

Maria Goeppert Mayer: Atoms, molecules and nuclear shells

The mathematical physicist's early work in atomic and molecular physics, and her unfamiliarity with the "fashions" in nuclear physics, gave her the ideal preparation for solving the puzzle of the nuclear "magic numbers."

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Maria Goeppert Mayer is frequently mentioned as an example of a woman who managed to make significant contributions to science in spite of tremendous obstacles. Robert Sachs, Mayer's first graduate student, has given a personal account of her life and career (PHYSICS TODAY, February 1982, page 46). Joan Dash has written¹ a longer biography of Mayer, focusing on her family life. In this article I look at the distinctive character of Mayer's scientific accomplishments.² As a mathematical physicist, she worked on a number of apparently unrelated topics during the years 1930–46. I will focus on her work during this period and show how it prepared her for the nuclear physics research for which she received the Nobel Prize in 1963.

Mayer received the prize for her "discoveries concerning nuclear shell structure." Hence I will first outline the essential characteristics of the nuclear shell model. Then I will look back to the period before the shell model and show how the emphasis in Mayer's early work in atomic and molecular physics put her in an ideal position to enter nuclear physics and quickly make a major contribution to the nuclear shell model.

The nuclear shell model

In the Bohr–Pauli model of the atom

an electron moves in a potential produced by the atomic nucleus and by the other electrons. The Bohr model, together with the Pauli exclusion principle, explains, among other things, why certain atoms with filled electron shells are particularly stable. The nucleus, by contrast, may be conceived of in two different ways. It may be treated as a homogeneous mass of material, such as a liquid drop, or, in analogy to the Bohr–Pauli atom, it may be treated as a collection of discrete particles. In the latter case, the individual nucleons—neutrons or protons—are considered to be independent particles with independent energy levels, spins and angular momenta, moving in a potential well produced by the action of all the remaining nucleons. This is the basis of the nuclear shell model. In it the interaction energy between individual nucleons is subsidiary to the energy of the central nuclear potential. Again by analogy to the Bohr atom, the primary evidence for the nuclear shell model is that nuclei with certain specific numbers of protons or of neutrons—2, 8, 20, 28, 50, 82 or 126—are particularly stable. Data collected from nuclear binding energies, radioactive-decay energies and isotopic abundances established this pattern. This evidence suggested that nucleons fill nuclear energy levels in a way similar to the way in which electrons fill atomic energy levels. The numbers of nucleons occupying filled shells, however, turned out to be different from those that the usual

methods of quantum mechanics predicted for standard central potentials, as the figure on page 46 shows.

Mayer discovered³ the solution to this puzzle in 1949. Responding to a question by Enrico Fermi, she suggested that there is a strong spin–orbit interaction that splits the energy levels of the nucleons. This interaction occurs for electrons in an atom, but the energy splitting is so small compared with the total electron binding energy due to the central potential that it is generally neglected, except for very heavy atoms. Until 1949, nuclear physicists generally assumed that the same is true of nucleons; Mayer's mathematical argument showing that a strong spin–orbit interaction would account for the magic numbers was a radical departure, and a major contribution to nuclear shell theory.

Mayer once used a simple analogy to explain spin–orbit coupling to her daughter. Think of a room full of couples waltzing. They are moving around the room in circles, each circle enclosed within another. Each circle corresponds to an energy level. In addition to orbiting, though, each couple is also spinning like a top. Now suppose that while orbiting counterclockwise some couples are spinning clockwise, and the rest counterclockwise. Those spinning counterclockwise will find the going easier than those spinning clockwise. As Mayer said,¹ "Everybody who has ever danced the fast waltz knows that it's easier to



Maria Goeppert Mayer (1906–72) with Max Born. After attending one of Born's lectures on quantum mechanics, Mayer changed her university major from mathematics to physics. (Courtesy AIP Niels Bohr Library.)

dance one way around than the other." Therefore, for a given circle of dancers, the energy necessary to orbit will be different for couples spinning in opposite senses. In the same way, nucleons of a given orbital angular momentum have two possible energies, depending on whether their spin is parallel or antiparallel to the orbital motion. This splitting of the energy level is called spin-orbit coupling. The figure on page 48 indicates how this effect can explain nuclear shells.

The nuclear shell model thus has two essential features:

- ▶ The identification of the shell structure itself, based on the evidence for nuclear stability and leading to the basic assumption that the nucleus can be described with a single-particle model.
- ▶ The assumption of a strong spin-orbit interaction in nucleons, accounting for the splitting of their energy levels.

