The phenomenon of the inelastic deformation of solids has in some respects been an aggravation, yet it has often been put to good use. The following article traces the progress in understanding of plastic flow as a major problem for the structural designer and the physicist alike.

plasticity

By James B. Kelley

In recent years the engineer has been treated to many strange sights, not the least of which have been the "flapping" of airplane wings and the "flow" of metals under circumstances which have come to be considered quite normal. The "flapping" phenomenon brings to mind a story concerning a very famous (but for the moment anonymous) theoretician in the field of structural design. Although the theoretician had for a long time commanded respect and been accorded great honors in engineering circles, he had seldom bothered with the application of his theories beyond checking their usefulness. However, it happened that he was invited as an expert to attend an important conference some distance from the university where he was teaching. Although he was somewhat reluctant to fly he finally gave in and undertook his first airplane trip. After sitting rigidly in his seat for some minutes he turned to his travelling companion and said,

"Look." His hand was pointing to the starboard wing which was moving slightly in the rough air, the metal skin showing noticeable ripples.

"Yes," his companion nodded. "Quite interesting, isn't it?"

"But, it's moving. Isn't that dangerous?"

"You're joking, Dr. ___."

The other shook his head. "No, I'm not joking."

"But Dr. ___, you proved conclusively in several of your papers that the condition we're observing may take place with complete safety."

Whether the theoretician was convinced of the validity of his own theories cannot be said, nor can it be said that he ever proved to be an inveterate air passenger. One thing this anecdote does show, and that is that regardless of what physics and mathematics may prove, human conceptions of how substances and processes should behave are frequently much stranger than the more rigorously derived (and perhaps, more valid) conceptions of physical phenomena. But if the rather strange behavior of thin, metal sheet construction causes a raised eyebrow, the inelastic, or plastic, behavior of metals can well cause a loud, "OH!"

Concept of Plasticity

Plasticity might be called solid viscosity. Whether this is a very helpful definition leaves some doubt in many minds, for while it is most certainly a

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manifestations of what we call viscosity, this latter quantity is itself not completely defined. We find viscosity called everything from "oiliness" to the resistance to shearing stress of a fluid. And while both of these definitions, as well as all those in between, convey some kind of meaning to a reader they leave a great deal unanswered for the investigator who would like to know something basic about the nature of physical substances.

It is well known that solids flow. Any solid matter has a definable melting point at which it passes from the solid to the liquid (or gaseous, for materials which sublimate) state. Even those substances which sublimate have three states, although the liquid may be unstable. When heat is applied to a solid in sufficient quantity the solid turns into a different material, not necessarily in its constituent composition but in the alignment of its crystals. So that we speak in metallurgical circles of *heat treatment* which has a marked effect on the physical properties of the material being treated. Not only does the temperature to which the material is raised have a considerable effect but also the manner in which it is cooled must be considered, for it is in this way that the metallurgist can cause the formation of a crystalline structure which he desires.

There is still another way of bringing about a change in crystalline arrangement and that is through the process known as *cold working*, where cold metal may be rolled or hammered into a different internal state. Since, according to the generally accepted theory, viscosity may be thought of as an internal stress which offers resistance to external pressure, this hot or cold working, which changes the crystalline structure, and hence the resistance to external pressure, causes the metal to flow even as a fluid might flow. Internally the crystals have slipped or slid, been crushed or deformed, twisted or warped. So great an effect does cold working have on metals that they must be reheated and cooled (annealed) so that the internal stress may be relieved and the brittleness caused by the cold working removed. Although cold worked metals gain in hardness and strength they lose considerably in ductility and resistance to shock. The inability to resist shock is caused by the large internal stresses which were set up.

The chief factor in determining the size of crystals in a metal is the length of time the metal is held at high temperatures, hence also the rate of cooling. An alloy which is cooled quickly tends to have small crystals. For example in Figure 1, an alloy of iron and chromium has been quickly cooled,

while the same alloy is shown in Figure 2 after it has been reheated and held awhile at red heat to grow larger grains. The specimen with large grains will in general be much weaker than the one with small grains. Therefore in cold working and subsequent annealing it is very important to prevent a type of heating and cooling which will permit the crystals to grow, thus weakening and softening the alloy.

Crystalline Structure

Before proceeding to a further discussion of some of the more practical problems of the plastic flow of metals it might be worthwhile to review briefly some of the typical crystals found in the more common materials and the limits of temperature and pressure which cause the formation of the crystals.

