Novel magnetic properties of solid helium-3

At low densities solid helium exhibits two distinct magnetically ordered phases that are easily accessible with present technology; studies of these systems are enhancing our understanding of nuclear magnetism and atomic exchange in quantum solids.

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Nuclear magnetic ordering in solid helium-3 provides the ultralow-temperature physicist with a unique opportunity to study cooperative magnetic phenomena in a quantum spin system. There are two known—and very different—magnetic phases of low-density solid He^3, and at higher densities ferromagnetism may exist. With ordering temperatures at low densities near 1 mK, these systems are easily accessible with modern demagnetization cryostats. Over the next few years experimental and theoretical studies of these systems will advance considerably our understanding of nuclear magnetism.

Nuclear-spin ordering in solid He^3 does not result from magnetic dipole interactions between nuclei, but rather from He^3 atoms hopping between adjacent lattice sites. We will call this phenomenon "exchange" without at this stage suggesting any particular geometry or mechanism mediating the process. The exchange theory was first proposed over 20 years ago,[1] and a substantial volume of experimental results support this view; yet our understanding of nuclear-spin ordering in solid He^3 is far from complete. Indeed, there is a stark contrast between our rates of progress in understanding superfluid liquid He^3 and in understanding spin-ordered solid He^3. The superfluid is truly an ideal material to study experimentally, as it exists without impurities or lattice defects and possesses an exceedingly high thermal conductivity. In addition, the BCS theory used to explain superfluidity in He^3 did not have to be created, but only borrowed. (See the article by Pertti Hakonen and Olli Lounasmaa on page 70.) The experimenter studying solid He^3, however, must overcome the severe experimental difficulties associated with cooling this poorly conducting material through its ordering transition, as well as the absence of any adequate theoretical methods to deal with the highly quantum mechanical system. Despite these problems, solid He^3 remains a very useful model magnetic system. To a very good approximation the spins are a true Heisenberg system (that is, the effective spin interactions are isotropic), and exchange is the only interaction important in the thermodynamics. By contrast, electronic magnetism is always complicated by many competing interactions and anisotropies (such as exchanges, magnetic dipole interactions and spin–orbit coupling), and nuclear magnetism in metals such as copper involves both indirect exchange (through the conduction electrons) and dipole interactions. But perhaps the most important advantage of solid He^3 is that the spin ordering occurs in a temperature range low enough that the spin system totally dominates the thermodynamic and transport properties, but still sufficiently high that it can easily be reached by external refrigeration.

Nuclear demagnetization cryostats such as this one in Douglas Osheroff's lab at AT&T Bell Labs provide a stable environment below 1 mK allowing detailed measurements of the ordered phases of solid He^3 over long times. In the photo Osheroff is preparing to transfer liquid helium into the cryostat. Figure 1

Exchange

Atomic exchange plays an important role in solid He^3 because of the light mass of the nuclei and the weak interparticle potential. Indeed, He^3 will not solidify under less than about 30 atmospheres of pressure, even arbitrarily close to absolute zero. At melting pressures the atoms in the solid vibrate wildly about their lattice sites with a mean displacement of about one-third the distance between nearest neighbors. Roughly once in every 40 000 zero-point oscillations, neighboring atoms will exchange lattice sites. Once the atoms are freed from particular lattice sites, we must take their indistinguishability and the Pauli exclusion principle seriously. Remarkably, in this limit of rare exchange the delicate changes in the energy of this motion coming from the Pauli constraints can be completely described in terms of an effective Hamiltonian for the *spin* coordinates of the nuclei—the "exchange" Hamiltonian. To understand how atomic exchange leads to nuclear-spin ordering, consider the simplest possible process, the exchange of lattice sites by two nearest neighbors in the body-centered cubic lattice of low-density He^3. Because each He^3 atom possesses a net spin of $\frac{1}{2}$ it is a fermion, and the wavefunction of the exchanging atoms must be odd under interchange of two of the particles. This wavefunction can be written as the product of a spin component and an orbital component; exactly one of these

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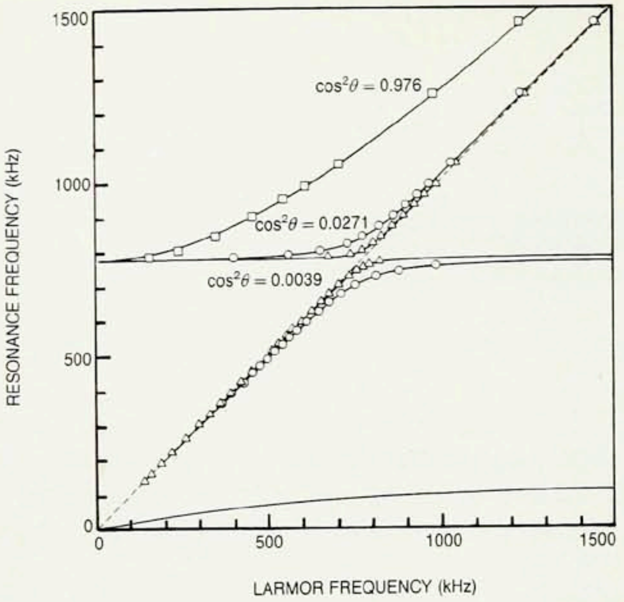