My thesis is that Mayer's early work led her quite naturally to this particular model.

Study in Göttingen

Maria Goeppert was born 28 June 1906, in Upper Silesia, which is now part of Poland. In 1910 her family moved to the university town of Göttingen. Her father, professor of pediatrics at the university, was the sixth generation of university professors in his family. In later years Mayer was always proud that she was the seventh generation to follow this tradition. Her

early schooling was typical for a child of the German middle class. She attended public elementary school until age 15 and then, in 1921, entered the Frauenstudium, a three-year college-preparatory school for girls. She passed the Abitur in 1924 and entered the University of Göttingen.

Goeppert began her university career as a mathematics major, influenced perhaps by the fact that David Hilbert was an old friend of her family. Sometime in 1927, however, she switched to physics after attending one of Max Born's seminars, where she was first exposed to the delights of quantum mechanics. Born became a close friend and advisor to Goeppert for the rest of his life, and he had a pronounced influence on the style of her physics. She always used a strictly mathematical approach to any physical problem, and she showed a definite preference for matrix over wave mechanics.

Goeppert's doctoral dissertation, written in 1929–30, involved a calculation⁴ of the probability that an electron will undergo a transition due to the simultaneous emission or absorption of two photons. This problem was suggested in part by some of James Franck's spectroscopic work. Goeppert's calculations indicated that the probability of such transitions was extremely low—so low, in fact, that they were not observed experimentally until the 1960s, after the invention of lasers. Laser experiments confirmed her predictions.

It was while she was at the university that Goeppert met Joseph Mayer, an American chemist who had just completed his doctoral degree with Gilbert Newton Lewis at Berkeley and was in Göttingen in 1929–30 on an International Education Board fellowship. The two were soon engaged to be married and Joe promptly assumed an extremely important role in Maria's scientific career. It was only through his support over the years that she was persuaded to continue in science. Before they were married Joe promised to hire a maid for Maria only if she finished her degree. They were married in January 1930 and she finished her PhD two months later. Joe's importance in encouraging Maria during the years when she found little support from academic science departments should not be allowed to overshadow the fact that he himself was a highly respected chemist and, until Maria received the Nobel Prize, by far the

better known of the two in the scientific world.

Office space at Johns Hopkins

After Mayer completed her degree, she and her husband came to America where he took up a position in the chemistry department at Johns Hopkins University. Although Maria Mayer was then one of the few people in America who was well trained in quantum physics, Johns Hopkins did not recognize what a resource it had. She received \$200 a year, initially to help a member of the physics department with his German correspondence, and later as research associate. She was, however, given office space and the opportunity to undertake, with Robert Williams Wood, the only experimental research of her entire career. The collaboration was not a complete success⁵—Wood distrusted some of Mayer's mathematical techniques, such as reducing data to exponential form—and although Mayer did construct one spectroscope that was still used at Johns Hopkins 15 years later, she never again considered abandoning theory for experiment.

Because there was no other opportunity to do atomic physics at Johns Hopkins, Mayer turned to chemical physics. Most of her work during her nine years at Johns Hopkins was carried out with the two people there who

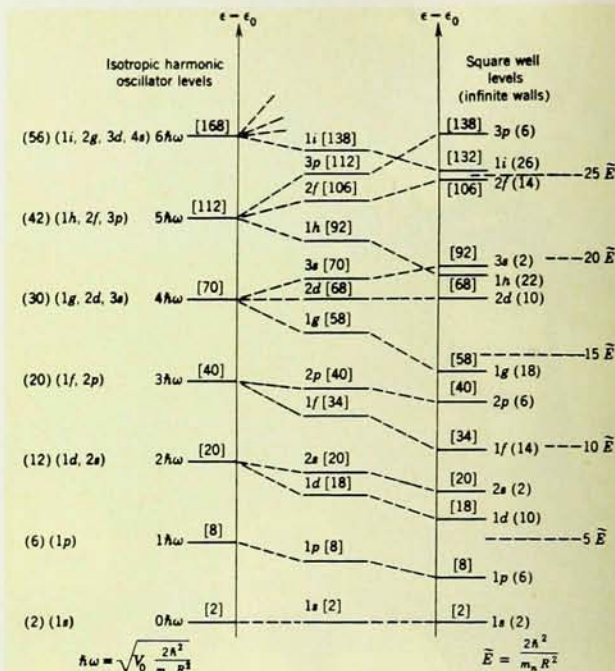


Fig. IV.1. Level system of the three-dimensional isotropic harmonic oscillator and the square well with infinitely high walls.

