reference frame

On two levels

Leo P. Kadanoff

Some of the most interesting situations in physics, and indeed in other sciences as well, concern the connections between two "levels of reality." How does the presumed world of strings connect with the more observable world of quarks and gluons? How do quantum problems "go to" their classical limits? How does the irreversibility of the macroscopic world connect with the known time-reversibility of microscopic description? In each of these cases, there is a tension between two levels of description. For each situation, different laws, formulations, conceptualizations, theories and experiments apply at each of the two levels.

The motion of particles in a fluid gives an interesting example of such a "two-level system." In this column I will discuss this motion using a variant of a model originally proposed by J. Hardy, O. de Pazzis and Yves Pomeau. (For some recent papers on the version of this model discussed here, see the references on page 9.) On level 1, the rules are that particles hop in a simple deterministic fashion upon a two-dimensional lattice. One obtains level 2 by averaging the velocities of many particles in a given region, thereby getting a continuum description of the system in terms of an averaged flow velocity $\mathbf{u}(\mathbf{r},t)$. In moving back and forth between the levels, one might wish to know how to derive the continuum equations from the rules or, conversely, how to derive the simplest rules that yield a prescribed continuum theory.

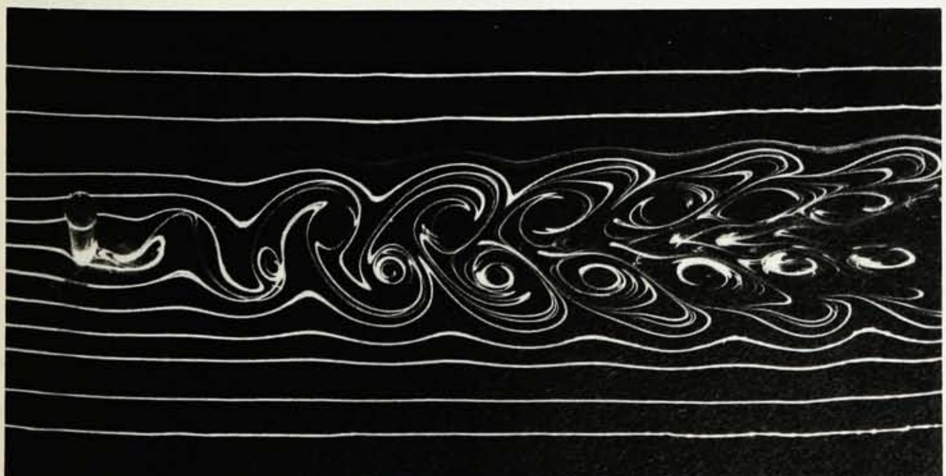
Real flows can be very rich and beautiful. Figure 1 shows a laboratory experiment involving the flow of a fluid past a cylinder. The flow is made visible by the injection of smoke particles into the fluid. In the depicted flow the fluid moves at a relatively low velocity. At higher speeds the pattern becomes even more intricate.

Figure 2 depicts a different world, containing particles in motion. The particles in this "hexagonal lattice gas" are distinguished by the six possible directions of their "momenta." The momenta are vectors of equal length directed parallel to the basic axes of the triangular lattice. One calculates the total momentum (or average velocity) for a group of particles by finding the sum (or average) of these momentum vectors for the individual particles. The model is an algorithm that tells you how the particles move about and change their momenta. The time development comes about through two types of steps. In the first type of step, which occurs between 2a and 2b, each particle moves one lattice constant in a direction determined by the direction of its momentum. This kind of step is immediately followed by a step of the next type, in which particles undergo collisions that conserve momentum and the number of particles. In the version depicted, the particles collide when the total momentum of the particles on a given site is zero. Then, as one can see by comparing figures 2b and 2c, these particles change their momenta by a counterclockwise rotation through 60°. The algorithm simply repeats the two steps again and again.

Of course, I am about to argue that the particle system of figure 2 will, on the continuum level, generate flows like the one in figure 1. How do I know? The references give both theoretical and "experimental" support for this belief. On the theoretical side, one knows real fluids are described by a set of relations called the Navier–Stokes

continued on page 9

Leo Kadanoff, a condensed-matter theorist, is John D. MacArthur Professor of Physics at the University of Chicago.



Fluid flow behind a circular cylinder (in a wind tunnel) is made visible by smoke filaments; the Reynolds number is about 300. Photograph by Peter Bradshaw (Imperial College, London), reproduced in Milton Van Dyke, *An Album of Fluid Motion* (Parabolic Press, Stanford, Calif., 1982). Figure 1

0031-9228 / 86 / 0900 07- 02 / $01.00

equations. These express the space–time dependence of the local fluid velocity $\mathbf{u}$ and pressure p in the form

$$\partial \mathbf{u}/\partial t = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \rho^{-1}\nabla p + \nu\nabla^2\mathbf{u}$$
$$\nabla\cdot\mathbf{u} = 0$$

The first of these equations is essentially an expression of local momentum conservation (or $\mathbf{F} = m\mathbf{a}$) for a situation in which the fluid's mass density ρ and its viscosity ν are constant. According to fluid mechanics, the density is held constant whenever flow velocities are very small in comparison with the speed of sound. The zero-divergence condition is then simply an expression of local mass conservation. The Navier–Stokes equations thus arise as a consequence of the real fluid's conservation laws. Because the "right" conservation laws are built into the lattice-gas model, in the appropriate limit the model should also obey the Navier–Stokes equations. (The symmetries are important too, but the hexagonal symmetry gives the correct continuum properties.)