Nuclear magnetic resonance spectrum for the three domains in a single crystal of solid He^3 at 0.4 mK. The symbols pair up the two modes in each domain, and the solid lines are theoretical fits with $\Omega_0 = 780$ kHz and the indicated values of $\cos^2\theta_i$ (see equation 2 in the text). Figure 2

components must be odd under exchange. Because an odd orbital component requires a node in the wavefunction, this state has a higher kinetic energy than the lowest even wavefunction. The spin component of the wavefunction in the lowest energy state must therefore be odd. If we step through a lattice that satisfies the condition that nearest neighbor spins be antiparallel (the antiparallel singlet state), we see an antiferromagnetic structure consisting of two simple cubic sublattices. This is often referred to as a "normal antiferromagnetic" phase.

It is important to understand that we are interested in the spin of a particular atom only as a label: Its dynamical properties are not involved in this description. Each atom maintains the same nuclear spin as it hops around the lattice. However, as it is easier to label the lattice sites than individual atoms, we describe the system's state in terms of the spin (up or down) at each lattice site. (The dynamics and intrinsic interactions of the spins *are* involved if we worry about how the system relaxes to the ground state.)

The full description of the spin system resulting from nearest neighbor pair exchange is a much studied theoretical model, the nearest neighbor spin-$\frac{1}{2}$ Heisenberg model, for which the effective Hamiltonian is

$$H = 2J\sum_{i,j} \mathbf{s}_i \cdot \mathbf{s}_j \qquad (1)$$

with $\mathbf{s}_i$ the spin on site i, and the sum goes over all nearest neighbor pairs in the lattice. Throughout most of the 1960s it was believed[2] that only this nearest neighbor pair exchange would be significant, and it appeared that the main interest in magnetism in He^3 might be as a simple pedagogical tool.

However, the behavior of solid He^3 in the disordered phase could not be quantitatively explained by this simple one-parameter model.[3] The first-order nature of the transition to the antiferromagnet and the enhanced magnetization at higher fields are also at odds with the model. The most direct experimental indication of how dramatically He^3 differs from the simple model came from nuclear magnetic resonance experiments in 1979, which showed that the low-temperature phase of He^3 is not even cubic![4,5]

Nuclear magnetic resonance

Nuclear magnetic resonance has turned out to be an enormously powerful probe of the ordered states of He^3, and to a large degree the assignment of the superfluid phases in the liquid resulted from such experiments. There are a number of experimental and theoretical reasons for this: Nuclear magnetic resonance is an easily controlled, contactless probe of the system. The resonant frequencies fall in an ideal range for measurements, and the lines are extremely narrow, allowing a precise measurement of resonant frequencies. Perhaps the most important reason however is that the shift of the resonance frequency away from the Larmor precessional frequency of free spins results from the tiny direct magnetic dipole energy (on the order of 0.1 μK/atom) and not from the much larger energies responsible for the ordering (exchange, Fermi and so forth). Only for collective motions of many atoms is this tiny energy not swamped by thermal fluctuations. Thus nmr is an experimentally convenient probe of collective states, and one that does not itself affect the thermodynamics of the state. The rather subtle correlations involved in evaluating a dipole energy (a combination of relative spin orientation and displacement vector between two atoms) has also turned out to be remarkably well tuned to the ordered states of He^3. The first nmr studies of the low-field phase were carried out almost simultaneously but independently by E. Dwight Adams and his group at the University of Florida[4] and by our group at AT&T Bell Laboratories[5] in 1979.

Both sets of experiments saw large shifts in the nmr frequency from the Larmor value. These shifts clearly showed that the sublattices in the low-field phase could not possess a cubic symmetry, for then the anisotropy of the dipole energy responsible for the shifts would vanish to first order.

Adams and his group self-cooled a mixture of solid and liquid He^3 by adiabatic solidification of the liquid. As a result, their work was limited to polycrystalline samples near the ordering temperature. In our work we used copper nuclear demagnetization as a refrigerant, which allowed us to maintain temperatures as low as 300 μK for two days. We also developed a technique to grow single crystals of solid directly into the ordered state, so that a more complete study was possible.