Shell occupation numbers, predicted for harmonic oscillator and square-well potentials. From Mayer's book with J. Hans D. Jensen, *Elementary Theory of Nuclear Shell Structure* (Wiley, New York, 1955).

were willing to work with a woman—her husband and the chemical physicist Karl Herzfeld. Because both worked in chemistry, her research went in this direction as well. Most of it involved the application of quantum mechanics to chemical problems, of which there was no shortage in the 1930s. It was while working with her husband that she learned to take experimental results seriously and to look for patterns concealed in numbers even in the absence of a theoretical interpretation.

Mayer wrote her best-known paper of this period with Alfred Sklar, a student of Herzfeld's. Herzfeld was interested in how chemical structure determines optical properties such as color, and he suggested this as a thesis topic for Sklar. Because the analysis of this problem required complex mathematical techniques, Herzfeld suggested that Mayer assist with the calculations. The complexity of most molecular systems makes it necessary to use various approximations to calculate molecular energy levels. The two most fruitful ones for determining molecular spectra were the Heitler-London-Slater-Pauling approximation and the Hund-Mulliken method. The first involves constructing wavefunctions for the molecule from linear combinations of orbitals of its individual constituent atoms. This assumes a high degree of interaction between the atoms, so that



Mayer and her daughter Marianne. Mayer came to the United States in 1930 after completing her doctorate at the University of Göttingen. (Courtesy AIP Niels Bohr Library, Stein Collection.)

they cannot be treated as individual units. Sklar used this method in his dissertation, published in 1937.

About a year later Sklar and Mayer published a paper in which they used the second method, the Hund-Mulliken approximation, to calculate⁶ the spectrum of the benzene molecule. Their central assumption was that one can determine the Hamiltonian for the molecule by summing the contributions of its individual atoms, with a correction term for their interaction energy. This resembles in many ways the independent-particle approximation for atoms, and Mayer's calculations gave her valuable experience for her later work. Her expertise was recognized; in 1939 Harold Urey listed it as one of the reasons for inviting⁷ the Mayers to Columbia University.

At Columbia during the war

Joseph Mayer lost his job with the chemistry department at Johns Hopkins in 1939, in part because of difficulties with administrators. He and Maria decided to accept Urey's invitation to Columbia University. By this time Enrico Fermi was also there, and Urey and Fermi became strong supporters of Maria Mayer and contributed considerably to her career. However, she was offered no position at all at Columbia, not even a nominal one, although Urey asked her to give some lectures in the

chemistry department so that she could have a title, Lecturer in Chemistry, and research opportunities.

Maria Mayer's primary research⁸ at Columbia was on a topic that Fermi suggested. In 1940 physicists identified one of the radioactive products of uranium as the transuranic element 93. This element had been shown to behave chemically like the rare earths; however, in other respects it differed somewhat from them, leaving open the possibility that it belonged to a new family of elements. This situation prompted Fermi to suggest that Mayer calculate the eigenfunctions for several rare-earth elements and for several transuranic elements, to see if their predicted chemical behaviors were the same. Fermi had made similar calculations for the rare earths while in Rome.

Using a statistical potential, Mayer first found that the energy and spatial extension of the eigenfunctions change abruptly between the two rare earths lanthanum (atomic weight 57) and neodymium (atomic weight 60). The binding energy of 4f electrons also increases some five times between the two. This behavior can be attributed in part to the decrease in strength of the Coulomb potential for atoms having so many electrons in partially filled shells; the electron-electron interaction energy contributes more to the total binding energy, and this is one

case in which the spin-orbit interaction becomes significant for electrons.

The same calculations for the eigenfunctions of elements 86, 91 and 93 revealed a similar pattern for the 5f electrons. Mayer's results were therefore consistent with the suggestion that element 93 and the following elements are chemically very similar to the rare earths. When the rest of the transuranic elements were eventually produced, Mayer's predictions concerning their chemical behavior were generally verified.