It is not at all uncommon for solids to occur in several solid states. Sulfur is a very good and rather common example of this. It undergoes very radical changes in crystalline structure above and below 96°C. Below this temperature the stable crystalline form of solid sulfur is called orthorhombic, while above 96°C the stable form is known as monoclinic. The orthorhombic crystal is almost self-explanatory, consisting as it does of three mutually perpendicular diagonals or axes of unequal lengths. Monoclinic crystals, on the other hand, are long, thin stick-like crystals. At a pressure of 20,000 pounds per square inch and 152°C sulfur has a triple-point where monoclinic, orthorhombic, and fluid sulfur coexist.

Most monatomic metals crystallize into cubic or hexagonal structures. These are shown in Figures 3 and 4. The table gives the crystal types for some of the simpler (crystallographically speaking) elements as well as those most commonly met in the engineering and physics laboratories.

Figure 1. Fractograph of stainless steel (iron containing 27% chromium) after rapid cooling from hot rolling operations. The illuminated area represents a cross section of ore entire grain, its small size requiring very high magnification. Magnified 2300 times.

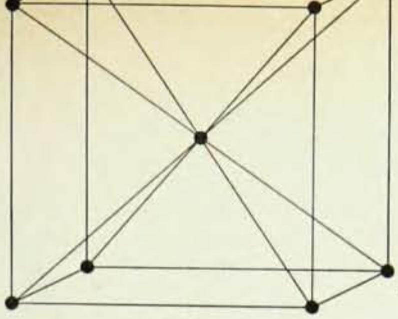


Figure 4(a). Body-centered cube

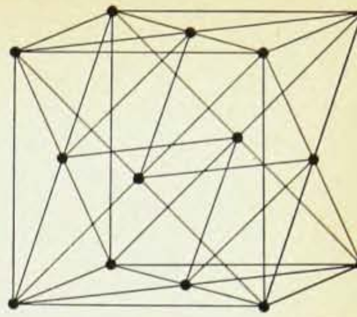


Figure 4(b). Face-centered cube

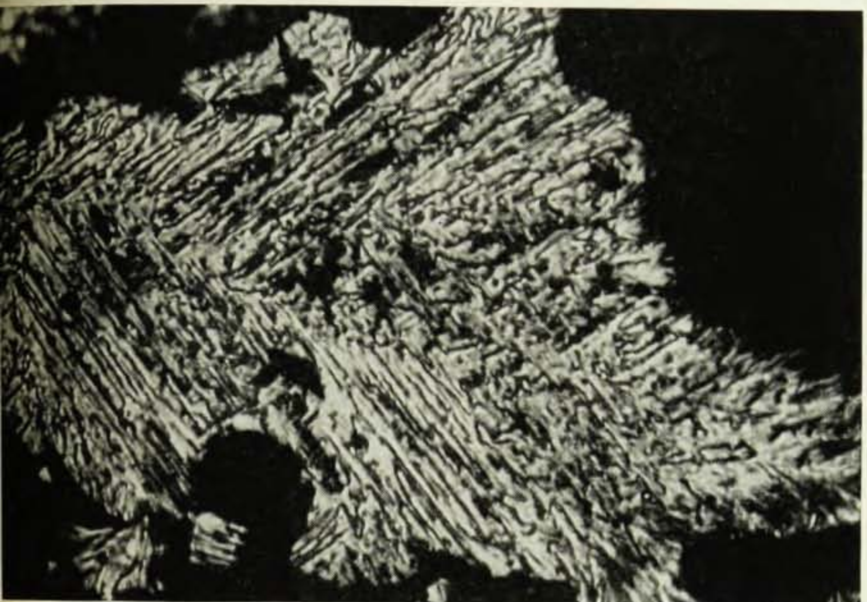


Figure 2. Same steel as in preceding Figure after an annealing treatment at 850°C. to grow large grains. The field again displays a single grain. Note the lower magnification, also the interesting fine detail in the fractographic pattern which probably discloses subtle processes of grain growth. Magnified 635 times.

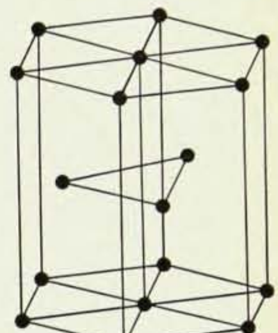


Figure 4(c). Hexagonal close-packed crystal

Figure 3. Fractograph of pure cast zinc exhibiting the hexagonal symmetry of the crystal by families of twinning planes intersecting the basal facet. The bands are at exactly 60° to one another. Magnified 80 times.