Figure 2 shows a nearly complete nmr spectrum from a single crystal of solid He^3 at about 0.4 mK. There are three pairs of resonances, one member of each pair above the Larmor frequency and one below. These resonances apparently originate from three separate magnetic domains in the single crystal: Indeed, we made many attempts to grow single crystals to ensure that there would be no spurious resonances, but we always saw three high-frequency resonances. The existence of just three domain orientations immediately suggests that the sublattice has a symmetry axis that can orient only along the three principal axes of the cubic structure—that is, that the arrangement is tetragonal and not cubic.

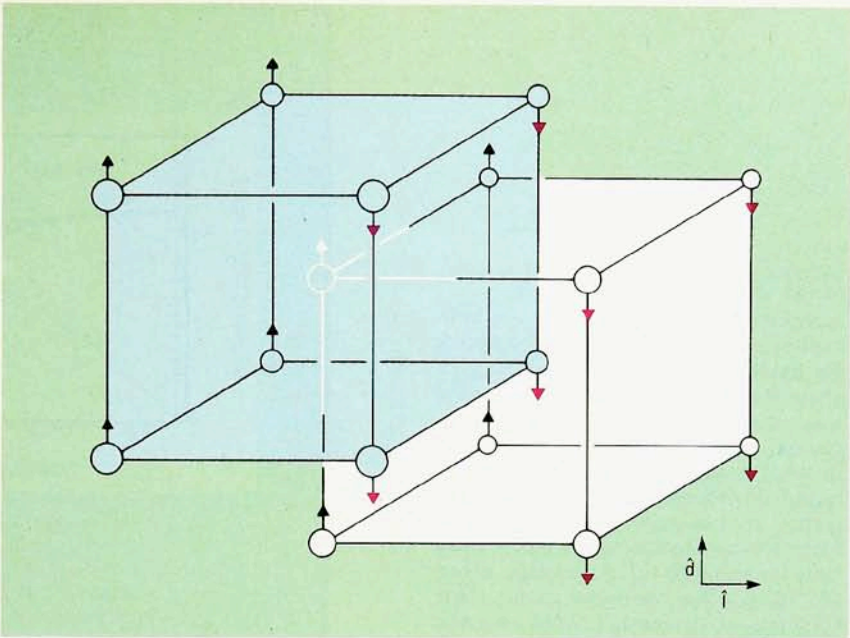
The nmr spectrum leads to precise deductions of *symmetries* of the spin alignments and sublattice structure.[6] For example, one of the modes coming from each domain has a vanishing frequency at zero field, which indicates a rotational symmetry of the sublattice structure; the existence of shifts at all tells us the symmetry is not cubic. From these data one can construct a simple, quantitative theory that is asymptotically exact for frequencies small compared with exchange frequencies. It involves a single temperature-dependent parameter Ω_0 that measures the magnitude of the dipole energy in the unperturbed antiferromagnetic states. The theory leads to two resonance modes for each sublattice, described by the roots of the following equation:

$$\omega^2 = \tfrac{1}{2}\left[\omega_L{}^2 + \Omega_0{}^2 \pm \sqrt{(\omega_L{}^2 - \Omega_0{}^2)^2 + 4\omega_L{}^2\Omega_0{}^2\cos^2\theta_i}\right] \quad (2)$$

Here $\cos\theta_i$ is the angle between the domain's symmetry axis and the magnetic field and ω_L is the Larmor frequency. The absolute values of the θ_i are not initially known, but if the three domain axes are mutually perpendicular, the sum of the three $\cos^2\theta_i$ terms must be 1. This indeed is accurately confirmed. These roots are plotted as the solid lines in figure 2. In fact, one can use these resonances to determine the precise orientation of the crystal axes with respect to the magnetic field save for a rotation about that field.

By applying an external field in two different directions, one can determine the orientation of the crystal uniquely, and with a precision that rivals an x-ray Bragg scattering determination.

Because Ω_0 depends on temperature—varying from about 520 kHz at the transition to the disordered phase to about 800 kHz at zero temperature—these resonances can be used as a precise contactless probe of the temperature of the He^3 spin system in other experiments such as measurements of magnon (spin wave) thermal conductivity, or perhaps in an effort to observe magnon second sound.



The probable antiferromagnetic state of low-field solid He^3, u2d2 in the notation used in the text. The spins are arranged in ferromagnetically aligned (100) type planes in the sequence up–up–down–down. The vector $\hat{d}$ is the spin symmetry axis, and $\hat{l}$ is the lattice symmetry axis; the dipole interaction orients $\hat{d}$ perpendicular to $\hat{l}$. Figure 3

To proceed with an identification of the magnetic structure at this point one must simply try various sublattice structures and see if they satisfy all the criteria set down by the nmr results. The only states known to satisfy all these criteria are those in which the planes of spins are normal to the (1,0,0) direction and are arranged in the following pattern: a set of N planes in which the spins are pointing up, followed by N planes with spins pointing down, repeated throughout the crystal. We denote such an ordering by the notation uNdN. The simplest state is obviously the u2d2 structure shown in figure 3. (The u1d1 phase is the cubic normal antiferromagnet.) We stress that nmr probes the global symmetries of the magnetic structure, but does not definitively probe the microscopic structure to determine it uniquely.

