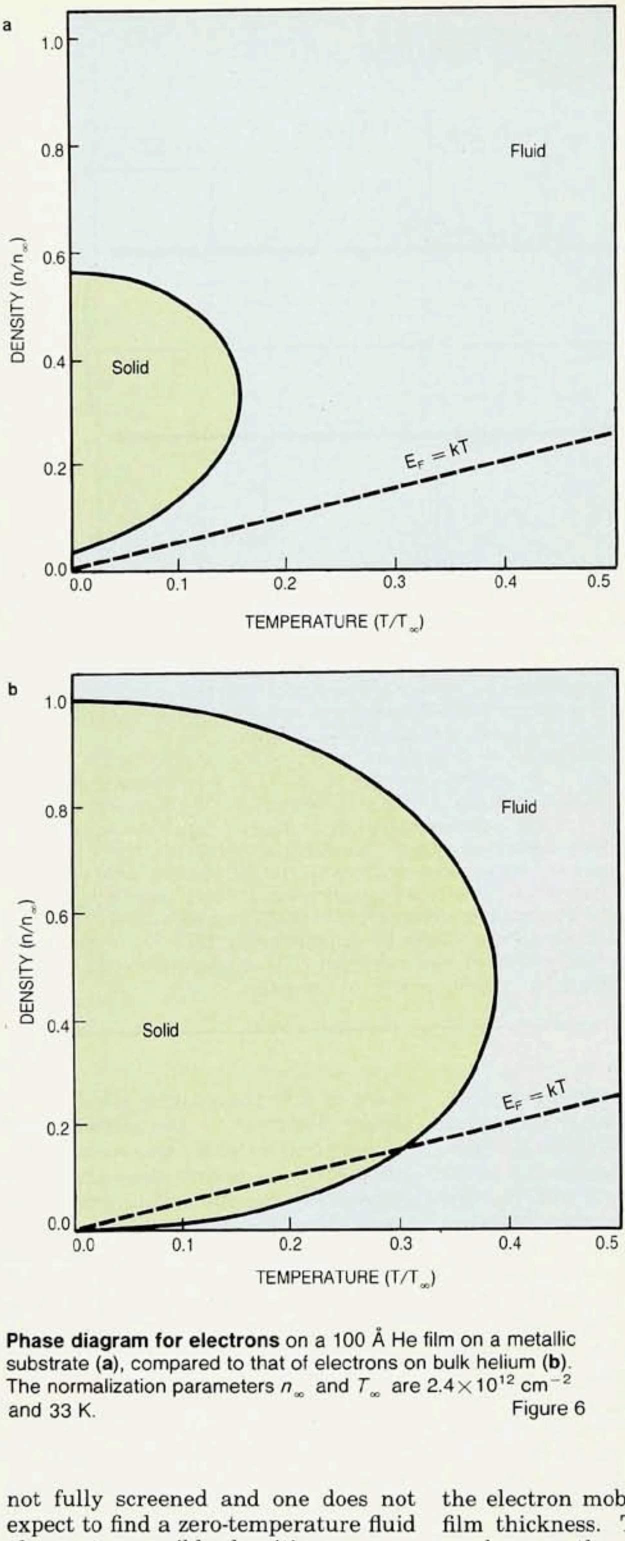
The fact that the phase diagram of figure 6a shows at very low temperatures a fluid phase of relatively low, and therefore accessible, density provides us with the exciting possibility of using electrons on the surface of a helium film to study quantum many-body effects in a two-dimensional degenerate Fermi fluid. One such experiment would be on the quantized Hall effect, although this would require the introduction of traps to create localized states. The electron–ripplon interaction is still present in the films, and it could well lead to an attractive electron–electron interaction via ripplon exchange, just as the electron–phonon interaction can lead to an attractive interaction between electrons in a three-dimensional metal. Such an attractive interaction could give rise to superconductivity in the two-dimensional array. These potentially exciting experiments have yet to be carried out. Preliminary measurements by Hong-Wen Jiang and Dahm with thin helium films on a dielectric (glass) substrate have shown a reduction in melting temperature that is qualitatively consistent with the ideas of Peeters and Platzman. With a dielectric substrate, however, the charges are

Experimental cell



The cross section of a typical experimental cell is shown in the figure above. It consists of a plane parallel plate capacitor semi-immersed in liquid helium with the helium surface parallel to the plates. A glow discharge in the helium vapor is used to charge the surface with electrons; a field emission or field ionization tip in the liquid is used to charge the surface with ions. A holding voltage (V_{dc}) is applied to the bottom electrode. Charges collect on the surface up to a maximum density such that the net electric field in the region between the charges and the neighboring electrodes vanishes.

The array of charges is probed by exciting and detecting the charge motion with electrodes (A, B, C) carved out of the capacitor plates. For example, a mobility measurement can be made by applying an ac voltage to electrode A. The response of the electrons to this excitation voltage (a current to the right) induces a capacitive signal on electrode C, with a time delay that is caused by the scattering of charges and their inertial mass. The mobility, proportional to the sheet conductivity, and the effective mass of the charges are determined from the in-phase and quadrature components, respectively, of the signal. Plasma modes are probed by measuring the response to an rf signal applied to the center electrode B or the guard electrode G. As one sweeps the rf frequency one observes resonances in the power absorption or in the pick-up on another electrode at frequencies corresponding to the plasma modes of the array.



Phase diagram for electrons on a 100 Å He film on a metallic substrate (**a**), compared to that of electrons on bulk helium (**b**). The normalization parameters n_∞ and T_∞ are 2.4×10^{12} cm^{-2} and 33 K. Figure 6

not fully screened and one does not expect to find a zero-temperature fluid phase at accessible densities.

Besides the possibility of studying the quantum limit of two-dimensional plasmas, experiments on electron arrays on thin helium films have also proved useful in the study of adsorption of helium on substrates. Recent experiments by Mikko Paalanen and Yasuhiro Iye and by Detlef Cieslikowski, Leiderer and Dahm show oscillations in the electron mobility as a function of film thickness. These oscillations occur because the electron scattering off the helium surface is reduced to its minimum value as each additional layer of helium is completed. Surprisingly, up to nine mobility maxima have been observed by Leiderer's group.

We hope that experiments now under way on electron arrays on thin helium films will guide our understanding of the quantum limit of two-dimensional plasmas, just as the experiments on low-density plasmas at the surface of helium that we have described in this article have been invaluable in our understanding of plasma oscillations, phonon modes, crystallization and melting in two dimensions.

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