ELEMENT	VALENCE	CRYSTAL TYPE *
Ag	1	f.c.c.
Al	3	f.c.c.
Au	1	f.c.c.
Be	2	h.c.p.
Cd	2	h.c.p.
Cu	1	f.c.c.
Fe	transitional	f.c.c. or b.c.c. **
Mg	2	h.c.p.
Mo	transitional	b.c.c.
Ni	transitional	f.c.c.
Pb	4	f.c.c.
Zn	2	h.c.p.

* b.c.c.—body-centered cube, f.c.c.—face-centered cube, h.c.p.—hexagonal close-packed.

** α-, β-, δ-iron crystals are body-centered. γ-iron is face-centered.

There are, in addition to the types of crystals shown, extremely complex crystalline structures which defy a concise geometrical classification. Such rather common elements as mercury, tin, bismuth, and tungsten have complicated lattices. Just as in the case of sulfur, metals may possess allotropy; and this is more pronounced in the transition metals. Bridgman (who, incidentally, showed five different crystalline structures for ice at pressures varying from one atmosphere to 20,000 atmospheres) showed that caesium, which at normal pressure had a body-centered lattice, changes to the hexagonal close-packed lattice at a pressure of about 160 pounds per square inch.

While solids take on these rather orderly characteristics, thus making it both easy and convenient to catalog them, liquids tend to be disordered and lacking in any such properties.

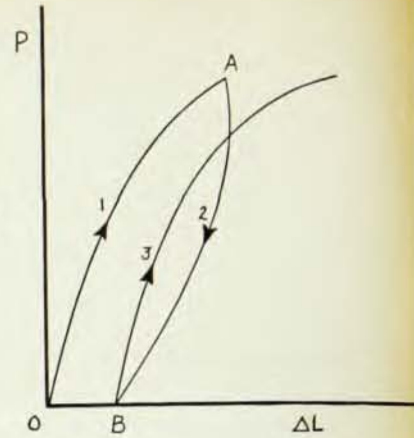
Plastic Yield

Plastic deformation may be said to imply slippage between crystal parts. In elastic deformation there is no actual change in crystalline structure, for by its very definition a metal undergoing elastic deformation must return to its original shape while maintaining its physical properties of strength, ductility, etc. The difference between the two types of deformation is shown very well in Figure 5. This curve is commonly known as the "elastic hysteresis" loop and occurs when a single crystal sample is distorted beyond the proportional limit to the point where sliding takes place within the crystal. Then after unloading, the sample is loaded again. By continued loading and unloading it may be possible to arrive at a condition of deformation in which there is no noticeable change in the permanent set at point B. The complete cycle formed by tracing the curves 1, 2, and 3 is then said to be elastic. This looping is by no means limited to a single crystal sample but is also found in polycrystalline metals and other nonmetals such as glass. The area within the closed portion of the curve represents the energy dissipated during a single cycle of loading, unloading, and reloading. If 1, 2, and 3 all coincide the case for the perfectly elastic material would result.

Many investigators, including Rowett, Hopkinson, Prandtl, Föppl, and Becker, have studied the amount and cause of the energy dissipation in the hysteresis loop. Rowett made measurements with steel which showed that this energy increases as the cube of the maximum stress during each cycle.

Among the principal factors which influence plastic yield the following might be considered the most significant: the *structure* of the material as dependent on the constitution of the substance, especially true in the case of alloys; the *absolute size* of the crystal grains which constitute a polycrystalline material; *imperfections and disturbance* within the crystal lattice, regardless of how they occur, i.e., inside the lattices, on the surfaces of the grains, or in the intercrystalline boundary which separates the grains; *temperature;* and *time.*

Figure 5. Elastic hysteresis loop



Generally speaking, foreign substances are important in crystalline structure and cause a strengthening of the structure. A notable exception to this is the cementite found in the pearlite in steel. Ordinarily it would be expected that the hard and brittle cementite (it ranks with glass in these respects) should spoil the excellent shock-resistant properties of pearlitic steel. Such is not the case at all—which fact is of no assistance in explaining the unexplainables concerning crystalline structure and its effect on the physical properties of substances.

The effect of temperature is both well and consistently established. As temperature increases the yield stress decreases. Aluminum, an extremely important structural material in aeronautical design, has very bad heat characteristics, dropping off to about half its yield stress at temperatures of several hundreds of degrees. The effect of time is equally apparent although perhaps a little more difficult to explain. An excellent example of how the crystalline form may change with time is given in the rocks found in and on the earth. Here relatively small stresses have been applied but their application has extended over many years, thus causing the formation of entirely new substances.