A direct quantitative test of any possible structure can, in principle, be made by comparing an estimate of the antiferromagnetic resonant frequency—calculated by summing all the dipole energies throughout the lattice—with the observed frequency Ω_0, but poorly known spin and lattice zero-point corrections prevent such a comparison from being very useful. Fortunately, other evidence supporting the u2d2 identification exists. In fact, even before the nmr experiments on the low-field phase were carried out, studies were begun to measure the magnetic structure directly with polarized neutron scattering. These experiments are exceedingly difficult because of the low temperatures required and because of the high neutron absorption cross section of the He^3 nucleus. After five years a French group at CNRS in Grenoble[7] has been able to observe a magnetic Bragg peak corresponding to a ($\tfrac{1}{2}$,0,0) wavevector that disappeared as the crystal warmed above the Néel temperature T_N. The crystal remained below T_N for only about 500 seconds while exposed to the neutron beam, and during that time only about 50 excess neutrons were scattered into the ($\tfrac{1}{2}$,0,0) Bragg peak. Other scattering directions were not probed in the Grenoble experiment. Although the neutron results thus appear to support the identification of the low-field phase with the u2d2 structure, we must regard this conclusion as preliminary.

Other experiments

There is clear experimental evidence of the richness of the magnetic properties of solid He^3 in the field–temperature phase diagram at melting pressure shown in figure 4. The antiferromagnetic phase exists only in a small corner of the diagram, below the Néel temperature, 1 mK, and a critical field H_c of 0.4 tesla. In this phase the magnetization is suppressed below that of the disordered paramagnet.

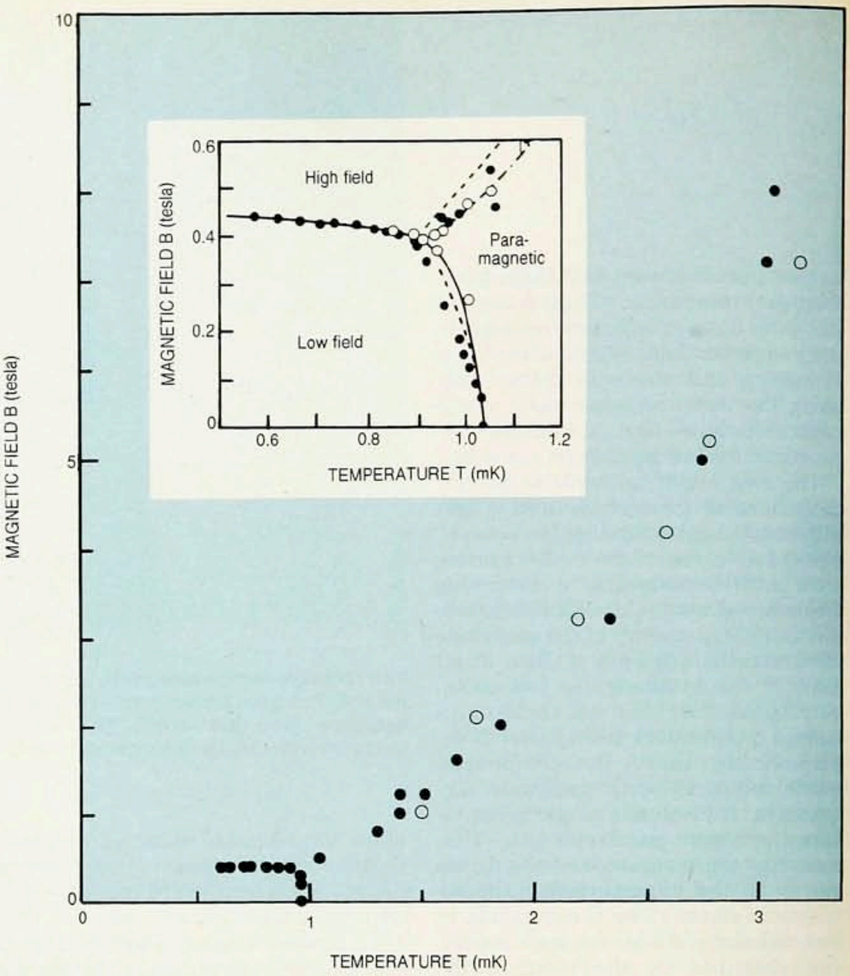
As Dwight Adams and his coworkers at the University of Florida discovered, for fields above a critical field H_{c1} of about 0.4 tesla at low temperatures the magnetization is dramatically enhanced above the paramagnetic value in what is now known to be a second, high-field ordered state.[8] The magnetization in this state is roughly three-fifths the saturation value at 0.4 tesla (extrapolating to full saturation above 15 tesla). We therefore have clear evidence of competing ferromagnetic and antiferromagnetic tendencies.