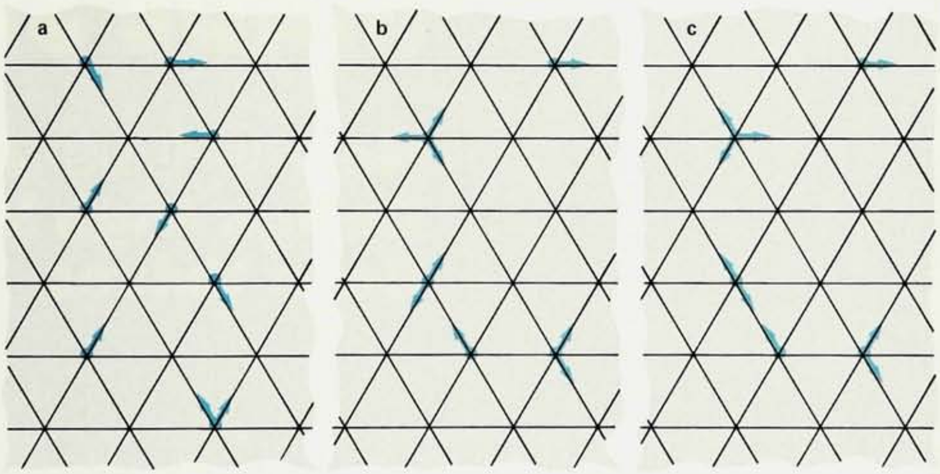
The other evidence for the connection between real fluids and the lattice gas comes from computer simulations. For example, figure 3 shows the result of a computer simulation of roughly the same physical situation as that depicted in figure 1. The two pictures show qualitatively similar patterns of swirls behind the moving cylinder. Work in progress is aimed at quantitative comparisons.

I find it fantastic and beautiful that the tiny, trivial world of the lattice gas can give rise to the intricate structures of hydrodynamic flow. The physical universe is also wonderfully simple at some levels, but overpoweringly rich in others.

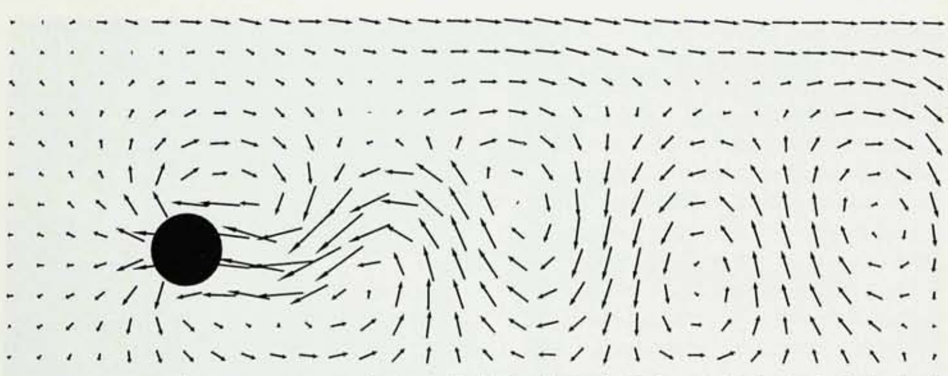
The exact connections between the two levels of this model system will be worked out in the next few years. In addition, the approach may turn out to be of some engineering and technical importance. Knowing flow patterns is necessary for designing an airplane, missile, ship or chemical plant, or for controlling air, water or soil pollution. Most predictions of complex flows will come from simulations. (Real experiments will also be quite necessary because they probably provide the best atmosphere for exploratory work.) It is possible that lattice-gas models might be an efficient tool for performing the required calculations. The relative efficiency of this method and of more standard methods of solving nonlinear partial differential equations is a matter for present debate and future investigation. The outcome of this competition among calculational methods may well depend upon future developments in parallel-processing hardware and software and in programming technique, and perhaps upon discovering more about the basic physics of flow processes.

Some recent references

- U. Frisch, B. Hasslacher, Y. Pomeau, Phys. Rev. Lett. **56**, 1505 (1986).
- S. Orszag, V. Yakhot, Phys. Rev. Lett. **56**, 1691 (1986).
- N. Margolis, T. Toffoli, G. Vichniac, Phys. Rev. Lett. **56**, 1694 (1986).
- J. Salem, S. Wolfram, in *Theory and Applications of Cellular Automata*, S. Wolfram, ed., World Scientific, Singapore (1986), p. 362. □



A lattice-gas model. Parts a, b and c show successive stages in the history of the gas; the arrows show the directions of the momenta. Note that more than one particle can occupy a lattice site. Figure 2



Flow behind a moving cylinder in the lattice-gas model of figure 2. The arrows show averaged local velocities; the Reynolds number is about 100. Simulation ©1986 by Bruce Nemnich, J. Salem and Stephen Wolfram. Figure 3