The precise explanation of all these phenomena is a little difficult to come by, but it is usually

Figure 6. Perfect crystal fragment outlined by crystal planes which first behaved in a ductile manner by twinning, as in Figure 3, and then became brittle and fractured. Specimen is 50–50 bismuth-antimony alloy. Magnified 185 times.

Figure 7. Stainless steel from Figure 2, fractured by riflefire at 350°C. The regular parallel markings are caused by slip on crystal planes which provides ductility at higher temperature even for high-velocity fracture. Magnified 525 times.

agreed that high pressure (stress), position change of the atoms due to heat agitation, translations of planes due to slip, and the formation of "twins" (whereby a part of a crystal shifts to a second position symmetrical with certain planes in the lattice of the remaining portion of the crystal) give some small indication of where an explanation may be found for plastic deformation. Illustrations for some of these factors appear in Figures 6, 7, and 8. Twinning, which showed itself as intersecting traces on the fracture of the previous Figure 3, may provide some ductile movement at first, and then may lead to fracture along the very planes which twinned. Figure 6 shows this behavior in a 50–50 bismuth antimony alloy, a splendid crystal fragment resulting from subsequent fracture along markings of the type in Figure 3.

As for slip and the effect of temperature, Figure 7 illustrates both. The same steel shown in Figures 1 and 2, where it was brittle in its behavior, becomes so ductile when heated that even rifle fire will not develop brittle fracture. The regular and parallel markings in Figure 7 show the phenomenon of slip on crystal planes.

Still further phenomenon are active, whose nature yet remains unknown. An example is the fractograph in Figure 8, showing a pattern for antimony fractured at 200°C.

On most metallic substances the effect of pressure is very slight until the pressure becomes quite large, or is applied for a long time. Bridgman and others have made a number of measurements on the compressibility of metals and found that the alkali metals have the greatest compressibilities with caesium being the most compressible (showing a one-third loss in volume at a pressure of 15,000 atmospheres).

Fatigue

Of very great interest to engineers and those concerned with practical use of metals is the phenomenon known as "fatigue", which was mentioned a little while back in connection with hot and cold working of metals. Fatigue fits in very well at this point because one of the factors involved in plastic yielding is time, and fatigue is a function of time. Metals apparently get tired if they are loaded too often or if they are forced to undergo repeated load reversals. In order to check for fatigue the laboratory man must run an endurance test in addition to a straight strength test on the specimen he has before him. In conducting such a test care is always taken to have the applied stress remain substantially below the ultimate yield stress. Then within a certain range of stresses the stress is reversed millions of times. At the beginning of such a test the stress falls off rapidly with the number of cycles, but after a few million cycles have been gone through, the stress levels off to some constant value. It is generally agreed that for most structural materials there is a very definite limiting stress range which can be withstood for an infinite number of cycles without failure. However, this new stress range may be (and usually is) at a much smaller value than the original range. Hence any normal design load, which because of fatigue is now excessive, will cause the structure to fail. The structural designer is therefore interested not merely in the load which his structure will withstand, but also how many reversals (cycles) it can go through, after how many cycles its maximum yielding stress will begin to become a constant, and what the value of this yield will be.

The cause of fatigue, which surely involves the crystalline structure of the material, is not very easily found. It seems to be fairly well established from microscopic examination after failure that the ordinary static test type of failure and the fatigue failure differ at least in appearance. Whereas a specimen which is pulled apart in a tension machine shows a clear indication of plastic flow in which the crystals have been stretched, such is not the case for a fatigue failure. However, returning to a study of the crystalline structure, it was found that while there was nonelongation of the crystals there was a definite slippage within a given crystal. Several investigators found that slip bands appeared on the surface of some of the crystals after a certain number of cycles. The fractograph in Figure 9 suggests further that the whole inner organization of the crystal changes. To date the subsequent explanations of the action of these slip bands have been inadequate, for neither the number nor the time of their appearance seems to have a pronounced effect on the endurance of the material.

Theories of Plasticity

Since the study of plasticity involves so many variables which are apparently tied up with the fundamental structure of matter itself, conflicting theories have evolved. At the present time there would appear to be two schools, the classical approach which thinks of plasticity in terms of flow and/or deformation, and the very recently proposed theories of Batdorf and Budiansky which deal with slip rather than flow.