Accompanying this delicate competition between opposing effects, we might expect a strong dependence of the H–T phase diagram on other external parameters. On the contrary, however, one finds that on varying the density (by applying pressure), the overall energy scale changes rapidly, but all quantities seem to scale in the same way. Thus all parameters T_i with the dimensions of a temperature vary with a "Grüneisen parameter"

$$\gamma_i = \frac{d(\ln T_i)}{d(\ln V)}$$

of about 18. Parameters measured include the transition temperature and various coefficients from the high-temperature expansion of thermodynamics properties.[9] The accuracy of the determination of γ_i is not high (typically the errors are ± 2), but the scaling behavior certainly remains a puzzle. The quantity that might be expected to differ most strongly from this trend—the low-temperature value of H_{c1}—has not yet been measured as a function of density.

A better understanding of the nature of the high-field phase will provide vital information on the magnetic interactions, and this is proving to be a tough experimental problem. There is still no definite experimental evidence on whether the crystal in this phase is simply an enhanced paramagnet, or in addition either has some sublattice structure (that is, is ferrimagnetic) or has a spontaneously broken symmetry transverse to the applied field (what one might call a transverse antiferromagnet). Nuclear magnetic resonance experiments have been carried out on single crystals in the high-field phase,



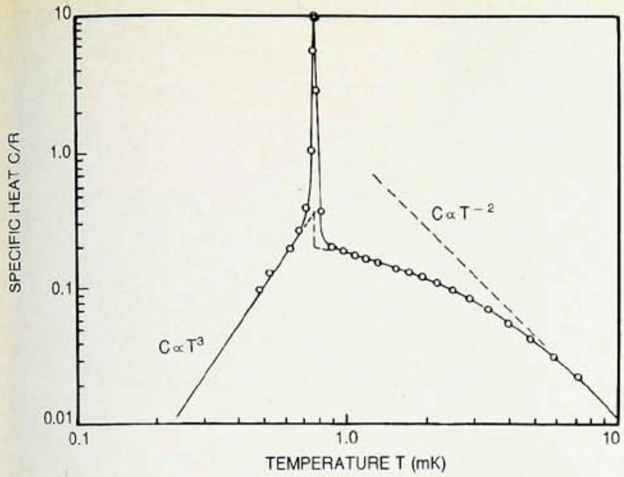
Experimental phase diagram of solid He^3 at melting pressure. The main figure is from A. Sawada, H. Yano, M. Kato, K. Iwahashi, Y. Masuda, *Phys. Rev. Lett.* **56**, 1587 (1986). The inset, showing more detail in the low-field behavior, is from reference 9. Figure 4

but here the magnetization is high and the experiment must be sensitive to frequency shifts two orders of magnitude less than the shifts caused by the demagnetizing field of the sample. These experiments show that if any anisotropies in the dipole energy exist, they must be at least an order of magnitude smaller than those in the low-field phase, suggesting that the high-field phase possesses a cubic symmetry. Recent experiments[10] investigating the specific heat in the high-field phase indicate a T^3 dependence consistent with spin waves in a transverse antiferromagnet rather than the exponentially small value expected if there is no transverse ordering, but these experiments need to be pushed to lower temperatures. Dennis Greywall (AT&T Bell Labs, Murray Hill) has measured the specific heat at 1 tesla, and has clear evidence of a second-order transition to the high-field phase with a λ-like anomaly, consistent with a transverse antiferromagnetic ordering (figure 5). Other circumstantial evidence, such as a very rapid magnetic relaxation, also points to a nontrivial ordering. Nonetheless, there is still no direct evidence to show the nature of the ordering. Unfortunately neutron scattering has not been done in this phase.

Theories

It is clear that the magnetic ordering in helium-3 results from the effect of the Pauli principle on atoms that are changing places in the lattice due to their zero-point motion. Despite enormous progress, the details of the atomic scrambling remain rather a puzzle.

As experimenters were making dramatic advances in characterizing the new phases, a group of theorists[11] was studying the consequences of multiple-exchange models. The motivating idea is that two-particle exchange is strongly suppressed by the steric hindrance of neighboring atoms in the crystal. (These are not point particles, but atoms with strongly repulsive hard



Specific heat C of solid He^3 near the melting pressure and in zero field. Notice the asymptotic $1/T^2$ behavior at large temperatures, and the anomalous decrease in the specific heat approaching 1 mK. The first-order transition to the antiferromagnetic phase shows up as a very sharp peak at about 0.8 mK. The T^3 specific heat at low temperatures is characteristic of spin waves in an ordered antiferromagnet. (Courtesy of Dennis Greywall.) Figure 5

cores that must squeeze by each other.) However, a cyclic exchange of more particles around rings of atoms in the lattice avoids this steric problem. Although suppressed by the greater distance moved by the *exchanging* particles, the neighbors need not move out of the way, and ring exchange may in fact be the dominant process.