Mayer's career changed dramatically after America entered the war. Urey asked her to work with his group at Columbia, the so-called Substitute Alloy Materials group, which dealt with problems of isotope separation. Although the Columbia group is best known for the development of the gaseous diffusion process, Urey was not sufficiently optimistic about that method to invest all of his resources in it. One alternative that he considered for separating uranium isotopes was a photochemical method, and this required a thorough understanding of the differences between the spectra of U^{235} and U^{238} . Because by then Mayer was recognized as an expert in spectral analysis, the photochemical separation problem was ideal for her. Urey placed her in charge of the theoretical work. Later she wrote⁹ to Born that she had started out as theoretical adviser to a minor project and found herself running an experimental group of 15 people, most of them chemists. She began by collecting and collating all of the available data on the spectra of uranium compounds from the published literature. She found that there were many gaps, and assigned her chemists the task of filling them in. The Atomic Energy Commission published these results after the war. They included a detailed analysis of the fluorescence and absorption spectra of the uranyl compounds, and an appendix with some 75 pages of spectroscopic data, most of which Mayer compiled.

In 1943 the Columbia group abandoned the photochemical-separation project as being too time consuming (it

TABLE II. Order of energy levels obtained from those of a square well potential by spin-orbit coupling.

Osc. no.	Square well	Spin term	No. of states	Shells	Total no.
0	1s	1s _{1/2}	2	2	2
1	1p	1p _{3/2}	4	6	8
		1p _{1/2}	2		
2	1d	1d _{5/2}	6	12	20
		1d _{3/2}	4		
	2s	2s _{1/2}	2		
		1f _{7/2}	8	8	28
3	1f	1f _{5/2}	6	22	50
		2p _{3/2}	4		
	2p	2p _{1/2}	2		
		1g _{9/2}	10		
4	1g	1g _{7/2}	8	32	82
		2d _{5/2}	6		
	2d	2d _{3/2}	4		
		3s _{1/2}	2		
	3s	1h _{11/2}	12		
5	1h	1h _{9/2}	10	44	126
		2f _{7/2}	8		
	2	2f _{5/2}	6		
		3p _{3/2}	4		
	3p	3p _{1/2}	2		
		1i _{13/2}	14		
6	1i	1i _{11/2}	12		

"Magic numbers" explained. This table from Mayer's 1950 *Physical Review* article (volume 78, page 16) takes spin-orbit coupling into account. Example: The occupation number for the 4th oscillator group is 70. However, the spin-orbit splitting of the 1h level lowers the energy of the 1h_{11/2} state enough so that it falls in the 4th oscillator group rather than the 5th, giving an additional 12 nucleons in that group, and shell closure at 82 nucleons.

has been revived in modern times with lasers), and Mayer was reassigned to the gaseous diffusion project. The attention of the entire theoretical group at Columbia was directed toward one set of problems centered around the compound UF₆. Uranium hexafluoride is the only compound of uranium that is a gas at moderate temperatures, and Mayer worked to establish the precise range of temperatures over which it is stable. From spectral measurements she determined the exact chemical structure of UF₆, and from that determined its thermodynamic properties. In part, she worked through analogies to similar compounds such as neptunium hexafluoride. Her research notes show¹⁰ that to calculate the spectrum of NpF₆, she had to take into account a significant spin-orbit coupling for its electrons. She was able to predict the behavior of UF₆ at various temperatures, and the gaseous diffusion project continued.

Shift to nuclear physics

By the end of the war Mayer was extremely familiar with the interpretation of atomic and molecular spectroscopic data in terms of electron shell structures. She was also completely familiar with the individual-orbital approximation, in which one assumes

that the interactions between electrons or atoms are small in comparison with the potential due to the atomic or molecular core. She had used this type of approximation in her calculations on molecular spectra with Sklar and in her analysis of the spectra of uranium compounds. Also by the end of the war, because of her calculations on the spectra of the transuranic elements and uranium hexafluoride, she had worked with complex atomic and molecular systems in which the energy of the spin-orbit interaction for electrons is comparable to that of the Coulomb interaction.

It was this theoretical background, coupled with the appreciation of experimental data that she had learned from her husband, that enabled Mayer, with virtually no prior experience in nuclear physics, to formulate one of the most fruitful models of the nucleus since 1937, after only three years in the field. I look briefly now at this transition in her work.