The classical theory, sometimes called the "rheological" approach to plasticity, divides elastic and plastic phenomena rather sharply. That is, metals act first in their commonly understood fashion and then quite suddenly change to act as though they were fluids. Mathematically such a transition is difficult, if not impossible to formulate, while physically it leaves many questions unanswered. In the rheological approach, yield stress of a material is defined as the shearing stress at which the material will begin to flow. Reiner, in his recent work, has advanced the theory that plastic flow is induced not by stress, but by something which is of the nature of the square of stress. Objection can be raised (and is!) to this theory by some very circuitous reasoning. Under a flow theory the concept of definite yield point is hard to visualize.

Basically there is one difference between elasticity and rheology (this is meant to include plastic deformation of metals). Whereas in elasticity the investigator is concerned primarily with the relations of stress to strain, in rheology he is very much concerned with a stress-strain-time relationship in which temperature plays an increasingly important part. For instance, a metal sample which would fail in a matter of minutes under a cyclical load at a temperature of several hundreds of degrees might withstand the same loading for centuries if kept at

Figure 8. Unsolved fractographic pattern for pure cast antimony fractured at 200°C. The deformation and fracture of this crystal have involved phenomena which are not yet solved. Magnified 65 times.

All photos courtesy Carl A. Zapffe, Baltimore, Md.

standard laboratory conditions. Since elasticity and plasticity can never be successfully divorced, the difference pointed out above is of course to some extent artificial. The questions of fatigue and creep (whereby a material changes its dimensions out of phase with the applied load, which is to say that the load is applied but there is a delay in movement and then a gradual rather than a sudden change) have concerned structural designers for many years. Thus the modern investigator into the nature and phenomena of plasticity is not necessarily operating in virgin territory.

Applications of Plasticity

One of the principal applications of plasticity in modern engineering is in the formation of the metals themselves. The internal stresses set up in the various metallurgical processes must be relieved and the crystalline structure so aligned that ductility (an important factor in materials which are to be drawn) and resistance to shock (almost all structural materials require this property to some degree) will not be sacrificed to strength. The conditions to which modern structures may be submitted are more varied than those experienced by structures of years past. As an example, consider the high speed airplane. At high velocities (say 400 m.p.h. or more) the metal skin of the airplane is very noticeably heated by friction as the airplane moves through the air. Since aluminum is a typical airplane structural material and since the strength properties of aluminum are noticeably affected for even relatively small (100°–200°C) rises in temperatures, the effects appearing in the case of high velocity aircraft are something quite new. A serious problem facing the designer of any interplanetary projectile is the effect of temperature on the projectile since it quite conceivably might disintegrate through a structural failure brought on by the elevated temperatures encountered on its surface, or since it might even "melt down" as a result of the heat.

Of more practical, or at least more immediate, concern are problems in the design of machine tools and machine parts. Machinery is in general subjected to both temperature rises and shock loading. This combination is extremely serious, as has already been pointed out, and can spoil, at best, the accuracy of the machine work which is being done, or can at worst cause a complete failure of the machine itself. It therefore becomes increasingly important to know something more about the fundamental nature of plastic deformation: how, when, and why it is occurring.

Conclusion

Any piece of metal which must withstand repeated and/or reversed loads over a long period of time will develop important fundamental changes in its internal crystalline structure. Hence, the theory of plasticity is important to virtually every designer working with metallic substances. The oversimplified (a personal opinion) theory of transforming elastic equations to plastic equations merely by substituting the coefficient of viscosity for the modulus of rigidity and substituting a velocity gradient for a displacement gradient may involve considerable errors of a conceptual nature. The analogy, and that is all it is, between fluid dynamics and the flow of solids is not supported by a rigorous derivation. In fact at the present time there is no stress-strain relationship which satisfactorily holds over both the elastic and the plastic ranges. Theory has for some time tended to separate the two into exclusive categories with Hooke's Law holding for the elastic range up to the proportional limit and nothing very definite holding beyond.

One must, therefore, come to the conclusion that structural theory, far from being "old stuff" and all worked out, is very definitely "new stuff" in which a great deal of research needs to be done.

Figure 9. Fractograph of an aluminum alloy (175T–6) in aircraft construction. A condition of fatigue from many millions of stress reversals has filled the entire body of the grain with fine parallel structures (see arrows) denoting readiness for fracture. These markings are a recent discovery and are not yet solved. Magnified 840 times.