M. Roger, J. H. Hetherington and J. M. Delrieu suggested[11] a simple multiple-exchange Hamiltonian that—qualitatively at least—accounts beautifully for the magnetic properties of solid He^3. The model Hamiltonian is made up of three-spin exchanges around rings, which consist of two nearest-neighbor hops and one next-nearest-neighbor hop, and a particular choice of four-spin exchanges consisting of nearest-neighbor hops around planar loops in the lattice as shown in figure 6. Two coefficients give the strengths of the three-spin and four-spin exchange terms in the Hamiltonian.

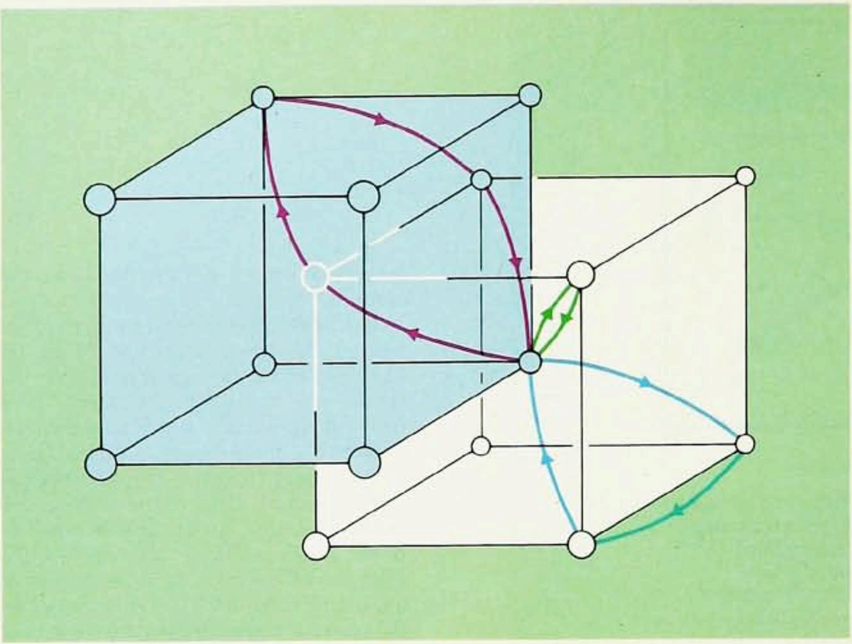
The model immediately yields the competition between ferromagnetism and antiferromagnetism, because, as a direct consequence of the Pauli principle, ring exchange of an even number of atoms leads to antiferromagnetic spin interactions and ring exchange of an odd number leads to ferromagnetism. A simple mean-field classical analysis of the model with chosen values of the relative strengths of the three-spin and four-spin exchange terms yields a phase diagram (figure 7a) remarkably similar to the experimental one. In particular, the low-field phase appears to be the u2d2 phase, and the model predicts a transition out of this phase to a "pseudoferromagnetic" phase—with magnetization of about $\frac{2}{3}$ saturation—as the field is raised. The pseudoferromagnetic phase has a particularly elegant explanation in this model: The nature of the phase is predicted to have the same sublattice structure as in the normal antiferromagnetic phase and hence no dipole energy anisotropy, but with the spins canted rather than antiparallel, so that there would be a net magnetization even in zero field if the u2d2 phase did not preempt it by a first-order transition. This state has been called the canted normal antiferromagnet.

In principle a quantitative test of multiple-exchange Hamiltonians is possible. One would have to determine the coupling parameters by a fit to the higher-temperature thermodynamic data and then calculate the low-temperature properties. Unfortunately the first step has proved difficult because of experimental problems where the theory is easy: It is difficult to extract the small corrections at the high temperatures where the interaction effects are small and the theory is good. The second step, where the experiments are well characterized, has proved extremely difficult theoretically, and there have not been any reliable results beyond the classical mean-field theory. One exception is the field at which the magnetization of the high-field phase reaches its saturation value, H_{c2}. This upper critical field can be calculated exactly using mean-field theory. Large quantum fluctuations, however, are likely to introduce errors of up to 50% into the classical theory.

The problem is the same one that has held up progress in many fields, for example lattice gauge theories or the condensed phases of fermions: It is proving extremely hard to calculate, even with sophisticated Monte Carlo techniques, properties of quantum systems in which the ground state wavefunction cannot be guaranteed positive. Bose systems have positive ground state wavefunctions, and have proved amenable to Monte Carlo attacks. Fermi systems necessarily have regions of either sign, and progress here is difficult. Spin systems may fall into either class. They can occasionally be transformed to the Bose class by a suitable change of basis, but this does not seem likely for the multiple-exchange Hamiltonians with competing interactions. The rather long range of the ring exchange also makes conventional approaches (such as the Padé approximations on high temperature series) very difficult.