After the war, the University of Chicago invited both of the Mayers to join its new Institute for Nuclear Studies. The physics department appointed Maria Mayer Voluntary Associate Professor of Physics (because of a supposed university antinepotism rule) and made her a member of the Institute. In

July 1946 the Argonne National Laboratory opened with the theoretical division under the direction of Robert Sachs, who had been Mayer's only graduate student at Johns Hopkins in the 1930s. Sachs offered his former professor a paid half-time position in the theoretical group at Argonne. Because both the Institute for Nuclear Studies and Argonne were devoted to nuclear physics, Mayer decided that the time had come to learn something about the subject. As a matter of fact, she learned most of her nuclear physics not from books, but from discussions with colleagues about current problems. This turned out to be most important, because it meant that she was relatively unfamiliar with many of the traditional beliefs in nuclear physics.

The first project that Mayer undertook at Chicago was a joint one with Edward Teller, an attempt to describe a possible mechanism for the origin of the elements. This work required a new list of all available isotopic abundances because more data were available after the war. The task fell to Mayer. In analyzing the data, she found that certain patterns emerged. It appeared that nuclei with 2, 8, 20, 28, 50, 82 or 126 neutrons or protons were unusually stable. She immediately recognized an analogy to electron shell structure, and she began to search for a nuclear potential that would predict the correct nucleon shell numbers. In 1948 Mayer published a paper in which she summarized¹¹ all of the data supporting the idea that nuclei are composed of shells, but she offered no theoretical interpretation. In this paper she referred to the seven nucleon numbers as "magic numbers"—a term that Eugene Wigner first applied to symbolize his view that the whole idea of nuclear shells was a charming fan-



Joseph Mayer and **Maria Goeppert Mayer** (center) with **Edward Teller** and **James Franck**. Mayer worked with Teller on her first project at the University of Chicago's Institute for Nuclear Studies. (Photograph by Francis Simon, courtesy AIP Niels Bohr Library.)

tasy. Mayer, however, liked the phrase and used it consistently thereafter.

Throughout the 1930s, various physicists attempted to formulate a nuclear shell model, based on just such evidence as that compiled by Mayer. These attempts, however, were largely discounted by nuclear physicists because they violated the fundamental assumption of the liquid-drop model, which was so successful in accounting for nuclear fission and other phenomena. This assumption was that the interaction between individual nucleons is so great that the independent-particle approximation is completely invalid—the nucleus as a whole possesses angular momentum and other collective properties, but the individual nucleons do not, and hence they cannot group themselves into shells.

However, with the publication of Mayer's 1948 paper on the magic numbers, containing so much new evidence for shell behavior, nuclear physicists again turned their attention to the possibility of deriving a model that could explain these patterns. Their attempts all involved assuming some kind of central potential. A harmonic oscillator or a square-well potential, for example, could account for the magic numbers up to but not beyond 20. Eugene Feenberg and Lothar Nordheim, as well as Mayer herself, tried¹² various modifications of these potentials, but any scheme that gave the magic numbers also involved some extremely questionable assumptions.

Finally, one day late in 1948 or early 1949, Mayer was discussing the problem with Fermi in her office. (They were in her office because Fermi would not let her smoke in his office, and she did not like to be without a cigarette for long.) Suddenly, Fermi was called away to answer a phone call, and as he left he asked, "Is there any evidence of

spin-orbit coupling?" Mayer immediately recognized that this was precisely the answer to her problem, and by the time Fermi returned she had calculated the splitting involved and the occupation numbers for the various energy levels. They corresponded exactly to the magic numbers. The figure on page 48 shows Mayer's schematic for this solution.

Mayer's interpretation of nuclear shell structure was accepted within a relatively short time. This rapid acceptance was aided by the simultaneous and independent discovery of the same model in Germany by Otto Haxel, J. Hans D. Jensen and Hans Suess. Starting from the same point of departure—an interest in the magic numbers as analogous to chemical data—the German group eventually found¹³ the solution of a strong spin-orbit splitting as well. Mayer received the Nobel Prize together with Jensen.

Mayer was particularly well prepared to formulate the shell model of the nucleus. Her experience analyzing spectroscopic data in terms of discrete energy levels for discrete particles and her experience with the strong spin-orbit interaction that occurs in heavy atoms and molecules were important to her contribution. But so was her unfamiliarity with what she later called the "fashions" in nuclear physics. She was, for example, not fully aware of the bias favoring a liquid-drop model over an independent-particle model. She said later that she was not aware that she was violating a long-standing belief when she suggested that spin-orbit coupling for nucleons is a strong effect.

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