To date there have been no reliable predictions of the low-temperature properties of multiple-exchange models going beyond classical mean-field theory. It is therefore difficult to know whether discrepancies between predictions based on classical mean-field theories and experiment are a sign of conceptual flaws in the model. The greatest discrepancy with the model containing only three-spin and four-spin exchange is in the field H_c. This field is remarkably low experimentally—about 0.4 tesla. By contrast, the thermal energy at the Néel temperature corresponds to the magnetic energy of the spin in a field of 1.4 T, and the spin-exchange theory yields a value for H_c of 1.2 T. It is unlikely that this factor of three discrepancy would be improved by quantum corrections to the classical result. As shown in figure 7b, the critical field can be brought down by adding the nearest-neighbor exchange back in—but at the expense of introducing additional ordered phases at higher temperatures, and these are not seen experimentally.

The multiple-exchange models have a major conceptual problem that becomes more serious as more exchange types are added to fit the data: If we calculate the exchange rate as the



Multiple particle exchange. The schematic diagram shows the paths of nearest neighbor two-spin, three-spin and four-spin planar exchange. Figure 6

frequency of zero-point motion reduced by a factor of 10^4 for tunneling or steric effects (exchange occurs roughly once in ten thousand oscillations), then one would expect the reduction factor to vary considerably for the different processes, so that only one exchange parameter should dominate. The coincidence of two or, even worse, three exchange constants having comparable values seems unreasonable. Furthermore, different exchange processes should vary differently with volume, depending on the relative importance of steric effects and path lengths. It seems increasingly clear that if a multiple-exchange Hamiltonian ultimately provides a good phenomenological fit to the magnetic properties, there must be additional physics on the microscopic scale, such as a particular lattice fluctuation that mediates a number of different exchange processes. D. M. Ceperley and G. Jacucci (Lawrence Livermore and University of Trento, Povo, Italy) have recently made Monte Carlo calculations of the exchange and indeed have found comparable two-, three- and four-particle exchange rates, but also important contributions from higher-order processes.[12]

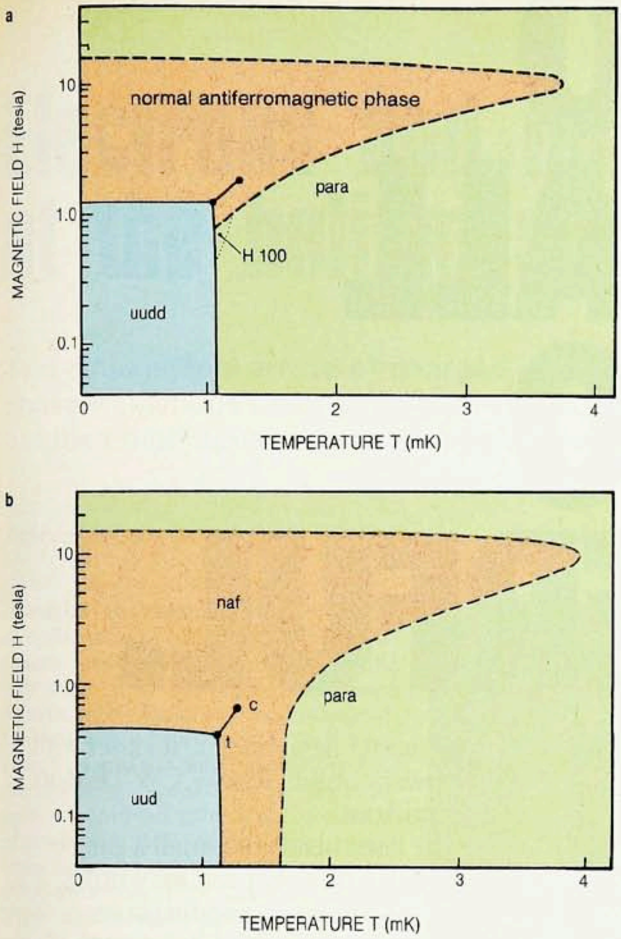
Helium-3 as a novel laboratory

With the advent of powerful new dilution refrigerators capable of providing several microwatts of refrigeration at 10 mK, and base temperatures at or below 4 mK, the environment necessary to study spin-ordered solid He^3 is becoming far more accessible. For instance, in the early compressional cooling studies one could remain below the spin ordering temperature for at most a few hours. In the AT&T Bell Labs nmr experiments,[5] one could remain below T_N for only about two days. In the new apparatus shown on page 23, which replaced the original Bell Labs demagnetization apparatus, it is currently possible to grow a single crystal of solid He^3 and maintain it below T_N for months. The large copper wire bundle in the center of the figure consists of 60 moles of copper across which an rms magnetic field of 6 tesla can be applied. Precooling the bundle to 6 mK (a process that admittedly takes several days) gives it a nuclear heat capacity of 150 J/K. This is to be compared with typical heat leaks of a few nanowatts. With such a powerful apparatus there is sufficient time to produce and study well-characterized samples.

Several potential problems remain. At least near T_N the equilibrium times in the solid are very long. The cooling power available today allows the experimenter to wait for equilibrium, but at the expense of his own time. In addition, the quality of the crystals used in such experiments is somewhat suspect. In those cases where the solid is grown as a bulk crystal at well below its ordering temperature, it is also well below all roughening transitions, and solid growth is usually supported only by crystal defects such as screw dislocations. So far, no one has attempted to cool bulk solid samples through T_N in the absence of liquid. Yet future studies of the magnetically ordered phases will, we believe, require bulk specimens—experiments done on He^3 in porous heat exchangers are too susceptible to strains and density inhomogeneities.

In the next few years we can expect to see studies aimed at probing the bulk transport properties of these unusual systems. For example, the enormous dependence of the exchange processes on molar volume produces an extraordinarily strong coupling between the ordered spin system and propagating sound waves. Heat transport in the solid at low temperatures should be dominated by magnon thermal conductivity, and there is already some evidence to suggest that the heat conductivity is indeed quite high. It is even within the realm of possibility that in this system, where the average spin wave velocity is only about 8 cm/sec, it will be possible to observe magnon second sound. Attempts at "hole burning" nmr experiments in both magnetic phases, where a field gradient is placed across the sample and only a narrow region of the sample is saturated, show exceedingly rapid magnetic recovery. Undoubtedly nmr experiments will continue to show exciting new behavior.

Of particular interest are the properties of interfaces involving the ordered solid phases. Spin-wave reflection at domain walls in the u2d2 phase can be studied by heat propagation across the domain wall. The probability that the ordered phases possess lattice distortions makes studies of the phase boundary between the high-field and low-field phases particularly exciting. Finally, the nature of the very low thermal boundary resistance between the superfluid liquid and the various ordered solid phases—while interesting in its own right—may shed some light on the general properties of a quantum solid–liquid phase boundary well below its roughening transition.



Predicted phase diagrams based on a classical-mean field calculation of multiple-exchange models with nearest-neighbor (J_{NN}), three-particle (J_t) and four-particle ring (K_p) exchange. The parameters are chosen to be consistent with high temperature measurements. **a:** A fit using just three- and four-spin exchange ($J_t = -0.10$, $K_p = -0.355$) gives a good qualitative description of the experiments, but the zero-temperature transition field H_c between the low- and high-field phases is too large. **b:** The predicted value of this field is reduced by adding the two-spin exchange ($J_{NN} = -0.377$, $J_t = -0.155$, $K_p = -0.327$), but at the expense of predicting additional phases above T_N that have not been seen experimentally. The value of H_c in **b** is very sensitive to the value chosen for J_{NN}. (From reference 1 and H. L. Stipdonk, J. H. Hetherington, *Phys. Rev. B* **31**, 4681, 1985). Figure 7

It is somewhat ironic that bulk solid He^3 will probably never prove useful in critical phenomena studies. It appears that either the transitions are first order or the thermal equilibration times near the transitions are unacceptably long. The long equilibration times can perhaps be circumvented by studying a sample contained in the pores of a heat exchanger, but this is simply trading thermal inhomogeneity for density inhomogeneity and strain fields.

We might hope ultimately to use the magnetic phenomena in He^3 as a probe of the motion of the atoms among the lattice sites in quantum solids. The same sort of motion presumably occurs in solid He^4, but there we do not have the up and down spin labels to follow the motion directly. It has been suggested that a high degree of atomic exchange may lead to a superfluid solid; investigations of He^3 may yield insights on this exotic phenomenon. Knowledge of the atomic exchange motions in the solid at conditions near the melting curve might lead to a better understanding of the quantum solidification transition and of the properties of strongly interacting Fermi liquids: He^3 provides a simple model that might be relevant to heavy-electron systems. There is already some evidence of anomalous behavior in the lowest-density solid.

Other phases of He^3 may also provide interesting quantum magnetic systems. The high-density hexagonal close-packed phase is expected to be dominated by three-spin exchange, leading to ferromagnetism. Thin films of He^3 on solid substrates, either in contact with the vapor or bulk liquid, yield interesting two-dimensional magnetism. Again the very small anisotropic interactions (small compared with isotropic exchange interactions) result in easily characterized, nearly ideal magnets. It is likely that the magnetic properties of this system dominate the heat transfer between liquid He^3 and a substrate, reducing the thermal boundary resistance to the anomalously low values observed in experiment.

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